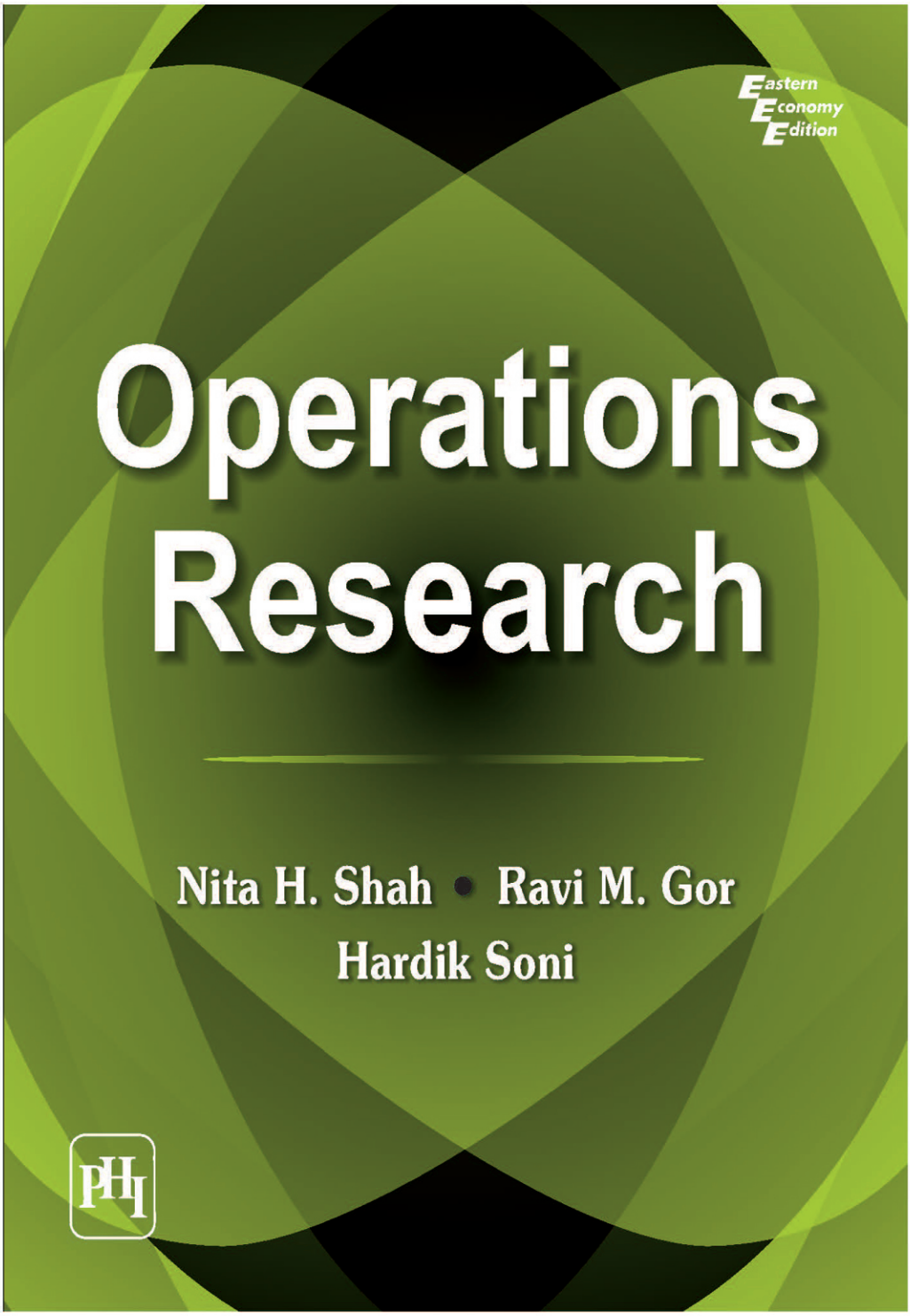
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Operations Research

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OPERATIONS RESEARCH

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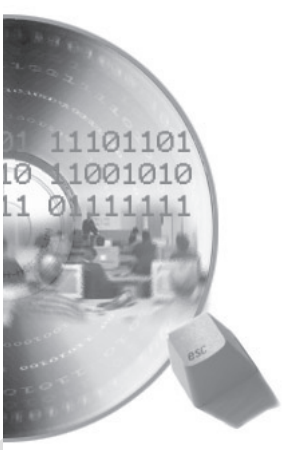

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Preface

Operations Research is a scientific approach to problem solving. It requires the formulation of mathematical, economical and statistical models for decision and control problem, to deal with the situations arising out of risk and uncertainty. In the process of various sub-systems of the organization, various decisions are involved, viz. strategic decisions, tactical decisions and operational decisions. Decision makers have to apply scientific techniques to analyze the firm's ongoing activities like production scheduling, inventory control, maintenance, replacement, etc. Thus, operations research may be defined as the application of scientific tools to decision making problems arising from operations involving integrated systems of people, machines and materials. The aim of operations research is to find the best possible course of action of a decision making problem with or without constraints.

For over 50 years, hundreds of thousands of students globally have been exposed to operations research in various disciplines. The implications of the various topics of operations research motivated us to come out with a book that meets the demands of forthcoming generations of students. The book caters to the needs of students of various disciplines where the subject is taught. However, we feel that the essential theory of operations research can best be understood and appreciated if it is presented from a mathematical viewpoint.

In this text, an attempt is made to provide the theoretical aspects of the subject with practical applications. It covers the following topics: linear programming, integer programming, goal programming, non-linear programming, geometric programming, transportation problems, assignment problems, decision analysis, inventory models, queuing theory, replacement models, network analysis, project management, sequencing problems, simulation and game theory.

The optimization of linear programming problem (LPP) by graphical method, simplex method and dual simplex method is presented. Sensitivity analysis is also carried out. Mathematical methods and solved examples are given in transportation and assignment problems. In LPP, the property of the continuity of the variable may lead to some practical difficulties, for example units cannot be produced in fractions, and rounding off the solution of LPP to the nearest integer may not conclude

to optimal solution. Hence, the concept of integer programming is introduced, followed by the cutting-plane algorithm, branch-and-bound algorithm, and zero-one implicit enumeration technique. The concept of achieving different goals in the order of priority to optimize the objective function is discussed in goal programming. Non-linear programming problem techniques are given to illustrate the Lagrangian method, Kuhn–Tucker conditions and quadratic programming. The various criteria of decision making under certainty, uncertainty and risk are discussed in decision analysis. These criteria, when extended to competitive situations among two opponents, lead to the theory of games.

Four types of basic inventory models are explained to study different inventory strategies. The concept of queuing system and its various disciplines are derived. The most crucial decision of replacing part of a machine or the machine as a whole is discussed in replacement models. Dynamic programming is then demonstrated in the context of inventory and queues. Under project management, different techniques of analyzing the time involved in completing a project and the related costs are presented after defining the prerequisites of networks. These tools are the key components for effective scheduling of activities which, in turn, reduce the project completion time and result in smooth resource allocation and resource levelling. The practical situations involving the knowledge of an optimum order of doing jobs are discussed in sequencing problems. In some of the complex problems, mathematical techniques could not be searched out. Therefore, simulation approach for tackling such problems is discussed.

In this book easy-to-use algorithms are presented, illustrating different techniques of operations research. Each chapter contains several fully worked-out problems and ends with exercises to give the students an opportunity to hone their understanding of the important concepts involved.

This book is designed for the students of MBA/PGDBM, MCA, MA/MSc (Mathematics, Statistics, Operations Research), MIT, MSc (IT), M.Com., CA, ICWA. It may be of great help to the students who are preparing for competitive examinations like Civil Services and UGC-NET. The concepts and results explained here can be applied to real-life industrial, business problems with some changes as per the requirements.

We are thankful to our students whose queries helped us in making the various concepts of operations research techniques more clear. We are also grateful to the editorial and production team of Prentice-Hall of India for their cooperation and sincerity.

Finally, we are very thankful to our family members for their constant support and encouragement, particularly during the prepartion of the manuscript. Dr. Ravi Gor conveys special thanks to his kids Tosha and Mandaar, who have missed him very much during the preparation of this book. Hardik Soni is thankful to D.M. Sikh, Director of Sardar Vallabhbhai Patel Education Trust, for his cooperation.

Suggestions and constructive comments from the readers for improvement of the book are most welcome.

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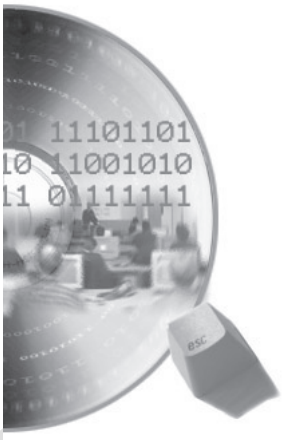
1 Why Operations Research?



1.1 INTRODUCTION

Operations Research (OR) is a scientific approach to analyzing problems and making decisions. OR professionals aim to provide rational bases for decision-making by seeking to understand and structure complex situations, and to use this understanding to predict system behaviour and improve system performance. Much of this work is done using analytical and numerical techniques to develop and manipulate mathematical and computer models of organizational systems composed of people, machines, and procedures.

Operations Research is a branch of mathematics used to provide a scientific base for management to take timely and effective decisions. It possibly avoids the dangers arising from decisions based on guesswork. The concept of management has basically two characteristics:

- *Multidimensional:* Because managerial problems and their probable solutions have repercussions in several fields such as human, economic, social and political fields.
- *Dynamic:* A manager will never remain static while prevailing in the business.

Hence, any manager, while taking decisions, considers all aspects in addition to economic aspect, to make his solution useful in every respect. The general approach is to analyze the problem in every aspect and implement the solution only if it does not violate human, social and political constraints.

Management problems can also be solved using quantitative approach. This approach requires the problem to be properly defined and thoroughly analyzed. This includes collecting data, facts, and information and then solving the problem in a rational and systematic way, based on analysis rather on mere guesswork, or using trial and error methods. Operations research is primarily concerned with helping managers and executives to arrive at better decisions.

Nowadays, managers are working in a dynamic environment. They require common sense, experience and commitment in making decisions as they have to deal with systems with complex

interrelationship of various factors among them, as well as equally complicated dependence of the criterion of effective performance of the system on these factors. The science of operations research helps them take value–based decisions in such a dynamic environment.

The scope of quantitative methods is very broad. They are applied in defining the problems and getting solutions of various organizations dealing with finance, manufacturing, services, etc.

1.2 ORIGIN OF OPERATIONS RESEARCH

Operations research is a *war baby* as its idea first germinated during the Second World War. The first problem to have been attempted in a systematic way was concerned with how to set the time fuse bomb to be dropped from an aircraft onto a submarine.

At the time of the Second World War, the military management in England invited a team of experts to analyze the problems related to the defense of the country. At that time, the resources available with England were limited and they targeted to win the war with these resources. Therefore, it was necessary to decide the most effective utilization of the available resources including the military resources. The experts were given the problem of resource utilization and asked to come out with a feasible solution. After much deliberation, these specialists came out with a method for solving the problem. They termed the method "Linear Programming". This method worked out well in solving the war problem. As the name indicates, the word *Operations* is used to refer to the problems of military, and the word *Research* is used for inventing a new method. After the Second World War, there was a scarcity of industrial material and industrial productivity reached its nadir. Industrial recession was there and to solve the industrial problem, the method *linear programming* was used to get the optimal solution. From then onwards, a lot of work has been done in the field, and today OR has numerous methods for solving different types of problems.

After the success of the British military, the United States military management started applying the techniques to various activities to solve military, civil and industrial problems. Some called it Operational Analysis, while some others called it Operations Evaluation, Operations Research, System Analysis, System Evaluation, Systems Research, Quantitative methods, and so on. But the most commonly used one is Operations Research. In the industrial world, the most important problem for which these techniques are used is how to optimize the profit or how to reduce the costs.

1.3 DEFINITIONS OF OPERATIONS RESEARCH

Here we give a few definitions to explain what exactly Operations Research is.

> *Operations research applies scientific methods to deal with the problems of a system where men, material and other resources are involved and the system under study may be industry, defense or business, etc.*

It also says that the manager has to build a scientific model to study the system which must be provided with the facility to measure the outcomes of various alternatives under various degrees of risk, which helps the managers to take optimal decisions.

> *Operational Research can be described as a scientific approach to the solution of problems in the management of complex systems.*

In a rapidly changing environment, an understanding is sought which will facilitate the choice of more effective solutions which, typically, may involve complicated interaction among people, materials, and money.

Operational Research in practice is a team effort, requiring the close cooperation among the decision-makers, the skilled OR analysts, and the people who will be affected by management action.

Because of wide scope and numerous applications of operations research, different mathematicians and researchers ended up giving different definitions. Although giving a precise definition of operations research is not possible, here we discuss some of them.

Operations Research is the art of winning wars without actually fighting.

— Aurther Clarke

This definition is oriented towards warfare. It means that the directions and guidance come from the minister or the king, according to which the war is fought and won.

Operations Research is the art of giving bad answers to problems where otherwise worse answers are given.

— T.L. Satty

This definition covers one aspect of decision making, i.e. choosing the best among the available alternatives. If the decisions are made on guesses, we may face the worse situation. But if the decisions are made on a scientific basis, it will help us to make better decisions. Hence, this definition deals with one aspect of decision-making and not clearly tells what operations research is.

Operations Research is Research into Operations.

— J. Steinhardt

This definition does not state anything in clear terms about the subject of Operations Research and simply says that it is a research into operations. Operations may here be referred as military activities or simply the operations that an executive performs in his organization while taking decisions. Research means finding a new approach. That is, when an executive is involved in performing his operations for taking decisions, he has to go for newer ways so that he can make a better decision for the benefit of his organization.

Operations Research is defined as scientific method for providing executive department quantitative basis for decisions regarding the operations under their control.

— P.M. Morse and G.E. Kimball

This definition suggests that Operations Research provides scientific methods to an executive to make optimal decisions. But it does not give any information about various models or methods. This suggests that executives can use scientific methods for decision-making.

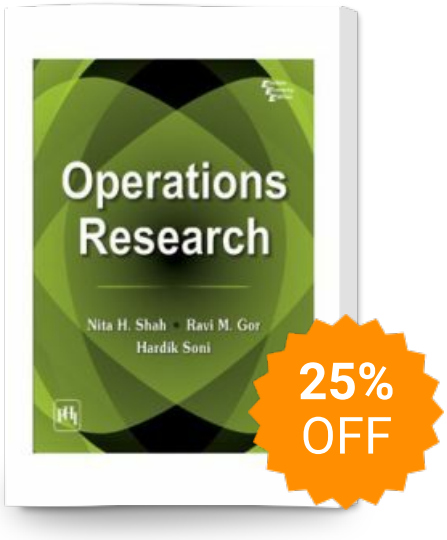
Operations Research is the application of scientific methods, techniques and tools to the operation of a system with optimum solution to the problem.

— Churchman, Ackoff and Arnoff

This definition clearly states that the Operations Research applies scientific methods to find an optimum solution to the problem of a system. A system may be a production system or information system or any system, which involves men, machine and other resources.

Operations Research is the application of the theories of Probability, Statistics, Queuing, Games, Linear Programming etc., to the problems of war, Government and Industry.

Operations Research



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