

FINITE ELEMENT METHOD IN STRUCTURAL ANALYSIS

A.S. Meghre
Ms K.N. Kadam

The image features two 3D finite element analysis (FEA) models. On the left is a tall, blue, lattice-structured tower. On the right is a green bridge structure with a curved arch and two vertical supports. The background is a yellow grid representing a mesh. The publisher's name is at the bottom.

KHANNA PUBLISHERS

FINITE ELEMENT METHOD
IN
STRUCTURAL ANALYSIS

FINITE ELEMENT METHOD IN STRUCTURAL ANALYSIS

A. S. MEGHRE, Ph. D.
Retired Professor of Applied Mechanics
Madhuban Housing Society, Camp,
Amravati (MS), 444602

Ms K. N. KADAM, Ph. D.
Assistant Professor in Applied Mechanics
Govt. College of Engineering,
Amravati (MS), 444604



KHANNA PUBLISHERS

Operational Office
4575/15, Onkar House, Ground Floor
Daryaganj, New Delhi-110002
Phones : 011-23243042, 23243043 & Mob. 9811541460

Published by :

Romesh Chander Khanna & Vineet Khanna
for **KHANNA PUBLISHERS**
2-B, Nath Market, Nai Sarak,
Delhi-110006 (India)

Visit us at : www.khannapublishers.in

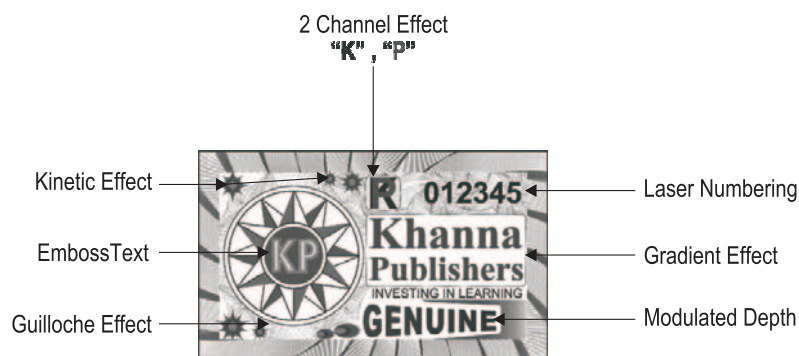
© 1979 and onward.

This book or part thereof cannot be translated or reproduced in any form without the written permission of both Authors and the Publishers. The right to translation, however, reserved with the Authors alone.

Hologram & Description

To all readers of our books, to prevent yourself from being frauded by pirated books. Please make sure that there is an Hologram on the cover of our books with the below specifications. If you find any book without the Hologram or Description, please mail us at khannapublishers@yahoo.in.

Thanking you



ISBN No. : 978-81-7409-283-0

First Edition : 2014

Preface to the First Edition

Finite element method started flourishing from 1960 or so. Since then, the method has acquired immense popularity and has found applications in various fields of engineering. Presently, the method has emerged as a powerful tool of analysis. This is because of various advantages associated with it. The method is versatile and can analyse any arbitrarily shaped structure having any combination of boundary conditions and subjected to different types of loads. It can be used for static linear, non-linear and dynamic analysis.

The method is applicable to solve problems in various fields of engineering. However, this book limits the presentation to structural engineering problems.

There are two different views of looking at the finite element method. Mathematicians treat it as a method of solving differential equation. Another view, by engineers, is to treat it as a means of obtaining deformed state of structure using minimum potential energy principle. These two approaches are, respectively, known as differential and variational approaches. This book presents the method as a variational approach. However, a brief description of differential approach, in terms of Galerkin finite element method, is given in last chapter.

Only displacement based finite elements are presented. Further, the scope of the book is limited to linear analysis of different structures and non-linear analysis of plates. Dynamic analysis is not included.

The finite element method is generally taught, in most universities, to senior level undergraduate students as optional subject and to first year graduate students as compulsory subject. This book written in simple and lucid language is intended as a text book for these students. It can also serve as a guide to practising engineers.

The finite element method involves tremendous amount of calculations which are beyond the capabilities of human beings. The availability of computing machine is, therefore, a must for its application. The study of method involves understanding theory, drafting corresponding program and execution of the program using computer. Hence, the knowledge of Fortran language and some experience in running the program is needed to understand the contents of the book.

It is assumed that the reader has already gone through a course on matrix methods of structural analysis. To a large extent, the programming strategies in stiffness matrix method and finite element method are similar except for the details of calculation of element stiffness and load matrices. As such, the previous knowledge of stiffness matrix method will greatly help the reader in understanding theory and step-by-step procedure in finite element method.

Both authors have taught the subject to the students over a period of time. The experience gained in teaching and supervising fem related projects has certainly influenced the mode of text in this book. Authors have observed that the students do show aversion to reading and understanding lengthy computer programs involving thousands of statements. As such, this book has completely avoided a single unified lengthy program with library of varieties of elements. Instead, separate small size programs are drafted for analysis of a particular type of structure using a particular element. All programs are transparent and easy to understand. The features of each program are specifically mentioned. Listings of all programs are given in Appendix. There are eleven such programs. For each program, atleast one data file is included in the text matter. It is expected that this will help the reader in preparing his own data files for solving additional problems. All programs are for educational purpose. These can not be considered as most general programs. Some have certain limitations which are clearly mentioned while describing features of the program.

The book is divided in thirteen chapters. Chapter 1 discusses various possible methods

of analysing continuum structures. This helps in deciding the location of finite element method in the cluster of methods.

Chapter 2 is a combination of heterogeneous topics which are referred to at a later stage of study of the method. Chapter 3 deals with plane stress analysis using constant strain triangle and rectangle. Chapter 4 introduces the concept of isoparametric elements. Chapter 4 to 10 discuss plane stress analysis, axisymmetric solids, 3D solids, thin plates, Mindlin plates, higher order thick plates and shells. Finite strip method is presented in chapter 11. Chapter 12 discusses geometric non-linear analysis of plates. Chapter 13 consists of miscellaneous additional topics.

A compact disc (CD) is provided with the book. The contents of CD are divided in four folders. Folder 1 i.e. FOLD1 contains listing of eleven FORTRAN source programs listed in Appendix. FOLD2 contains some typical data files. FOLD3 includes executable files for the source programs in FOLD1. FOLD4 is used to present four typical output files. In all programs, output file is labeled as OUT.TXT file. For the purpose of giving four different output files, the files are renamed as OUT1.TXT, OUT2.TXT etc.

Authors are grateful to their family members for their patience and understanding during the period of drafting and printing of the book. Without their cooperation, this endeavour could not have come to fruition. Authors extend their thanks to Er. Atul S. Shinde for artwork.

Authors have great pleasure in presenting the book to the readers and hope that it will be received favourably.

A.S. Meghre
(Ms) K. N. Kadam

Contents

Chapters	Pages
1. Introduction	1–34
1.1 General	1
1.2 Solution of Governing Differential Equation	4
1.2.1 Exact Solution	5
1.2.2 Series Solution	6
1.2.3 Methods Based on Residual $R(x)$	7
1.2.4 Point Collocation Method	7
1.2.5 Zone Collocation	8
1.2.6 Least Square Method	9
1.2.7 Galerkin Method (I)	10
1.2.8 Galerkin Method (II)	11
1.2.9 Finite Difference Method	13
1.2.10 Galerkin Finite Element Method	14
1.3 Methods Using Minimum Potential Energy Principle	14
1.3.1 Rayleigh Method	15
1.3.2 Rayleigh-Ritz Method	16
1.3.3 Review of Stiffness Matrix Method	17
1.3.4 Stiffness Method as a Variational Approach	20
1.3.5 Concept of Finite Element Method	24
1.3.6 Finite Strip Method	26
1.3.7 Finite Difference Energy Method	27
1.3.8 Discrete Energy Method	27
1.4 Boundary Element Method	28
1.5 Rayleigh-Ritz Method and Finite Element Method	28
1.6 Stiffness Matrix Method and Finite Element Method	29
<i>Exercises I</i>	29
<i>Answers to Exercises</i>	33
2. Some Preliminaries	35–101
2.1 Theory of Elasticity	35
2.1.1 Cartesian Coordinates	35
2.1.2 Cylindrical Coordinates	40
2.1.3 Strain Energy	42
2.1.4 Potential of Loads	45
2.1.5 Total Potential Energy	46
2.1.6 Theorems	46
2.1.7 Saint-Venant's Principle	47
2.2 Numerical Integration	47
2.2.1 One Dimensional Integration	48
2.2.2 Two Dimensional Integration (Rectangular Region)	50
2.2.3 Three Dimensional Integration	52
2.2.4 Integration in Triangular Region	52

<i>Chapters</i>	<i>Pages</i>
2.3 Interpolation and Simple Shape Functions	56
2.3.1 One Dimensional Interpolation	56
2.3.2 Two Dimensional Interpolation	62
2.3.3 Three Dimensional Interpolation	76
2.3.4 Conditions for Convergence	78
2.4 Stress Smoothing And Extrapolation	80
2.4.1 One Dimensional Extrapolation	80
2.4.2 Two Dimensional Extrapolation	82
2.5 Some Programming Aspects	82
2.5.1 Structure Stiffness Matrix in Full Form	83
2.5.2 Structure Stiffness Matrix in Half Band Form	86
2.5.3 Sky Line Storage of Stiffness Matrix	89
2.5.4 Programming Strategy	91
2.5.5 Variable Dimensioning	93
2.6 Solution of Simultaneous Algebraic Equations	95
2.6.1 Gauss Elimination Method	96
<i>Exercises II</i>	99
<i>Answers to Exercises</i>	101
3. Two Dimensional Analysis	102–135
3.1 General	102
3.1.1 Guidelines for Discretisation	102
3.2 Constant Strain Triangle (CST)	102
3.2.1 Stiffness Matrix	102
3.2.2 Element Load Matrix	104
3.2.3 Computer Program CONST.FOR	108
3.2.4 Examples using CONST.FOR	112
3.3 Linear Strain Triangle (LST)	116
3.3.1 Stiffness Matrix	116
3.3.2 Element Load Matrix	118
3.4 Rectangle	119
3.4.1 Stiffness Matrix	119
3.4.2 Element Load Matrix	121
3.4.3 Computer Program RECT.FOR	122
3.4.4 Examples using RECT.FOR	123
3.5 Higher Order Rectangular Elements	128
3.5.1 Eight Noded Serendipity Rectangle	128
3.5.2 Nine Noded Lagrangian Rectangle	130
3.6 Four Noded Annular Sector Element	130
<i>Exercises III</i>	132
<i>Answers to Exercises</i>	135
4. Two Dimensional Analysis Using Isoparametric Elements	136–170
4.1 Natural or Local Coordinates	136
4.2 Isoparametric Elements	137

<i>Chapters</i>	<i>Pages</i>
4.3 Uniqueness of Mapping	140
4.4 Rules	140
4.4.1 Rule 1	140
4.4.2 Rule 2	141
4.4.3 Rule 3	141
4.5 Four Noded Isoparametric Quadrilateral Element	142
4.5.1 Stiffness Matrix	142
4.5.2 Element Load Matrix	146
4.5.3 Examples	147
4.5.4 Computer Program QUAD1.FOR	151
4.5.5 Examples using QUAD1.FOR	152
4.6 Quadrilateral With Incompatible Displacements-Quad2	157
4.6.1 Formulation	157
4.6.2 Examples using QUAD2.FOR	159
4.7 Improved Quadrilateral with Incompatible Displacements-Quad3	160
4.7.1 Formulation	160
4.7.2 Computer Program QUAD3.FOR	163
4.7.3 Examples using QUAD3.FOR	163
4.8 Higher Order Elements	164
4.8.1 Eight Noded Serendipity Element	164
4.8.2 Nine Noded Lagrangian Element	166
4.8.3 Six Noded Isoparametric Triangle	166
<i>Exercises IV</i>	167
<i>Answers to Exercises</i>	169
5. Analysis of Axisymmetric Solids	171–186
5.1 Triangular Ring Element	171
5.1.1 Stiffness Matrix	171
5.1.2 Element Load Matrix	173
5.2 Isoparametric Four Noded Ring Element	174
5.2.1 Stiffness Matrix	174
5.2.2 Element Load Matrix	176
5.2.3 Computer Program AXISOLID.FOR	178
5.2.4 Examples using AXISOLID.FOR	179
5.3 Higher Order Ring Elements	183
<i>Exercises V</i>	183
<i>Answers to Exercises</i>	185
6. Three Dimensional Stress Analysis	187–202
6.1 Constant Strain Four Noded Tetrahedron	187
6.1.1 Stiffness Matrix	187
6.1.2 Element Load Matrix	188
6.2 Linear Strain Tetrahedron	189
6.2.1 Stiffness Matrix	189
6.2.2 Element Load Matrix	191

<i>Chapters</i>	<i>Pages</i>
6.3 Eight Noded Isoparametric Hexahedron	192
6.3.1 Stiffness Matrix	192
6.3.2 Element Load Matrix	194
6.3.3 Improved Eight Noded Hexahedron with Incompatible Displacements	197
6.4 Twenty Noded Isoparametric Hexahedron	191
6.5 Triangular Prism Family	199
<i>Exercises VI</i>	201
<i>Answers to Exercises</i>	201
7. Analysis of Thin Plates	203–229
7.1 Introduction	203
7.2 Classification of Plates	203
7.3 Thin Plate Theory	203
7.4 Displacement Function for Thin Plate Element	205
7.5 Twelve D.O.F. Rectangular Element (ACM)	206
7.5.1 Stiffness Matrix	206
7.5.2 Element Load Matrix	210
7.5.3 Check on Compatibility of ACM Element	211
7.5.4 Computer Program ACM.FOR	212
7.5.5 Examples using ACM.FOR	212
7.6 Sixteen d.o.f. Rectangular Element	214
7.6.1 Stiffness Matrix and Load Matrix	214
7.6.2 Check on Compatibility	215
7.7 Curved Quadrilateral Element (12 d.o.f.)	216
7.7.1 Formulation	216
7.7.2 Example	219
7.8 Other Curved Quadrilateral Elements	220
7.9 Triangular Elements	220
7.9.1 Six d.o.f. Constant Moment Triangle	220
7.9.2 Some Simple Triangular Elements	222
7.10 Annular Sector Element	223
<i>Exercises VII</i>	225
<i>Answers to Exercises</i>	226
8. Analysis Order Thick Plates	230–250
8.1 Mindlin Plate Theory	230
8.2 Shear Locking	232
8.3 Four Noded Quadrilateral Element	236
8.4 Computer Program MINPL4.FOR	239
8.5 Examples Using MINPL4.FOR	239
8.6 Higher Order Elements	243
8.7 Heterosis Element	244
<i>Exercises VIII</i>	246
<i>AnswerS to Exercises</i>	249

<i>Chapters</i>	<i>Pages</i>
9. Higher Order Thick Plates	251–265
9.1 Introduction	251
9.2 Nine Noded Lagrangian Isoparametric Element	251
9.3 Thick Circular Plates	257
9.3.1 Formulation	257
9.3.2 Computer Program HOTCIR.FOR	261
9.3.3 Examples using HOTCIR.FOR	262
<i>Exercises IX</i>	263
<i>Answers to Exercises</i>	264
10. Analysis of Shells	266–290
10.1 General	266
10.2 Shell as an Assemblage of Flat Elements	266
10.3 Curved Shell Element for Cylindrical Shell	269
10.4 Axisymmetric Shell	272
10.4.1 Formulation	272
10.4.2 Computer Program AXISHL.FOR	275
10.4.3 Examples using AXISHL.FOR	276
10.5 Solid Elements For Shell Analysis	279
10.6 Eight Noded Ahmad Shell Element	280
<i>Exercises X</i>	288
<i>Answers to Exercises</i>	290
11. Finite Strip Method	291–331
11.1 Thin Rectangular Plate - Lower Order Strip	291
11.1.1 Formulation	291
11.1.2 Computer Program FSM.FOR	296
11.1.3 Examples using FSM.FOR	298
11.2 Higher Order Strip	299
11.3 Strip Using Spline Along Strip Width	301
11.4 Spline Finite Strip Method (SFSM)	307
11.5 Fem-cum-FDEM Finite Strip	312
11.6 Finite Strips for General Thin Plates	316
11.6.1 Spline Finite Strip for General Plates	316
11.6.2 FEM-cum-FDEM Strip for General Plates	316
11.7 Thin Annular Sector Plate	316
11.8 Finite Strip for Rectangular Mindlin Plate	321
11.9 Finite Strip for Thin Cylindrical Shell	325
<i>Exercises XI</i>	329
<i>Answers to Exercises</i>	331
12. Non-Linear Analysis of Plates	332–364
12.1 General	332
12.2 Non-Linear Analysis of Thin Rectangular Plate	333
12.2.1 Tangent Stiffness Matrix	333

<i>Chapters</i>	<i>Pages</i>
12.2.2 Element Load Matrix	340
12.2.3 Algorithm	340
12.2.4 Convergence Criteria	341
12.2.5 Equilibrium Equation	341
12.3 Thin Annular Sector Plate	342
12.4 Thin Circular Plate – Axisymmetric Loads	347
12.4.1 Tangent Stiffness Matrix	347
12.4.2 Element Load Matrix	350
12.4.3 Computer Program NLACIR.FOR	351
12.4.4 Examples using NLACIR.FOR	352
12.5 Mindlin General Plates	354
12.6 Mindlin Circular Plate - Axisymmetric Loads	358
<i>Exercises XII</i>	363
<i>Answers to Exercises</i>	363
13. Additional Topics	365–386
13.1 Galerkin Finite Element Method	365
13.1.1 Using Original Differential Equation	365
13.1.2 Weak Form of Weighted Residual Statement	370
13.2 Functional And Extremisation	376
13.3 Patch Test	380
13.4 Lagrangian Multiplier	381
13.5 Preprocessing and Post Processing	382
13.6 Available Software Packages	382
<i>Exercises XIII</i>	385
<i>Answers to Exercises</i>	386
Appendix	387–510
References	511–515
Index	516–519

1.1 GENERAL

Structural engineers are required to analyse and design various types of structures. The knowledge of displacements, strains and stresses is important to them to check serviceability and safety of the structures. There are different varieties of structures such as trusses, frames, buildings, bridges, irrigation structures, folded plates, box-girders etc. In the earlier days, analysis was carried out by introducing certain simplifying assumptions so as to bring the analysis within the reach of hand calculation methods.

One possible way of classifying varieties of structures in civil engineering field is to put them in two categories as

- (i) Discrete element or skeletal structures
- (ii) Continuum structures.

The discrete element structures are formed by joining discrete structural members at their ends forming joints. Each member may be straight or curved. For such members, sectional dimensions are small compared to the length dimension. Pin jointed trusses, frames, grids, arched portals and space frames are skeletal structures. Some of these are shown in Fig. 1.1. The members of the skeletal structures are simple members and their properties namely force-displacement relations can be found easily using standard structural mechanics formulae. The analysis of a skeletal structure is no more a difficult job. For large size structures, stiffness matrix method is now widely used tool of analysis.

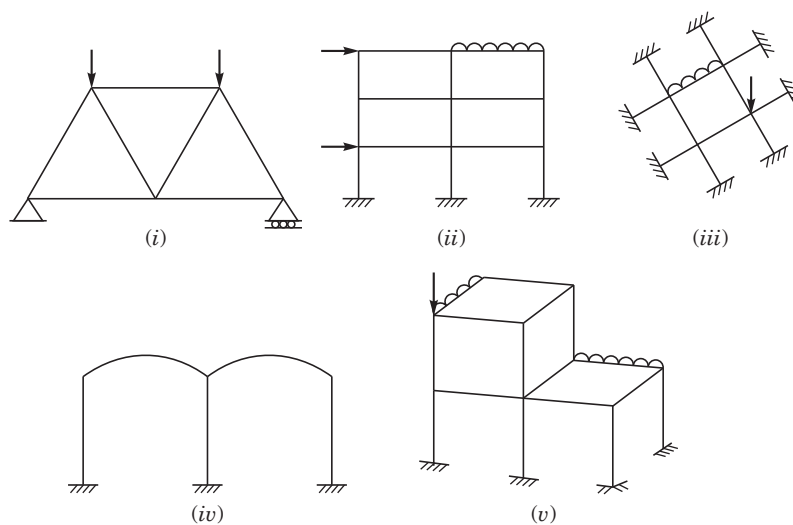


Fig. 1.1. Skeletal structures (i) Plane truss (ii) Plane frame (iii) Grid (iv) Arched portal (v) Space frame.

The continuum structures are formed from surface elements and solids. Plates, stiffened plates, shells, folded plates, box-girders are some examples of the continuum structures. Some of these are shown in Fig. 1.2.

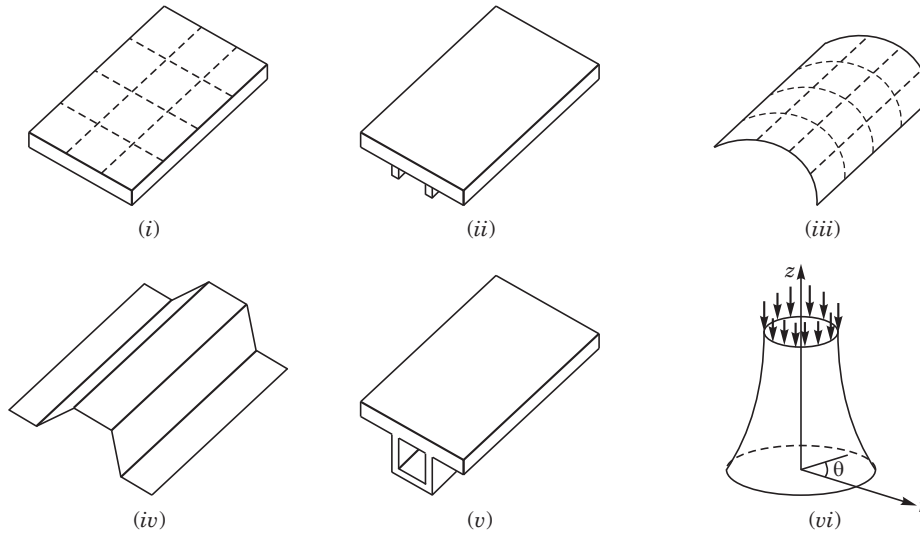


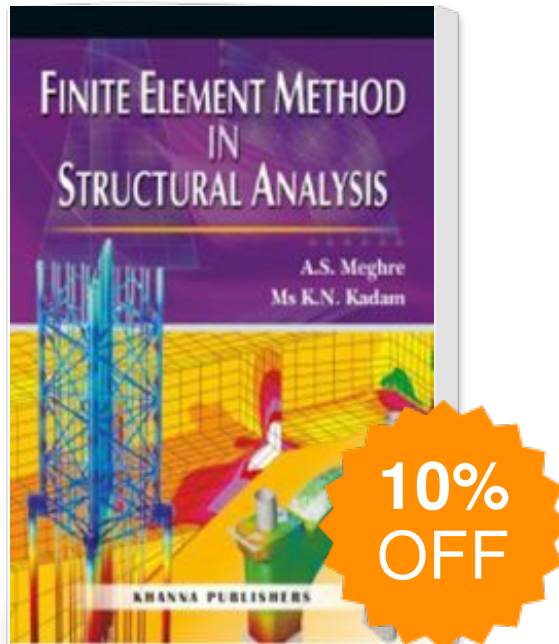
Fig. 1.2. Continuum structures (i) Plate (ii) Stiffened (iii) Shell (iv) Folded plate (v) Box girder (vi) Axisymmetric solid.

The analysis of a continuum structure is a difficult job. To begin with, the behaviour of the structure is described in terms of a governing differential equation or equations. This is not so difficult. The general procedure is to express equilibrium of an infinitely small element of the structure in terms of stress-resultants and external forces acting on the element. The stress-resultants are written in terms of generalised strains and the strains are expressed as derivatives of displacements. Finally, combining all relations, it is generally possible to derive a governing differential equation in terms of displacements. This may be a single equation or a set of simultaneous differential equations. The analysts' aim is to obtain the solution of the governing differential equation(s) satisfying kinematic and force boundary conditions. Obviously, the first attempt is to obtain exact or analytical solution. In analytical solution, algebraic or trigonometric or hyperbolic expressions are obtained for field variable and other dependant variables. These expressions are continuous over the entire structure domain and it is possible to calculate the quantities of importance at any location in the structure using appropriate coordinates of the location.

An alternate approach of obtaining solution of governing differential equation is to express the field variable (w) as a combination of linearly independent trial or approximate functions satisfying boundary conditions such as $\bar{w} = \sum C_i \phi_i$. An error function $R(x)$ arises when \bar{w} is substituted in differential equation. The values of the constants C_i associated with trial functions ϕ_i are obtained from the solution of simultaneous algebraic equations which are formed using any one of the following procedures :

- (i) Equate residual or error function $R(x)$ to zero at selected locations known as collocation points (Point collocation).
- (ii) Equate integral of the residual function $R(x)$ over sub-domains or zones of the structure to zero (Zone collocation).

Finite Element Method In Structural Analysis



Publisher : KHANNA
PUBLISHERS

ISBN : 9788174092830

Author : A. S. Meghre, Ms
K. N. Kadam

Type the URL : <http://www.kopykitab.com/product/22301>



Get this eBook