FINITE ELEMENT METHOD IN STRUCTURAL ANALYSIS

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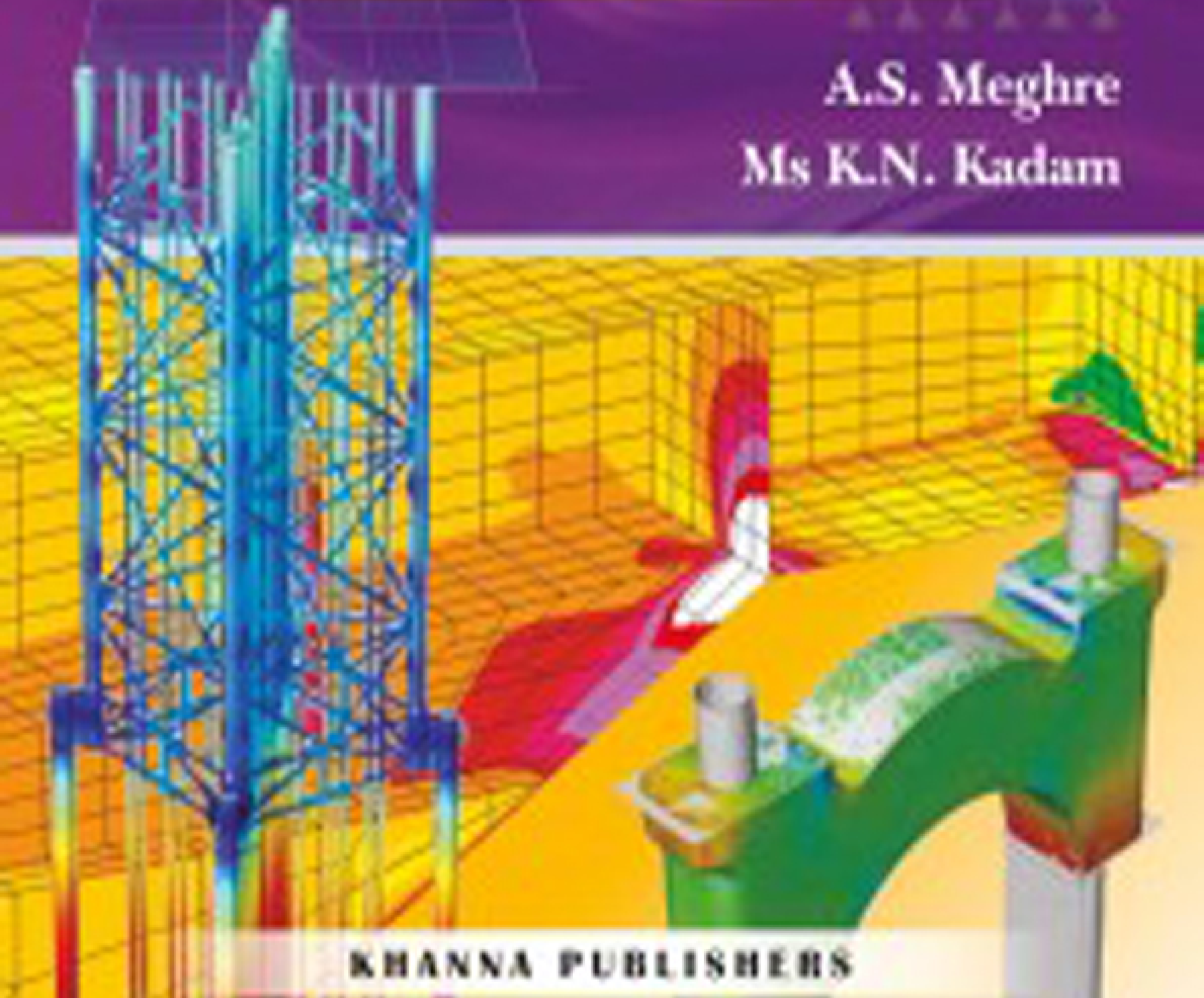

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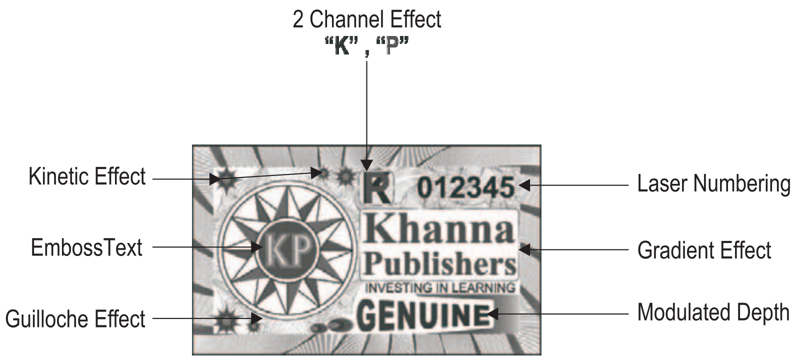

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Preface to the First Edition

Finite element method started flourishing from 1960 or so. Since then, the method has acquired immense popularity and has found applications in various fields of engineering. Presently, the method has emerged as a powerful tool of analysis. This is because of various advantages associated with it. The method is versatile and can analyse any arbitrarily shaped structure having any combination of boundary conditions and subjected to different types of loads. It can be used for static linear, non-linear and dynamic analysis.

The method is applicable to solve problems in various fields of engineering. However, this book limits the presentation to structural engineering problems.

There are two different views of looking at the finite element method. Mathematicians treat it as a method of solving differential equation. Another view, by engineers, is to treat it as a means of obtaining deformed state of structure using minimum potential energy principle. These two approaches are, respectively, known as differential and variational approaches. This book presents the method as a variational approach. However, a brief description of differential approach, in terms of Galerkin finite element method, is given in last chapter.

Only displacement based finite elements are presented. Further, the scope of the book is limited to linear analysis of different structures and non-linear analysis of plates. Dynamic analysis is not included.

The finite element method is generally taught, in most universities, to senior level undergraduate students as optional subject and to first year graduate students as compulsory subject. This book written in simple and lucid language is intended as a text book for these students. It can also serve as a guide to practising engineers.

The finite element method involves tremendous amount of calculations which are beyond the capabilities of human beings. The availability of computing machine is, therefore, a must for it's application. The study of method involves understanding theory, drafting corresponding program and execution of the program using computer. Hence, the knowledge of Fortran language and some experience in running the program is needed to understand the contents of the book.

It is assumed that the reader has already gone through a course on matrix methods of structural analysis. To a large extend, the programming strategies in stiffness matrix method and finite element method are similar except for the details of calculation of element stiffness and load matrices. As such, the previous knowledge of stiffness matrix method will greatly help the reader in understanding theory and step-by-step procedure in finite element method.

Both authors have taught the subject to the students over a period of time. The experience gained in teaching and supervising fem related projects has certainly influenced the mode of text in this book. Authors have observed that the students do show aversion to reading and understanding lengthy computer programs involving thousands of statements. As such, this book has completely avoided a single unified lengthy program with library of varieties of elements. Instead, separate small size programs are drafted for analysis of a particular type of structure using a particular element. All programs are transparent and easy to understand. The features of each program are specifically mentioned. Listings of all programs are given in Appendix. There are eleven such programs. For each program, atleast one data file is included in the text matter. It is expected that this will help the reader in preparing his own data files for solving additional problems. All programs are for educational purpose. These can not be considered as most general programs. Some have certain limitations which are clearly mentioned while describing features of the program.

The book is divided in thirteen chapters. Chapter 1 discusses various possible methods of analysing continuum structures. This helps in deciding the location of finite element method in the cluster of methods.

Chapter 2 is a combination of heterogeneous topics which are referred to at a later stage of study of the method. Chapter 3 deals with plane stress analysis using constant strain triangle and rectangle. Chapter 4 introduces the concept of isoparametric elements. Chapter 4 to 10 discuss plane stress analysis, axisymmetric solids, 3D solids, thin plates, Mindlin plates, higher order thick plates and shells. Finite strip method is presented in chapter 11. Chapter 12 discusses geometric non-linear analysis of plates. Chapter 13 consists of miscellaneous additional topics.

A compact disc (CD) is provided with the book. The contents of CD are divided in four folders. Folder 1 i.e. FOLD1 contains listing of eleven FORTRAN source programs listed in Appendix. FOLD2 contains some typical data files. FOLD3 includes executable files for the source programs in FOLD1. FOLD4 is used to present four typical output files. In all programs, output file is labeled as OUT.TXT file. For the purpose of giving four different output files, the files are renamed as OUT1.TXT, OUT2.TXT etc.

Authors are grateful to their family members for their patience and understanding during the period of drafting and printing of the book. Without their cooperation, this endeavour could not have come to fruition. Authors extend their thanks to Er. Atul S. Shinde for artwork.

Authors have great pleasure in presenting the book to the readers and hope that it will be received favourably.

A.S. Meghre
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Introduction

1.1 GENERAL

Structural engineers are required to analyse and design various types of structures. The knowledge of displacements, strains and stresses is important to them to check serviceability and safety of the structures. There are different varieties of structures such as trusses, frames, buildings, bridges, irrigation structures, folded plates, box-girders etc. In the earlier days, analysis was carried out by introducing certain simplifying assumptions so as to bring the analysis within the reach of hand calculation methods.

One possible way of classifying varieties of structures in civil engineering field is to put them in two categories as

(*i*) Discrete element or skeletal structures

(*ii*) Continuum structures.

The discrete element structures are formed by joining discrete structural members at their ends forming joints. Each member may be straight or curved. For such members, sectional dimensions are small compared to the length dimension. Pin jointed trusses, frames, grids, arched portals and space frames are skeletal structures. Some of these are shown in Fig. 1.1. The members of the skeletal structures are simple members and their properties namely force-displacement relations can be found easily using standard structural mechanics formulae. The analysis of a skeletal structure is no more a difficult job. For large size structures, stiffness matrix method is now widely used tool of analysis.

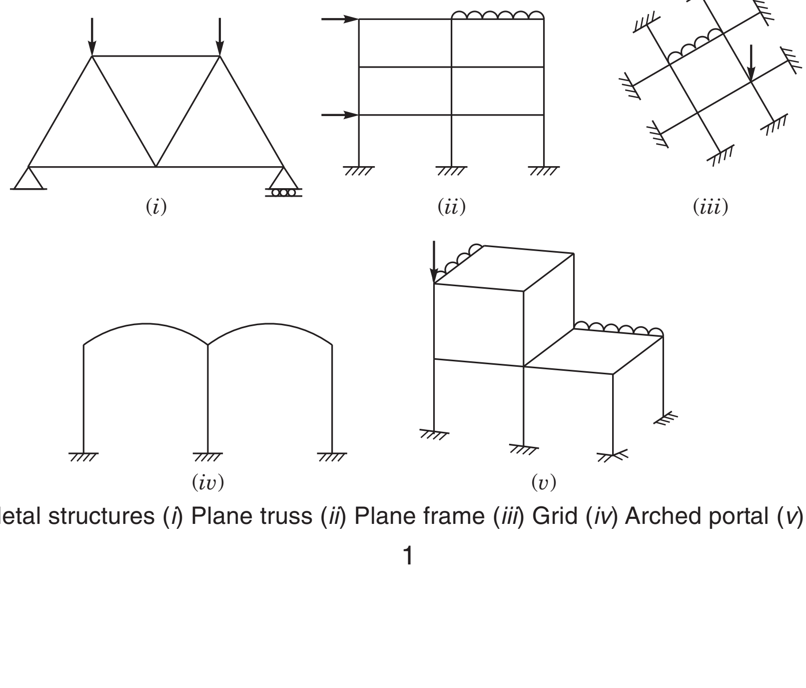

Fig. 1.1. Skeletal structures (*i*) Plane truss (*ii*) Plane frame (*iii*) Grid (*iv*) Arched portal (*v*) Space frame.

The continuum structures are formed from surface elements and solids. Plates, stiffened plates, shells, folded plates, box-girders are some examples of the continuum structures. Some of these are shown are in Fig. 1.2.

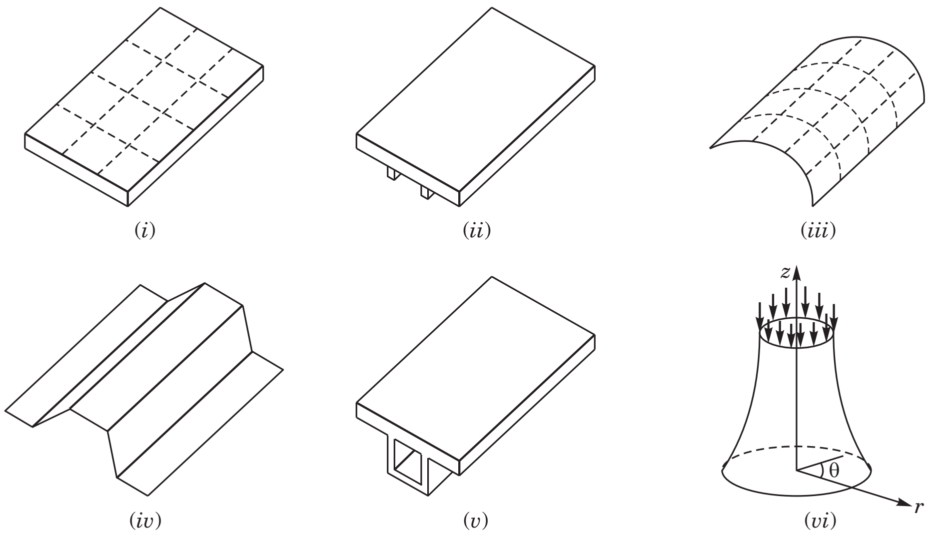

Fig. 1.2. Continum structures (*i*) Plate (*ii*) Stiffened (*iii*) Shell (*iv*) Folded plate (*v*) Box girder (*vi*) Axisymmetric solid.

The analysis of a continuum structure is a difficult job. To begin with, the behaviour of the structure is described in terms of a governing differential equation or equations. This is not so difficult. The general procedure is to express equilibrium of an infinitely small element of the structure in terms of stress-resultants and external forces acting on the element. The stress-resultants are written in terms of generalised strains and the strains are expressed as derivatives of displacements. Finally, combining all relations, it is generally possible to derive a governing differential equation in terms of displacements. This may be a single equation or a set of simultaneous differential equations. The analysts' aim is to obtain the solution of the governing differential equation(s) satisfying kinematic and force boundary conditions. Obviously, the first attempt is to obtain exact or analytical solution. In analytical solution, algebraic or trigonometric or hyperbolic expressions are obtained for field variable and other dependant variables. These expressions are continuous over the entire structure domain and it is possible to calculate the quantities of importance at any location in the structure using appropriate coordinates of the location.

An alternate approach of obtaining solution of governing differential equation is to express the field variable (w) as a combination of linearly independent trial or approximate functions satisfying boundary conditions such as $\overline{w} = \sum C_i \, \phi_i$. An error function $R(x)$ arises when $\overline{w}$ is substituted in differential equation. The values of the constants C_i associated with trial functions ϕ_i are obtained from the solution of simultaneous algebraic equations which are formed using any one of the following procedures :

(*i*) Equate residual or error function $R(x)$ to zero at selected locations known as collocation points (Point collocation).

(*ii*) Equate integral of the residual function $R(x)$ over sub-domains or zones of the structure to zero (Zone collocation).