NEW SYLLABUS

A TEXT BOOK OF

DISCRETE MATHEMATICS

MT 212 (A)

S. R. PATIL
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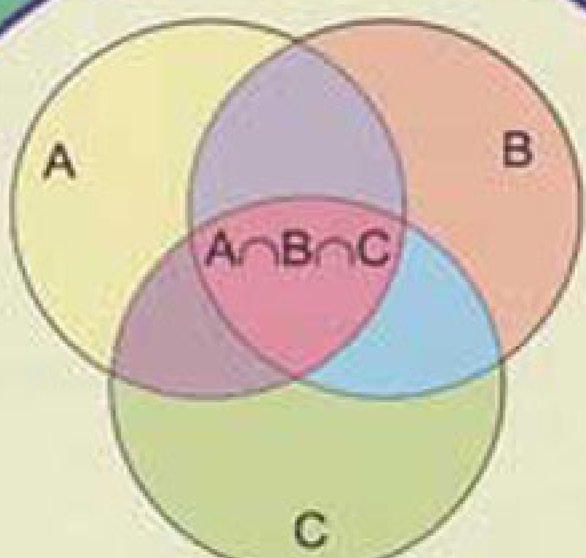
S. Y. B. Sc. MATHEMATICS • PAPER II (A) - SEMESTER I



$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

NIRALI PRAKASHAN

A Book Of

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As Per Revised Syllabus Effective from June 2014

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Preface …

We have great pleasure in presenting this text book on **Discrete Mathematics (MT 212 (A)) Semester – I** to the students of S.Y.B.Sc. class.. This book is written according to the new revised syllabus of Savitribai Phule Pune University to be implemented from June 2014.

We have taken utmost care to present the matter systematically. The book contains several selected solved examples and an ample number of graded problems in the exercises.

We are thankful to **Shri Dineshbhai Furia, Shri Jignesh Furia, Shri M. P. Munde**, Mrs. Anagha Medhekar, Mr. Santosh Bare, Mrs. Prachi Sawant and the staff of Nirali Prakashan, for the great efforts that they have taken to publish the book in time.

We welcome the valuable suggestions from our colleagues' and readers for the improvement of the book.

PUNE **AUTHORS**

JUNE 2014

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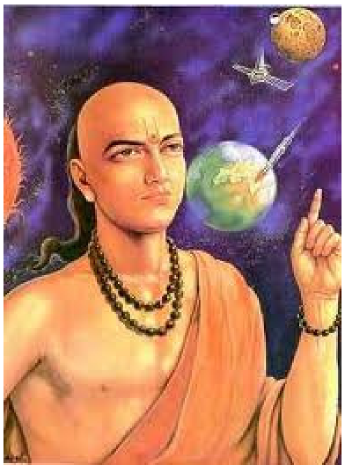
Logic and Proofs



Bhāskaracharya

Bhāskara (also known as **Bhāskarāchārya** ("Bhāskara the teacher") and as **Bhāskara II** to avoid confusion with Bhāskara I) (1114–1185), was an Indian mathematician and astronomer. He was born near Vijjadavida (Bijapur in modern Karnataka). Bhāskara is said to have been the head of an astronomical observatory at Ujjain, the leading mathematical center of medieval India. He lived in the Sahyadri region (Patnadevi, in Jalgaon district, Maharashtra).

Bhāskara and his works represent a significant contribution to mathematical and astronomical knowledge in the 12^{th} century. He has been called the greatest mathematician of medieval India. His main work Siddhānta Shiromani, (Sanskrit for "Crown of treatises,") is divided into four parts called Lilāvati, Bijaganita, Grahaganita and Golādhyāya. These four sections deal with arithmetic, algebra, mathematics of the planets, and spheres respectively. He also wrote another treatise named Karana Kautoohala.

Bhāskara's work on calculus predates Newton and Leibniz by over half a millennium. He is particularly known in the discovery of the principles of differential calculus and its application to astronomical problems and computations. While Newton and Leibniz have been credited with differential and integral calculus, there is strong evidence to suggest that Bhāskara was a pioneer in some of the principles of differential calculus. He was perhaps the first to conceive the differential coefficient and differential calculus.

Some of Bhaskara's contributions to mathematics include a proof of the Pythagorean theorem by calculating the same area in two different ways and then canceling out terms to get $a^2 + b^2 = c^2$. In Lilavati, solutions of quadratic, cubic and quartic indeterminate equations are explained, solutions of indeterminate quadratic equations (of the type $ax^2 + b = y^2$), etc.

1.1 Propositional Logic

In XII class, we have already studied the algebra of propositions. We review in this article the results very briefly.

Statements (Propositions) :

Definition : A declarative sentence which is either true or false (but not both) at the same time is called a statement or proposition. For example, consider the following sentences.

(i) The square of a negative integer is negative.

(ii) 19 is a prime number.

(iii) 2 + 2 = 6.

(iv) The earth rotates round the sun.

(v) If integer is even then, its square is even.

In above examples (i), (iii) are false but (ii), (iv) and (v) are true.

Hence, all of them are statements (propositions).

Consider the following examples :

(i) He is Engineer.

(ii) $x + 5 = 7$

(iii) Bring that pen !

(iv) What a beautiful painting !

(v) Where are you going ?

(vi) Do you speak English ?

(vii) $x > 100$.

In above sentences, some sentences are neither true nor false and in some sentences we are unable to judge whether it is true or false. Hence, above all sentences are not propositions (statements).

Logical Variables and Constants :

Let us consider the statements :

p : It is raining

q : 7 is a prime number

r : The sun rises in the west

s : The room temperature is not more than 26°C.

Here, we observe that the truth values of p and s depend upon the situation, while the truth values of q and r are always true and false respectively. Here, the statements p and s are called *logical variables* and the statements q and r are called *logical constants*.

Thus, a statement whose truth value changes according to circumstances is called a *logical variable*.

A statement whose truth value never changes is called as *logical constant*.

Logical Connectives :

Definition :

Two or more propositions can be joined using the connectives 'and', 'or', 'not', 'but', 'while', 'if, ... then ...' and 'if and only if'. These connectives are called *logical connectives*.

Simple statement (Definition) : A statement which does not contain any logical connective is called as *simple statement*.

Following are some examples of simple statement :

(i) There are seven days in a week.

(ii) Tiger is a wild animal.

(iii) 9 is a perfect cube.

(iv) $\sqrt{2}$ is a rational number.

(v) Pune is in Gujarat.

Compound statement (Definition) : A statement which contains logical connectives is called a *compound statement*.

Following are some examples of compound statement :

(i) If you score 95% at H.S.C. exam, then you may get admission to engineering.

(ii) Sachin Tendulkar is a cricketer and Ashok Kumar was a good singer.

(iii) $-1 \leq \cos \theta$ and $\cos \theta \leq 1$.

(iv) The sun is shining and it is cold.

(v) It will rain tomorrow or it will snow tomorrow.

(vi) If you drive, then I will walk.

Negation :

The negation of a statement is obtained either by introducing the word 'not' at a proper place or by prefixing the statement with the phrase 'It is not the case that'.

Definition : If 'p' is any statement then 'not p' is called the *negation* of the statement p and is denoted by '$\neg p$ or $\sim p$'.

If the truth value of p is true, then the truth value of $\neg p$ is false. If the truth value of p is false, then the truth value of $\neg p$ is true.

Example : If p : I am going for a walk.

$\neg p$: I am not going for a walk.

or $\neg p$: It is not the case that I am going for a walk.

Example : If p : It is cold.

$\neg p$: It is not cold.

Example : If p : Today is Friday.

$\neg p$: Today is not Friday.

$\neg (\neg p)$: It is not that today is not Friday.

$\therefore$ $\neg (\neg p)$: Today is Friday.

We see that $\neg (\neg p) \equiv p$.

Conjunction (and) :

Definition : If p and q are two statements, then the compound statement 'p and q' is called the *conjunction* of p and q; and is denoted by '$p \wedge q$'.

Disjunction (or) :

Definition : If p and q are two statements, then the compound statement 'p or q' is called the disjunction of p and q; and is denoted by '$p \vee q$'.

Note : 'p or q' means either **'p or q'** or **'both p and q'**. Therefore, 'or' is many times written as 'and/or'. Thus, $p \vee q$ always means **'p and/ or q'**. We say it is inclusive 'or'.

Another use of 'or' is exclusive 'or' and it is denoted by $p \oplus q$; which means either p or q but not both.

Conditional Statement (If ... then) :

Definition : If p and q are two statements, then the compound statement 'If p then q' is called the *conditional statement* or *implication* and is denoted by '$p \rightarrow q$'. We read this as 'p implies q'. Throughout this chapter, we use the symbol $\rightarrow$ for implication. In the conditional statement $p \rightarrow q$, the statement p is called anticident and the statement q is called consequent.

Bi-conditional Statement (If and only if) :

Definition : If p and q are two statements, then the compound statement 'p if and only if q' is called a **bi-conditional statement**. It is denoted by $p \leftrightarrow q$.

Example : $3 > 2$ if and only if $0 < 3 - 2$.

Example : Two lines are parallel if and only if they have the same slope.

Example : An integer is even if and only if it is divisible by 2.

Converse, Inverse and Contrapositive of a Conditional Statement :

If p and q are two statements, then

(i) The conditional statement $q \rightarrow p$ is called *converse* of the conditional statement $p \rightarrow q$.

(ii) The conditional statement $\neg p \rightarrow \neg q$ is called *inverse* of the conditional statement $p \rightarrow q$.

(iii) The conditional statement $\neg q \rightarrow \neg p$ is called *contrapositive* of the conditional statement $p \rightarrow q$.

Example : Give the converse, inverse and contrapositive of the conditional statement 'If you are good in Mathematics, then you are good in Physics'.

Solution : Let p : You are good in Mathematics.

q : You are good in Physics.

Conditional statement : $p \rightarrow q$.

(i) Converse : $q \to p$.

If you are good in Physics, then you are good in Mathematics.

(ii) Inverse : $\neg p \to \neg q$

If you are not good in Mathematics, then you are not good in Physics.

(iii) Contrapositive : $\neg q \to \neg p$.

If you are not good in Physics, then you are not good in Mathematics.

Propositional Form (Propositional Function) :

Definition : An expression constructed from logical variables p, q, r, ... and the logical connectives ($\wedge$, $\vee$, $\neg$, $\to$, $\leftarrow$) is called a *propositional form* [or well formed formula (wff)]. We denote such propositional form by P (p, q, r, ...).

Following are some examples of propositional forms :

(i) $\neg (p \vee q) \to p$

(ii) $(p \to q) \leftrightarrow (p \wedge \neg q)$

(iii) $p \wedge (p \to q) \to q$

(iv) $p \wedge q \to q \vee \neg p$

(v) $(p \vee q) \leftrightarrow [q \vee (r \to p)]$.

A propositional form has no fixed truth value. It is only when the statement variables in a form are assigned definite truth values, that we obtain the truth value of the propositional form.

Hence, the truth value of a statement assumes the truth value 'true' or the truth value 'false', depending upon the truth values assigned to the statement variables, appearing in the propositional form.

If the statement is true, then its truth value is denoted by T.

If the statement is false, then its truth value is denoted by F.

Truth Tables :

Definition : A table giving truth values of a compound statement for all possible combinations of truth values of its simple components is called a *truth table*.

Note :

(i) The compound statement $p \wedge q$ is true if and only if both p and q are true, otherwise it is false.

(ii) The compound statement $p \vee q$ is false if and only if both p and q are false, otherwise it is true.

(iii) The compound statement $p \to q$ is false, if and only if p is true and q is false, otherwise it is true.

(iv) The compound statement $p \leftrightarrow q$ is true if and only if p and q have the same truth values, otherwise it is false.

Below we prepare the truth tables for each of the following (i) Negation of a statement. (ii) Conjunction of a statement. (iii) Disjunction of a statement. (iv) Conditional statement. (v) Bi-conditional statement. (vi) Exclusive or.

Solution : (i) Negation ($\neg$p); not p :

p	**$\neg$p**
T	F
F	T

(ii) Conjunction (p $\wedge$ q) : p and q :

p	**q**	**p $\wedge$ q**
T	T	T
T	F	F
F	T	F
F	F	F

(iii) Disjunction ($\vee$) (or) : p or q :

p	**q**	**p $\vee$ q**
T	T	T
T	F	T
F	T	T
F	F	F

(iv) Conditional (p $\to$ q) : p implies q :

p	**q**	**p $\to$ q**
T	T	T
T	F	F
F	T	T
F	F	T

(v) Bi-conditional (p $\leftrightarrow$ q) : p implies and implied by q :

p	**q**	**p $\leftrightarrow$ q**
T	T	T
T	F	F
F	T	F
F	F	T

(vi) Exclusive or (p ⊕ q) :

p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

It should be noted that conjunction, disjunction, implication (conditional) and bi-conditional statements are basic compound statements. Any compound statement is obtained by using these statements. The students are therefore advised to memorize the above five truth tables.

The following truth table shows the truth values of a given conditional statement together with its converse, inverse and contrapositive.

p	q	¬p	¬q	Conditional $p \to q$	Converse $q \to p$	Inverse $\neg p \to \neg q$	Contrapositive $\neg q \to \neg p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

The conditional statement plays an important role in mathematical reasoning. We use various terminologies to express $p \to q$. They are

p implies q

If p, then q

q if p

p is sufficient for q

q is necessary for p

q unless not p, p only if q, etc.

Illustrative Examples

Example 1.1 : Let p : It rains

and q : The atmospheric humidity increases.

Write the following statements in symbolic form :

(a) Atmospheric humidity increases only if it rains.

(b) Sufficient condition for it to rain is that atmospheric humidity increases.

(c) Necessary condition for it to rain is that atmospheric humidity increases.

(d) Whenever atmospheric humidity increases it rains.

Solution : p : It rains

q : Atmospheric humidity increases

(a) Given statement is 'atmospheric humidity increases only if it rains'.

∴ It is q only if p.

Symbolically it is $q \to p$.

(b) Given statement is 'sufficient condition for it to rain is that atmospheric humidity increases'.

∴ It is q is sufficient for p.

Symbolically it is $q \to p$.

(c) Given statement is 'necessary condition for it to rain is that atmospheric humidity increases'.

∴ It is q is necessary for p.

Symbolically it is $p \to q$.

(d) Given statement is 'whenever atmospheric humidity increases it rains'.

∴ It is p whenever q.

Symbolically it is $q \to p$.

Example 1.2 : Construct a truth table for the following statement forms :

(a) $\neg (p \wedge q) \wedge (p \leftrightarrow q)$

(b) $(\neg p \to r) \wedge (p \leftrightarrow q)$

(c) $\neg r \to ((\neg q \vee p) \leftrightarrow r)$

Solution : (a)

p	**q**	**$p \wedge q$**	**$\neg(p \wedge q)$**	**$p \leftrightarrow q$**	**$\neg (p \wedge q) \vee (p \leftrightarrow q)$**
T	T	T	F	T	T
T	F	F	T	F	T
F	T	F	T	F	T
F	F	F	T	T	T

(b)

p	**q**	**r**	**$\neg p$**	**$\neg p \to r$**	**$p \leftrightarrow q$**	**$(\neg p \to r) \wedge (p \leftrightarrow q)$**
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	T	F

(c)

p	q	r	¬q	¬r	¬q ∨ p	(¬q ∨ p) ↔ r	¬r → ((¬q ∨ p) ↔ r)
T	T	T	F	F	T	T	T
T	T	F	F	T	T	F	F
T	F	T	T	F	T	T	T
T	F	F	T	T	T	F	F
F	T	T	F	F	F	F	T
F	T	F	F	T	F	T	T
F	F	T	T	F	T	T	T
F	F	F	T	T	T	F	F

Example 1.3 : Write the converse, inverse and contrapositive of the following implications.

(a) If it rains tonight, then I will stay at home.

(b) I come to class whenever there is going to be a quiz.

Solution : (a) p : It rains tonight

q : I stay at home

Given statement is in symbolic form as p → q.

Now, ¬p : It does not rain tonight

¬q : I will not stay at home.

Converse (q → p) : If I stay at home, then it will rain tonight.

Inverse (¬p → ¬q) : If it does not rain tonight, I will not stay at home.

Contrapositive (¬q → ¬p) : If I do not stay at home, it will not rain tonight.

(b) p : I come to class.

q : There is a quiz.

Given statement in symbolic form is q → p.

¬p : I do not come to class.

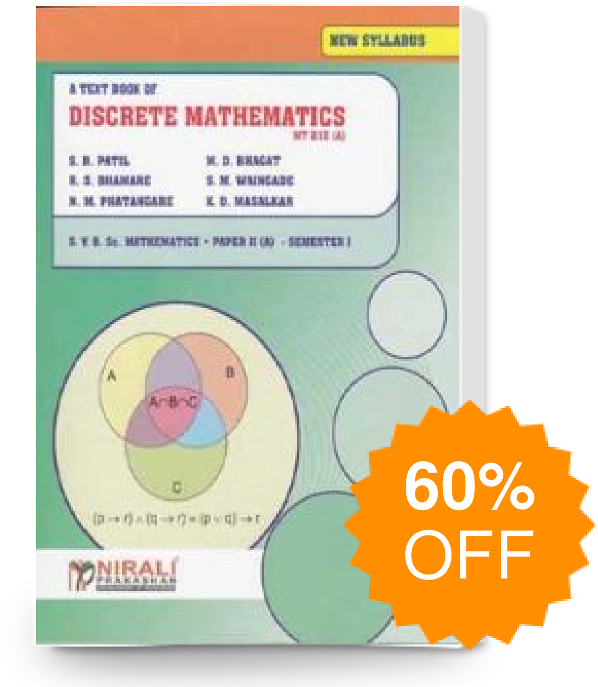
¬q : There is no quiz.

Converse (p → q) : If I come to class, then there is a quiz.

Inverse (¬q → ¬p) : If there is no quiz, I do not come to class.

Contrapositive (¬p → ¬q) : If I do not come to class, then there is no quiz.

A Textbook Of Discrete Mathematics



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