NEW SYLLABUS

OPERATIONS RESEARCH

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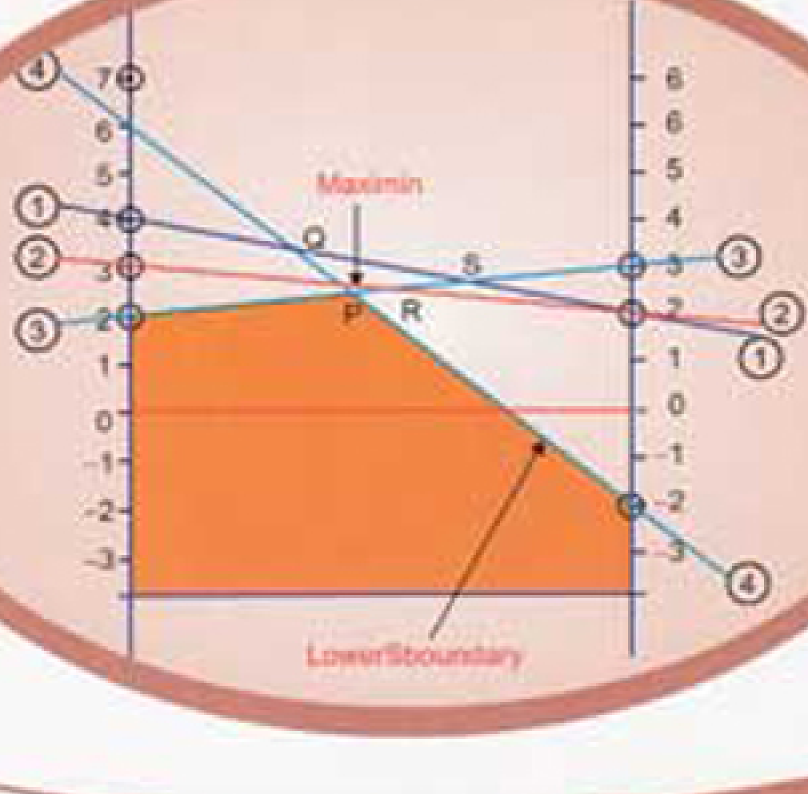
S.Y.B.Sc.: COMPUTER SCIENCE : MATHEMATICS (MTC 222) : PAPER-II : SEMESTER-II



NIRALI PRAKASHAN
ADVANCEMENT OF KNOWLEDGE

A Book Of

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For Second Year B. Sc. (Computer Science)
MTC : 222 : Mathematics – Paper-II : Semester-II
As Per Savitribai Phule Pune University
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Preface ...

We have great pleasure in presenting this text book on **'Operations Research'** to the students of S.Y.B.Sc. (Computer Science) class. Mathematics Paper-II [MTC : 222 Semester–II]. This book is written according to the new revised syllabus of Savitribai Phule Pune University to be implemented from June 2014.

We have taken utmost care to present the matter systematically. The book contains several selected solved examples and an ample number of graded problems in the exercises.

We are thankful to **Shri Dineshbhai Furia, Shri Jignesh Furia,** Mrs. Anagha Kaware, Mr. Santosh Bare, Mrs. Anjali Mule and the staff of Nirali Prakashan, for the great efforts that they have taken to publish the book in time.

We welcome the valuable suggestions from our colleagues' and readers for the improvement of the book.

PUNE **AUTHORS**
NOVEMBER 2014

Syllabus ...

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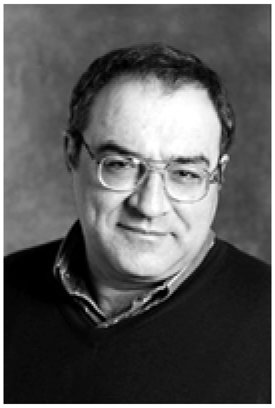
Chapter **1**...

Modeling with Linear Programming



Leonid Khachiyan

Leonid Genrikhovich Khachiyan (Armenian: Լեոնիդ Գենրիխովիչ Խաչիյան; Russian: Леонид Генрихович Хачиян; May 3, 1952 – April 29, 2005) was a Soviet mathematician of Armenian descent who taught Computer Science atRutgers University. He was most famous for his Ellipsoid algorithm (1979) for linear programming, which was the first such algorithm known to have a polynomial running time. Even though this algorithm was shown to be impractical due to the high degree of the polynomial in its running time, it has inspired other randomized algorithms for convex programming and is considered a significant theoretical breakthrough.

1.1 Linear Programming Problem (L.P.P.)

Linear programming uses mathematics to describe the problem of concern. The term linear means that all the mathematical functions in this model are linear and the word "programming" is synonym with planning. Thus, linear programming deals with that section of programming problems for which all relations among the variables are linear. The general linear programming problem optimizes a linear function of several variables subject to a system of equalities and/or inequalities. The optimization of a linear function includes either a maximization or a minimization of this function. For example, in an industrial plant the objective may be viewed as maximizing gains or productive time or minimizing cost or idle time. The optimization suggests the common goal of achieving the best solution to the linear programming model.

1.1.1 Some Definitions

Objective Function : The linear function to be optimized is called "*objective function*" of the L.P.P.

Constraints : The linear conditions in terms of equalities and/or inequalities imposed upon the variables to be determined are called *constraints* or *restrictions of L.P.P*.

Decision Variables : The unknown variables in terms of which "objective function" is expressed are called '*decision variables*' of the L.P.P.

1.1.2 Applications of Linear Programming

There are several fields of applications of operations research like military, industries, insurance companies, banks, agriculture, planning etc.

In industries all mega companies have several departments within the company itself, like Production department to minimize the cost of production, Marketing department to maximize the amount of selling product and to minimize the cost of selling, Finance department to minimize the capital required to maintain any level of business. There are other departments like research and development. In all these departments, co-ordination is maintained and linear programming helps each department to optimize their requirements and make overall work of the company effective and the best output.

Linear programming is used in the fields like, Diet Problem, Blending problem, Transportation problems. In diet problem, linear programming provides optimal food mix for meeting the nutritional needs of human beings, animals or a broiler at the least cost. Linear programming is also used in allocation problems, a problem which involves allocation (allotment) of available resources to the jobs (or activities) that are to be done. Allocation problems can be further subdivided into the following three types of problem :

(i) Linear Programming Problems (L.P.P.)

(ii) Transportation Problems (T.P.)

(iii) Assignment Problems (A.P.)

Let us illustrate the definition of linear programming problem by an example.

(i) Suppose a factory decides to manufacture two kinds of products say A and B. Let the profit per unit of product A and product B be ₹ 5 and ₹ 7 respectively. Each unit of product A requires 6 machine hours and that of product B requires 5 machine hours. Each unit of product A requires 10 units of raw material, whereas each unit of product B requires 6 units of raw material. The maximum available machine hours and material units are 220 and 320, respectively. A maximum of 100 units are required of product B. Determine the number of units to be manufactured of products A and B. This problem can be represented in mathematical linear equalities and/or inequalities.

For this, suppose x_1 and x_2 denote the number of units to be produced of products A and B respectively. The above information can be summarized in tabular form :

	Product A	Product B
Number of units produced (decision variables)	x_1	x_2
Machine hours required	$6x_1$	$5x_2$
Raw material required in units	$10x_1$	$6x_2$
Maximum requirements	No unit	100
Profit obtained	$5x_1$	$7x_2$

From above information we see that the objective function of this problem is to maximize the net profit $5x_1 + 7x_2$ for the two products. This maximum profit is subject to the conditions :

(i) The machine hours constraint, i.e.

$6x_1 + 5x_2 \leq 220$, since the total number of machine hours available is 220.

(ii) The raw material constraint, viz

$10x_1 + 6x_2 \leq 320$, as the total number units of raw material available is 320.

(iii) The maximum requirement constraints, viz.

$x_1 \geq 0$, since negative product has no physical meaning and $x_2 \leq 100$, since the maximum number of units required that of product B is 100.

This problem is usually written in the following form;

maximise : $z = 5x_1 + 7x_2$

Subject to

$$6x_1 + 5x_2 \leq 220$$
$$10x_1 + 6x_2 \leq 320$$
$$x_2 \leq 100$$
$$x_1 \geq 0, \quad x_2 \geq 0$$

where, z is the value of the objective function.

The above illustration also explains how to formulate (or formulation) a given linear programming problem.

1.2 Linear Programming Formulation

Let us see some more examples, where we formulate the given problem in mathematical model, at the same time we see the various areas of the applications of L.P.P.

Illustrative Examples

Example 1.1 : A company sells two different products A and B. The company makes a profit of ₹ 30 and ₹ 40 per unit on the products A and B respectively. The two products are produced in a common production process and are sold in two different markets. The production process has a capacity of 35,000 man-hours. It takes two hours to produce one unit of A and 3 hours to produce one unit of B. The market has been surveyed and company feels that the maximum number of units of A that can be sold is 12000 units and the maximum of B is 8000 units. Assuming that the products can be sold in any circumstances, formulate the L.P.P.

Solution : Suppose company produces x_1 units of product A and x_2 units that of B. Then the profit function will be $30x_1 + 40x_2$. Subject to the conditions that $2x_1 + 3x_2 \leq 35,000$ and $x_1 \leq 12000$, $x_2 \leq 8000$, with $x_1 \geq 0$, $x_2 \geq 0$. Thus LPP formulation is

Maximize : $Z = 30x_1 + 40x_2$

Subject to

$$2x_1 + 3x_2 \leq 35000$$
$$x_1 \leq 12000$$
$$x_2 \leq 8000$$

and $x_1 \geq 0, \; x_2 \geq 0.$

Example 1.2 : Suppose in a hospital, it is decided that each patient should be given at least 3, 4, 5 units of nutrients say A, B, C respectively. Suppose there are 4 foods say f_1, f_2, f_3 and f_4 available. Let the following table shows the nutrients A, B, C present per unit in the foods f_1, f_2, f_3 and f_4.

Nutrients	Foods				Requirement of nutrients in units
	f_1	f_2	f_3	f_4	
A	0.5	1	3	1.5	3
B	1	2	0	2.5	4
C	2	1.5	0.5	0	5

Suppose the cost per unit of the foods f_1, f_2, f_3 and f_4 is ₹ 1, ₹ 2, ₹ 3, ₹ 0.5 respectively. The problem is to find the best diet (the food combination) that can be supplied at minimum cost, satisfying the daily requirements of the patient.

Solution : Suppose x_1, x_2, x_3 and x_4 be the quantities of the foods f_1, f_2, f_3 and f_4 respectively, that constitute the best daily diet. Then problem is

Minimize $Z = x_1 + 2x_2 + 3x_3 + 0.5x_4$

Subject to

$$0.5x_1 + x_2 + 3x_3 + 1.5x_4 \geq 3$$
$$x_1 + 2x_2 + 2.5x_4 \geq 4$$
$$2x_1 + 1.5x_2 + 0.5x_3 \geq 5$$
$$\text{with} \quad x_1, x_2, x_3, x_4 \geq 0.$$

Example 1.3 : A firm can produce three types of clothes say A, B and C. The clothes are made of three colours of wools say, red, green and blue. One unit of cloth A needs 2 meters of red wool and 3 meters of blue wool; one unit of cloth B requires 3 meters of red wool, 2 meters of green and 2 meters of blue wool and one unit of cloth C requires 5 meters of green wool and 4 meters of blue wool. The firm has only a stock of 800 meters of red wool, 1000 meters of green wool and 1500 meters of blue wool. Suppose that the profit per unit of clothes A, B and C is ₹ 3, ₹ 4 and ₹ 5 respectively. Determine how the firm should use the available material, so as to maximize the income from the finished clothes.

Solution : Suppose the firm produces x_1, x_2 and x_3 units of clothes A, B and C respectively. Thus red wool required will be $2x_1 + 3x_2$ meters, $3x_1 + 2x_2 + 4x_3$ meters of blue wool and $2x_2 + 5x_3$ meters of green wool. Also the profit of the company will be

$3x_1 + 4x_2 + 5x_3$.

Thus the problem is

Maximize : $Z = 3x_1 + 4x_2 + 5x_3$

Subject to

$$2x_1 + 3x_2 \leq 800$$
$$2x_2 + 5x_3 \leq 1000$$
$$3x_1 + 2x_2 + 4x_3 \leq 1500$$
$$\text{and} \quad x_1, x_2, x_3 \geq 0.$$

Example 1.4 : A factory manufactures three products. These products are processed through three distinct stages. The time required to manufacture a unit of each of three products and the daily capacity of the stages are given by the following table :

Stage	Time per unit (minutes)			Stage Capacity min/day
	Product 1	Product 2	Product 3	
1	2	3	4	520
2	4	6	–	460
3	–	5	2	490

It is required to determine the daily number of units to be produced of each product, given that the profits per unit of products 1, 2 and 3 are 3, 5 and 6 respectively. Suppose that all the amounts produced are absorbed by the market.

Solution : Let x_1, x_2, x_3 be the number of units to be manufactured of products 1, 2, 3 respectively. As all the units manufactured are absorbed by the market, the net profit becomes $3x_1 + 5x_2 + 6x_3$. For each of the three stages the total time consumed by all three products should not exceed the capacity of the stage. So we have

$$2x_1 + 3x_2 + 4x_3 \leq 520$$
$$4x_1 + 6x_2 \leq 460$$
$$5x_2 + 2x_3 \leq 490$$

Therefore the problem is to

Maximize : $Z = 3x_1 + 5x_2 + 6x_3$

Subject to $2x_1 + 3x_2 + 4x_3 \leq 520$

$4x_1 + 6x_2 \leq 460$

$5x_2 + 2x_3 \leq 490$

and $x_1, x_2, x_3 \geq 0$.

Example 1.5 : A paper mill received three orders for paper rolls with the widths and length indicated in the following table :

Order No.	Width (meters)	Length (meters)
1	5	10,000
2	7	30,000
3	9	20,000

Rolls are produced in the mill in two standard widths 10 and 20 meters which are slit to the sizes specified by the orders. There is no limit on the lengths of the standard rolls. The objective is to determine the production schedule that minimizes the firm losses while satisfying the given demand.

Solution : Let x_{ij} be the length of the i^{th} roll (i = 1 for 10 meter and i = 2 for 20 meter roll) which is cut according to the j^{th} pattern. The table below shows the possible patterns for both standard rolls.

Width	i = 1 (10 mt)			i = 2 (20 mt)					
	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}
5 mt.	2	0	0	4	2	2	1	0	0
7 mt.	0	1	0	0	1	0	2	1	0
9 mt.	0	0	1	0	0	1	0	1	2
Time loss in mts	0	3	1	0	3	1	1	4	2

Let s_1, s_2 and s_3 be the surplus lengths produced of the rolls with widths 5, 7 and 9 respectively. Then

Minimize : $Z = 3x_{12} + x_{13} + 3x_{22} + x_{23} + x_{24} + 4x_{25} + 2x_{26} + 5s_1 + 7s_2 + 9s_3$

Subject to

$2x_{11} + 4x_{21} + 2x_{22} + 2x_{23} + x_{24} - s_1 = 10,000$

$x_{12} + x_{22} + 2x_{24} + x_{25} - s_2 = 30,000$

$x_{13} + x_{23} + x_{25} + 2x_{26} - s_3 = 20,000$

$x_{ij} \geq 0, \quad s_r \geq 0$ for all i, j and r.

Exercise (1.1)

1. A firm produces three products. These products are processed on three different machines. The time required to manufacture one unit of each of these three products and daily capacity of the three machines is given in the table below :

Machine	Time per unit (minutes)			Machine Capacity (minutes/day)
	Product-1	Product-2	Product-3	
M_1	2	3	2	440
M_2	4	–	3	470
M_3	2	5	–	430

Determine the daily number of units to be produced for each product to maximize the profit, if the profit per unit for product 1, 2 and 3 is ₹ 4, ₹ 3 and ₹ 6 respectively. It is assumed that all products produced are consumed in the market. Formulate the problem.

2. A chemical company produces two products X and Y. Each unit of product X requires 3 hours on operation - I and 4 hours on operation - II, while each unit of product Y requires 4 hours on operation – I and 5 hours on operation - II. The total available time for operation - I and II is 20 hours and 26 hours respectively. The production of each unit of product Y also results in two units of a by-product Z at no extra cost.

   Product X sells at profit of ₹ 10 per unit, while Y sells at profit of ₹ 20 per unit. By-product Z brings a unit profit of ₹ 6, if sold; in case it cannot be sold, the destruction cost is ₹ 4 per unit. Market survey indicates that more than five units of Z can be sold. Determine the quantities of X and Y to be produced, keeping Z in mind, so that the profit earned is maximum. Formulate the L.P.P.

3. A person wants to decide the constraints of a diet which will fulfil his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given below :

Food type	Yield per unit			Cost per unit
	Proteins	Fats	Carbohydrates	
1	p_1	f_1	c_1	d_1
2	p_2	f_2	c_2	d_2
3	p_3	f_3	c_3	d_3
4	p_4	f_4	c_4	d_4
Maximum daily requirements	P	F	C	

Formulate the L.P.P.

4. Three products are processed through three different operations. The time in minutes required to process per unit of each product, the daily capacity of operations (in minutes per day) and profit per unit of each product are given :

Operation	Time per unit in minutes			Operation capacity minutes per day
	Product-1	Product-2	Product-3	
1	2	3	2	450
2	4	0	3	480
3	1	5	0	440
Profit per unit	4	3	6	–

Formulate the L.P.P.

5. A company can produce three types of cloth say A, B and C. Three kinds of wool are required for it, say R, G and V. One unit length of type A cloth needs 2 meters wool of type R, 3 meters of type V; one unit of length of type B cloth needs 3 meters wool of type R, 2 meters of type G and 2 meters of type V; and one unit of type C cloth needs 5 meters of wool of type G and 4 meters of type V. A company has only a stock of 80 meters of R, 100 meters of G and 150 meters of V wool. The profit per unit of type A, B and C is ₹ 3, ₹ 5 and ₹ 4 respectively. Determine how the firm should use the available material so as to maximize the income from the finished cloth.
6. A company is contracted to receive 60,000 kg of ripe tomatoes at ₹ 7 per kg from which it produces both canned tomato juice and tomato paste. The canned products are packaged in cases of 24 cans each. A single can of juice requires 1 kg of fresh tomatoes, whereas that of paste requires $\frac{1}{3}$ kg only. The company's share of the market is limited to 2000 cases of juice and 6000 cases of paste. The wholesale prices per case of juice and paste stand at ₹ 300 and ₹ 200 respectively. Formulate the problem.
7. A firm manufactures two products A and B on which the profit earned per unit are ₹ 3 and ₹ 4 respectively. Each product is processed on two machines M_1 and M_2. Product A requires one minute of processing time on M_1 and *two* minutes on M_2, while B requires one minute on M_1 and one minute on M_2. Machine M_1 is available for not more than 450 minutes, while machine M_2 is available for 600 minutes during any working day. Formulate the L.P.P.
8. A company has two bottling plants, say S and M. Each plant produces three drinks, say A, B and C. The number of bottles produced per day are as follows :

Drink	Plant-S	Plant-M
A	1500	1500
B	3000	1000
C	2000	5000

A market survey indicates that during a month of April there will be a demand of 20,000 bottles of drink A, 40,000 bottles of B and 44,000 bottles of C. The operating costs per day for plants S and M are ₹ 600 and ₹ 400. Formulate the L.P.P., so as to minimize the production cost, while still meeting the market demand.

9. A firm uses lathes, milling machines and grinding machines to produce two machine parts. The following table represents machining times required for each part, the machining times available on different machines and profit on each machine part.

Time of machine	**Time required for (in minutes)**		**Time available in minutes**
	Part – I	**Part – II**	
Lathes	12	6	3000
Milling machines	4	10	2000
Grinding	2	3	900
Profit per unit	₹ 40	₹ 100	

Formulate L.P.P.

Answers (1.1)

1. Maximize $Z = 4x_1 + 3x_2 + 6x_3$

Subject to

$$2x_1 + 3x_2 + 2x_3 \leq 440$$
$$4x_1 + 3x_3 \leq 470$$
$$2x_1 + 5x_2 \leq 430$$
$$x_1, x_2, x_3 \geq 0.$$

2. Maximize $Z = 10x_1 + 20x_2 + 6x_3 - 4x_4$

Subject to

$$3x_1 + 4x_2 \leq 20$$
$$4x_1 + 5x_2 \leq 26$$
$$-2x_2 + x_3 + x_4 = 0$$
$$x_3 \leq 5$$

where $x_1, x_2, x_3, x_4 \geq 0.$

3. Minimize $Z = x_1 d_1 + x_2 d_2 + x_3 d_3 + x_4 d_4$

Subject to

$$x_1 p_1 + x_2 p_2 + x_3 p_3 + x_4 p_4 \geq P$$
$$x_1 f_1 + x_2 f_2 + x_3 f_3 + x_4 f_4 \geq F$$
$$x_1 c_1 + x_2 c_2 + x_3 c_3 + x_4 c_4 \geq C.$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

4. Maximize $Z = 4x_1 + 3x_2 + 6x_3$

Subject to

$$2x_1 + 3x_2 + 2x_3 \leq 450$$
$$4x_1 + 3x_3 \leq 480$$
$$x_1 + 5x_2 \leq 440$$
$$x_1, x_2, x_3 \geq 0.$$

5. Maximize $Z = 3x_1 + 5x_2 + 4x_3$

Subject to

$$2x_1 + 3x_2 \leq 80$$
$$2x_2 + 5x_3 \leq 100$$
$$3x_1 + 2x_2 + 4x_3 \leq 150$$

$x_1, x_2, x_3 \geq 0.$

6. Maximize $Z = 300x_1 + 200x_2$

Subject to

$$24x_1 + 8x_2 = 60,000$$
$$x_1 \leq 2000$$
$$x_2 \leq 6000$$

x_1, x_2 are number of cannes of juice and paste respectively and $x_1, x_2 \geq 0$.

7. Maximize $Z = 3x_1 + 4x_2$

Subject to

$$x_1 + x_2 \leq 450$$
$$2x_1 + x_2 \leq 600$$

$x_1, x_2 \geq 0.$

8. Minimize $Z = 600x_1 + 400x_2$

Subject to

$$1500x_1 + 1500x_2 \geq 20,000$$
$$3000x_1 + 1000x_2 \geq 40,000$$
$$2000x_1 + 5000x_2 \geq 44,000.$$

$x_1, x_2 \geq 0.$

9. Maximize $Z = 40x_1 + 100x_2$

Subject to

$$12x_1 + 6x_2 \leq 3000$$
$$4x_1 + 10x_2 \leq 2000$$
$$2x_1 + 3x_2 \leq 900$$

$x_1, x_2 \geq 0.$

1.3 Graphical Solution to Linear Programming Problem (L.P.P.)

Example 1.6 : Formulate mathematical model of the following L.P.P., and solve it by graphically.

A company produces both interior and exterior house points for wholesale distribution. The two basic raw materials, A and B are used to manufacture the paints. The maximum availability of A is 6 tons a day; that of B is 8 tons a day. The daily requirements of the raw materials per ton of interior and exterior paints are summarized in the following table :

	Tons of raw material per ton of paint		Maximum availability
	Exterior	**Interior**	
Raw material A	1	2	6
Raw material B	2	1	8