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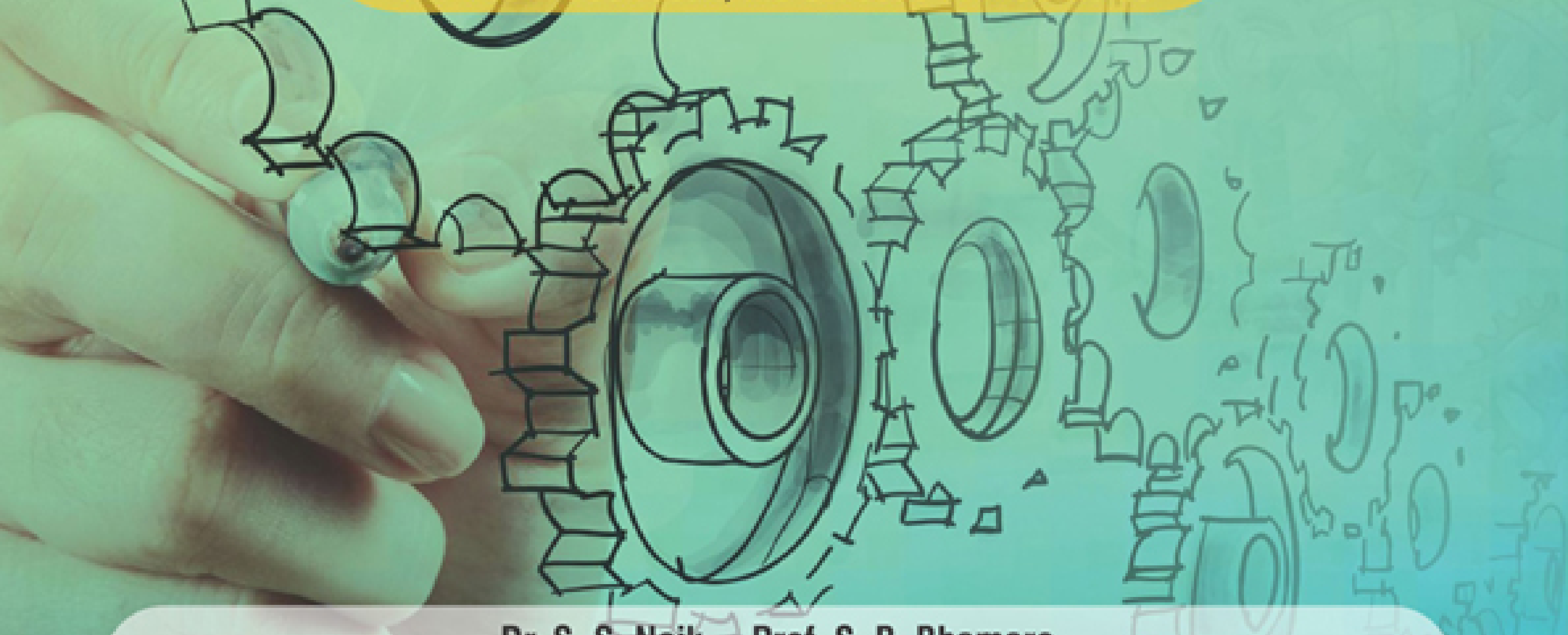

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Second Year Engineering

Semester – I

Mechanical Sandwich, Production, Production Sandwich Industrial and Automobile Engineering

According to New Revised Credit System Syllabus of Savitribai Phule Pune University, Pune.

EFFECTIVE FROM ACADEMIC YEAR JUNE 2016

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Engineering Mathematics - III

Second Year Engineering (Semester – I)

Mechanical Sandwich, Production, Production Sandwich Industrial and Automobile Engineering

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Preface

This Book Engineering Mathematics-III is intended to be a textbook for students of Second Year Engineering In most sciences, one generation years down what another has built and what one has established another undoes. In Engineering Mathematics-III, each generation adds a new story to the old structure. Keeping this in mind, this book is written to have a better introduction of the Engineering Mathematics-III. This book is presented with simple but exact explanation of subject matter, application of each topic to real life, engineering problems, large number of illustrative examples followed by well graded exercise. We have tried to be rigorous and precise in presenting the concepts in very simple manner. We hope that the students will not only learn some powerful concepts, but also will develop their ability to understand the concept and apply it properly to solve engineering problems. We feel that faculty member will also enjoy reading this book which is enriched with application of each topic.

Acknowledgment

The authors acknowledge the help of colleagues and friends for the warm relationship which provides a source of energy in our endeavours.

We are grateful to our family members for the encouragement and constant cooperation and assistance in creation of this book.

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We are certainly thankful to the students of engineering who are a constant source of our enthusiasm and encouragement in our endeavours.

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SYLLABUS

Unit - I : Linear Differential Equations (LDE) and Applications (09 Hrs.)

LDE of nth order with constant coefficients, Method of variation of parameters, Cauchy's & Legendre's Differential Equations, Simultaneous & Symmetric simultaneous Differential Equations. Modeling of problems on bending of beams, whirling of shafts and mass spring systems.

Unit - II : Transforms (09 Hrs.)

Laplace Transform (LT): LT of standard functions, properties and theorems, Inverse LT, Application of LT to solve LDE.

Fourier Transform (FT): Fourier integral theorem, Fourier transform, Fourier Sine & Cosine transform, Inverse Fourier Transforms.

Unit – III : Statistics and Probability (09 Hrs.)

Measure of central tendency, Standard deviation, Coefficient of variation, Moments, Skewness and Kurtosis, Correlation and Regression, Probability, Probability distributions: Binomial, Poisson and Normal distributions, Population and sample, Sampling distributions, t-distribution, Chi-square distribution.

Unit – IV : Vector Differential Calculus (09 Hrs.)

Physical interpretation of Vector differentiation, Vector differential operator, Gradient, Divergence and Curl, Directional derivative, Solenoidal, Irrotational and Conservative fields, Scalar potential, Vector identities.

Unit - V : Vector Integral Calculus and Applications (09 Hrs.)

Line, Surface and Volume integrals, Work-done, Green's Lemma, Gauss's Divergence theorem, Stoke's theorem. Applications to problems in Fluid Mechanics, Continuity equations, Streamlines, Equations of motion, Bernoulli's equation.

Unit - VI : Applications of Partial Differential Equations (PDE) (09 Hrs.)

Basic concepts, modeling of Vibrating String, Wave equation, one and two dimensional Heat flow equations, method of separation of variables, use of Fourier series. Solution of Heat equation by Fourier Transforms, Two-dimensional wave equation.

Recommended by SPPU Text Books and Reference Books

Text Books :

1. Advanced Engineering Mathematics, Ninth edition, by Erwin Kreyszig (Wiley India).
2. Advanced Engineering Mathematics, seventh edition, by Peter V. O'Neil (Cengage Learning).

Reference Books :

1. Advanced Engineering Mathematics, 2e, by M. D. Greenberg (Pearson Education).
2. Advanced Engineering Mathematics, Wylie C.R. & Barrett L.C. (McGraw-Hill, Inc.)
3. Higher Engineering Mathematics by B. S. Grewal (Khanna Publication, Delhi).
4. Applied Mathematics (Volumes I and II) by P. N. Wartikar & J. N. Wartikar (Pune Vidyarthi Griha Prakashan, Pune).
5. Higher Engineering Mathematics by B.V. Ramana (Tata McGraw-Hill).
6. Advanced Engineering Mathematics with MATLAB, 2e, by Thomas L. Harman, James, Dabney and Norman Richert (Brooks/Cole, Thomson Learning).

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CONTENTS

Solved University Question Papers

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Unit 1

Linear Differential Equations (LDE) and Applications

Syllabus :
LDE of n^{th} order with constant coefficients, Method of variation of parameters, Cauchy's & Legendre's Differential Equations, Simultaneous & Symmetric simultaneous Differential Equations. Modeling of problems on bending of beams, whirling of shafts and mass spring systems.

1.1 Introduction :

Linear differential equations of higher order arise in modelling physical and engineering problems such as in the theory of electrical circuits mechanical vibrations etc. Linear differential equations with constant coefficients can be solved by finding their complimentary solution and particular integrals where as some standard linear differential equations with variable coefficients can be reduced by suitable substitutions to linear differential equations with constant coefficients, then using methods of solutions of LDE with constant coefficients the general solution of such equations can be obtained.

1.1.1 Definition :

A linear differential equation of n^{th} order is defined as,

$$\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + a_2(x)\frac{d^{n-2}y}{dx^{n-2}} + \ldots.. + a_n(x)\, y = F(x) \qquad \ldots.(1)$$

where $a_1(x), a_2(x) \ldots.. a_n(x)$ and F(x) are functions of x only.

(a) If the coefficients $a_1(x), a_2(x) \ldots. a_n(x)$ are all constants, then equation (1) becomes

$$\frac{d^n y}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + \ldots.. + a_n\, y = F(x) \qquad \ldots.(2)$$

Which is Linear Differential equation of n^{th} order with constant coefficients.
(i.e. LDE of n^{th} order with constant coefficients)

(b) If F(x) = 0, then

$$\frac{d^n y}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + \ldots.. + a_n y = 0 \quad \ldots. (3)$$

is called the homogeneous equation of (2) or complimentary equation of (2) ≡ C.E. = (3)

(c) The general solution of C.E. which contains n arbitrary constants equal to the order of D.E. is called C.F. Complimentary Function.

(d) The general solution of (2) contains one more part the particular integral part which has no arbitrary constant and it satisfies the given equation :

Hence general solution of (2) is

(e) $y = \text{C.F.} + \text{P.I} = Y_c + Y_p$

1.2 To Find Complimentary Function, C.F. = Y_c :

Let $D \equiv \frac{d}{dx}$ be the differential operator

so that $D^2 = \frac{d^2}{dx^2}$, $D^3 = \frac{d^3}{dx^3}$ …… $D^n = \frac{d^n}{dx^n}$

then (2) can be written as,

$$(D^n + a_1D^{n-1} + a_2D^{n-2} + \ldots.. + a_n)y = F(x) \quad \ldots. (4)$$

To find the complimentary function we solve the complimentary equation.

$$(D^n + a_1D^{n-1} + a_2D^{n-2} + \ldots.. + a_{n-1}D + a_n)y = 0 \text{ (C.E.)} \quad \ldots. (5)$$

to solve C.E, equate coefficient of y to zero i.e. $D^n + a_1D^{n-1} + \ldots.. + a_n = 0$. Auxiliary equation.

If $m_1, m_2, \ldots.., m_n$ are roots of A.E. then $(D - m_1)(D - m_2) \ldots.. (D - m_n) = 0$

- **Depending on the nature of roots $m_1, m_2, \ldots.., m_n$ we have the following four cases of C.F. :**

a) Case I : If all the roots are real and different then,

$$\text{C.F.} = Y_c = c_1e^{m_1x} + c_2e^{m_2x} + \ldots.. + c_ne^{m_nx}$$

b) Case II : If r roots are real and repeated or equal say $m_1 = m_2 = \ldots.. = m_r = m$

r < n and other roots are different, then C.F. is

$$Y_c = (c_1 + c_2x + c_3x^2 + \ldots. + c_rx^{r-1})\, e^{mx} + c_{r+1}e^{m_{r+1}x} + \ldots. + c_4e^{m_4x}$$

In particular if three roots are real and repeated say $m_1 = m_2 = m_3 = m$, and all other are real and different then,

C.F. is $Y_c = (c_1 + c_2x + c_3x^2)\, e^{mx} + c_ue^{m_ux} + \ldots. + c_ne^{m_nx}$

c) Case III : If a pair of roots are complex say $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$ and all others are real and different then

C.F. is $Y_c = (c_1\cos\beta x + c_2\sin\beta x)e^{\alpha x} + c_3e^{m_3x} + \ldots.. + c_ne^{m_nx}$

d) **Case IV :** If two pairs of roots are

Complex and all others are real and different,

Say $m_1 = m_3 = \alpha + i\beta, \quad m_2 = m_4 = \alpha - i\beta$

m_5, m_6 ….. are real and different then

C.F. is $Y_c = e^{\alpha x}(c_1 + c_2x)\cos\beta x + (c_3 + c_4x)\sin\beta x + c_5e^{m_5x} + \ldots + c_ne^{m_nx}$

1.3 Particular Integral (P.I. = Y_p) :

From equation (2)

$(D^n + a_1 D^{n-1} + a_2D^{n-2} + \ldots + a_{n-1}D + a_n)\, y = F(x)$ which can be written as,

$$f(D)\,y = F(x), \text{ where}$$

$$f(D) = D^n + a_1D^{n-1} + \ldots + a_n \text{ , particular integral is defined as}$$

P.I. $= Y = \frac{1}{f(D)}F(x)$ which is a function of x, free from arbitrary constant, and satisfies the given LDE with constant coefficients; i.e. $f(D)\left\{\frac{1}{f(D)}F(x)\right\} = F(x) \Rightarrow f(D)y = F(x)$. Depending upon the nature of F(x) there are different methods to find the particular integrals, which are short methods and general methods.

If $F(x) = e^{ax}$ then

P.I. $= y = \frac{1}{f(D)}e^{ax}$

Case I : If $f(a) \neq 0$, i.e. replace D by a

then P.I. $= y = \frac{1}{f(a)}e^{ax}$

$$\Rightarrow \quad \frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}$$

$$= \frac{1}{t(a)}e^{ax}$$

Case II : If $f(a) = 0$, the above case fails r times then $f(D) = (\theta - a)^r f_1(D)$

where, $f_1(a) \neq 0$, so that

P.I. $= y = \frac{1}{(D-a)^r}\frac{e^{ax}}{f_1(D)}$

$$= \frac{1}{f_1(a)}\frac{1}{(D-a)^r}e^{ax}$$

$$= \frac{1}{f_1(a)}\frac{x^re^{ax}}{r!}$$

Illustrative Examples

Example : 1

Solve : $(D^2 + 4)y = e^x$

Solution : Let $(D^2 + 4)y = e^x$ …. (1)

For finding C.F. the complimentary function.

Consider it's C.E. complimentary equation

$(D^2 + 4)\,y = 0$ …. (2)

Equating to zero the coefficient of y

i.e. $D^2 + 4 = 0$ A.E. (Auxiliary Equation)

$\Rightarrow \quad m^2 + 4 = 0$

$\Rightarrow \quad m^2 = -4 \qquad \Rightarrow \quad m = \pm\sqrt{-4} = 0 \pm 2i$ Note

$= \alpha \pm \beta i$ Note

It has a pair of complex roots with $\alpha = 0$ and $\beta = 2$

$\therefore$ C.F. is $y_c = e^{\alpha x}(c_1\cos\beta x + c_2\sin\beta x)$

$= e^{0x}(c_1\cos 2x + c_2\sin 2x) \qquad (\because e^{0x} = 1)$

$\therefore \quad y_c = c_1\cos 2x + c_2\sin 2x$ …. (3)

To find particular integral P.I. = y_p :

By definition P.I. is $y_p = \dfrac{1}{f(D)}F(x) = \dfrac{1}{D^2+4}e^{1x}$

Here, $F(x) = e^x = e^{ax}$ type

with a = 1

$\therefore \quad y_p = \dfrac{1}{D^2+4}e^{1x} = \dfrac{1}{1^2+4}e^{1x}$ replace D by a = 1

$= \dfrac{1}{1+4}e^x = \dfrac{1}{5}e^x$

$y_p = \dfrac{1}{5}e^x$ …. (4)

General solution of (1) is

G.S. $= y_c + y_p$ = C.F. + P.I. = y

$\therefore$ $$\boldsymbol{y = c_1\cos 2x + c_2\sin 2x + \frac{1}{5}e^x}$$

Example : 2

$(D^3 + 1)y = 3 + e^{-x} + 5e^{3x}$

Solution : Let $(D^3 + 1)\,y = 3 + e^{-x} + 5e^{3x}$ …. (1)

For finding C.F.

Consider C.E.

$$(D^3 + 1)\, y = 0$$

$$\Rightarrow \quad D^3 + 1 = 0 \quad \text{A.E.}$$

OR

$$m^3 + 1 = 0 \qquad \left(\because D^3 + a^3 = (D + a)(D^2 - aD + a^2) \quad D^3 + 1^3 = (D + 1)(D^2 - D + 1) \right.$$

$$\Rightarrow \quad (m + 1)(m^2 - m + 1) = 0 \qquad \text{and}$$

$$\therefore \quad m + 1 = 0$$

$$\text{or} \quad m^2 - m + 1 = 0 \qquad \left. ax^2 + bx + c = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$$

$$\Rightarrow \quad m = -1 \quad \text{or} \quad m = \frac{1 \pm \sqrt{1 - 4 \times 1 \times 1}}{2 \times 1}$$

$$= \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}\, i}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i \qquad \textit{Complex roots}$$

$$= \alpha \pm \beta i$$

$\therefore$ $m_1 = -1$ is real root and $m_2 = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$m_3 = \frac{1}{2} - \frac{\sqrt{3}}{2} i$ are complex roots.

$$\therefore \quad \text{C.F. is} \quad y_c = c_1 e^{-1x} + e^{\frac{1}{2}x}\left(c_2 \cos \frac{\sqrt{3}}{2} x + c_3 \sin \frac{\sqrt{3}}{2} x \right) \qquad \text{…. (2)}$$

For finding particular Integral :

$$\text{P.I.} = y = \frac{1}{f(D)} F(x) = \frac{1}{(D^3 + 1)} (3e^{0x} + e^{-x} + 5e^{3z})$$

$$= 3 \frac{1}{D^3 + 1} e^{0x} + \frac{1}{D^3 + 1} e^{-x} + 5 \frac{1}{D^3 + 1} e^{3x}$$

$$= 3 \frac{1}{0 + 1} e^{0x} + \frac{1}{(D + 1)(D^2 - D + 1)} e^{-x} + 5 \frac{1}{3^3 + 1} e^{3x}$$

$$= 3 + \frac{1}{(-1)^2 - (-1) + 1} \frac{1}{(D + 1)} e^{-x} + \frac{5}{28} e^{3x}$$

$$= 3 + \frac{1}{3} \frac{x^1}{11} e^{-x} + \frac{5}{28} e^{3x}$$

$$\therefore \quad y_p = 3 + \frac{x}{3} e^{-x} + \frac{5}{28} e^{3x} \qquad \text{….. (3)}$$

$$\therefore \quad \text{G.S.} = y = y_c + y_p$$

$$= \mathbf{c_1 e^{-x} + e^{\frac{x}{2}}\left(c_2 \cos \frac{\sqrt{3}}{2} x + c_3 \sin \frac{\sqrt{3}}{2} x \right) + 3 + \frac{x}{3} e^{-x} + \frac{5}{28} e^{3x}}$$

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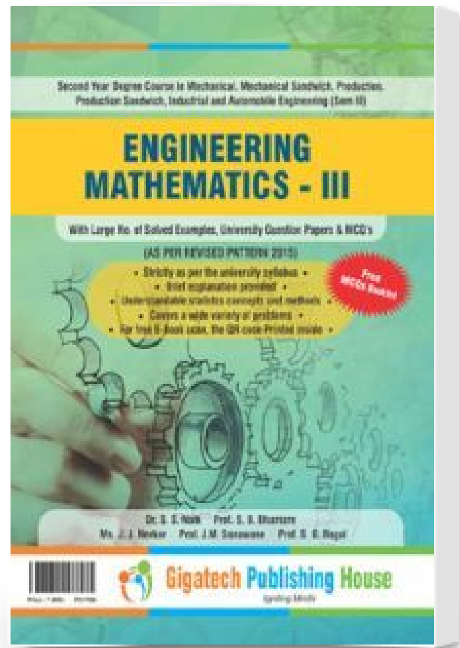

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