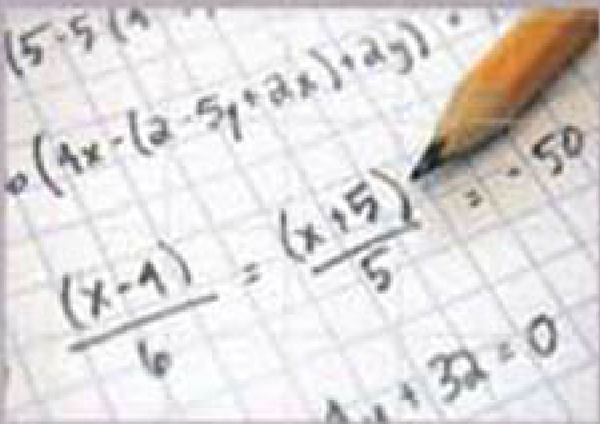
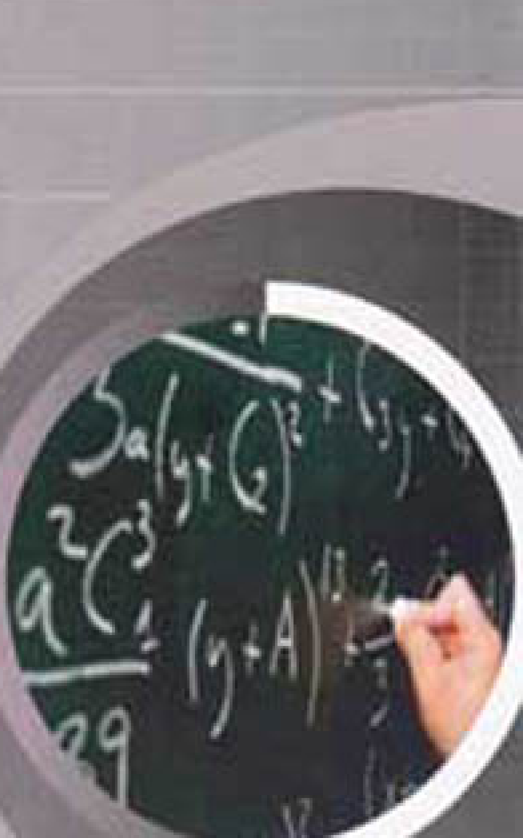
As Per PCI Regulations

First Year B. Pharm. • Semester-I

REMEDIAL MATHEMATICS

Dr. P. K. Sharma





NIRALI PRAKASHAN
ADVANCEMENT OF KNOWLEDGE

A Text Book Of

REMEDIAL MATHEMATICS

As Per PCI Regulations

FIRST YEAR B. PHARM
Semester I

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This book is dedicated to my beloved daughter ' Kanishka Sharma'

– P. K. Sharma

Preface

It is indeed a matter of great pride for us that, the *Pharmacy Council of India (PCI), New Delhi has framed Bachelor of Pharmacy (B. Pharm.)* course regulations 2014. The Remedial Mathematics is a very important subject at First year of the course. It gives me a great pleasure to present this book in the hand of students. The book is strictly written as per syllabus framed by PCI under Section 6, 7 & 8 of Regulation 2014 and I had made an attempt to make it simple and understandable to the students. The sequence of syllabus content is designed unit wise imparting better understanding of subject matter.

The objective of this book is to understand Mathematics in a very simple way for the students of Pharmacy. This book contains a sufficient number of solved examples and figures (where it is required). All the important questions asked in various universities are included in this book and broadly covers the latest syllabus of B. Pharm.

I am thankful to Mr. Jignesh Furia, Santosh Bare, Anagha Kaware, Anjali Mulye of Nirali Prakashan, Pune and Staff of Nirali Prakashan for bringing out nicely printed book.

PUNE **Dr. P. K. SHARMA**

Syllabus

UNIT - I **(06 Hours)**

- **Partial Fraction :**

  Introduction, Polynomial, Rational fractions, Proper and Improper fractions, Partial fraction, Resolving into Partial fraction, Application of Partial fraction in chemical kinetics and pharmacokinetics.

- **Logarithms :**

  Introduction, Definition, Theorems/Properties of logarithms, Common logarithms, Characteristics and Mantissa, Worked examples, Application of logarithm to solve pharmaceutical problems.

- **Function :**

  Real valued function, Classification of real valued functions.

- **Limits and Continuity :**

  Introduction, Limit of a function, Definition of limit of a function ($\varepsilon - \delta$ definition), $\lim\limits_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$, $\lim\limits_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$.

UNIT - II **(06 Hours)**

- **Matrices and Determinants :**

  Introduction matrices, Types of matrices, Operation on matrices, Transpose of a matrix, Matrix multiplication, Determinants, Properties of determinants, Product of determinants, Minors and co-factors, Adjoint or adjugate of a square matrix, Singular and non-singular matrices, Inverse of a matrix, Solution of system of linear of equations using matrix method, Cramer's rule, Characteristic equation and roots of square matrix, Cayley-Hamilton theorem, Application of matrices in solving pharmacokinetic equations.

UNIT - III **(06 Hours)**

- **Calculus :**

  Differentiation : Introductions, Derivative of a function, Derivative of a constant. Derivative of a product of a constant and a function, Derivative of the sum of or difference of two functions, Derivative of the product of two functions (product formula) – Without proof, Derivative of x^n w.r.t. x, where n is any rational number, Derivative of e^x, Derivative of $\log_e x$, Derivative of a^x, Derivative of trignometric functions from first principles (without proof), Successive differentiation, Conditions for a function to be a maximum or a minimum at a point. Application.

UNIT - IV **(06 Hours)**

- **Analytical Geometry :**

  Introduction : Signs of the co-ordinates, Distance formula.

  Straight line : Slope or gradient of a straight line, Conditions for parallelism and perpendicularity of two lines, Slope of a line joining two points, Slope – intercept form a straight line.

  Integration : Introduction, Definition, Standard formula, Rules of integration, Method of substitution, Method of partial fractions, Integration by parts, Definite integrals, Application.

UNIT - V **(06 Hours)**

- **Differential Equations :**

  Some basic definitions, Order and degree, Equations in separable from, Homogeneous equations, Linear differential equations, Exact equations, Application in solving pharmacokinetic equations.

- **Laplace Transform :**

  Introduction, Definition, Properties of Laplace transform, Laplace transforms of elementary functions, Inverse Laplace transforms, Laplace transform of derivatives, Application to solve linear equations, Application in solving chemical kinetics and pharmacokinetics equations.

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Contents

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UNIT I

Chapter...1

PARTIAL FRACTION

♦ **OBJECTIVES** ♦

On successful completion of the learning of this chapter students should be able to :

- **Define "polynomial" and classify it.**
- **Write rational functions as the sum of functions by the process of partial fraction decomposition.**
- **Evaluate integral using partial fraction.**

1.1 INTRODUCTION

Splitting up of a a given fraction into simpler fractions called partial fractions.

Let us consider a fraction $\frac{2x + 5}{(x + 2)(x + 3)}$.

We can write the given fraction as

$$\frac{2x + 5}{(x + 2)(x + 3)} = \frac{1}{x + 2} + \frac{1}{x + 3}$$

The fractions $\frac{1}{x + 2}$ and $\frac{1}{x + 3}$ are called the partial fractions of the given fraction $\frac{2x + 5}{(x + 2)(x + 3)}$.

In this chapter, we study the methods of resolving a given fraction into partial fractions.

1.2 POLYNOMIAL

An algebraic expression of the form $P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_n$ is a polynomial of degree n (highest power of x) in x. Where $a_0 \neq 0, a_1, a_2, \ldots a_n$ are real constants.

Example : $f(x) = x^3 + 2x^2 + 5x + 9$ is a polynomial of degree 3 in x.

1.3 RATIONAL FRACTION

The quotient $\frac{P(x)}{Q(x)}$ of two polynomials P(x) and Q(x), $Q(x) \neq 0$ is called a rational fraction.

Example : $\dfrac{x^2 + 2x + 3}{x^3 + 3x^2 + 3x + 1}$ is a rational fraction.

Here, $P(x) = x^2 + 2x + 3$ and $Q(x) = x^3 + 3x^2 + 3x + 1$ and $Q(x) \neq 0$ also.

1.4 PROPER AND IMPROPER FRACTIONS

The rational fraction $\dfrac{P(x)}{Q(x)}$ is said to proper, if the degree of numerator P(x) is less than the degree of denominator Q(x).

Example : $\dfrac{2x + 1}{x^2 + 6x + 5}$

Here, $P(x) = 2x + 1$, degree of $P(x) = 1$

and $Q(x) = x^2 + 6x + 5$, degree of $Q(x) = 2$

$\therefore$ $\dfrac{2x + 1}{x^2 + 6x + 5}$ is a proper fraction.

If the degree of numerator is greater than or equal to the degree of denominator, then the fraction is known as improper fraction.

Example 1 : $\dfrac{x^2 + 3x + 2}{x^2 - 3x + 9}$

Here, $P(x) = x^2 + 3x + 2$, degree = 2

and $Q(x) = x^2 - 3x + 9$, degree = 2

$\because$ Degree of numerator P(x) is equal to the degree of denominator Q(x).

$\therefore$ The given fraction is improper.

Example 2 : $\dfrac{x^3 + 3x^2 + 6x + 9}{x^2 + 2x - 1}$

Here degree of numerator = 3 and degree of denominator = 2.

$\therefore$ The given fraction is improper.

Note : Improper fraction can be convert into proper fraction by dividing numerator by denominator.

1.5 PARTIAL FRACTION

Any proper rational fraction $\dfrac{f(x)}{g(x)}$ can be expressed as the sum of rational fraction, each having a simple factor of g(x). Each such fraction is called a partial fraction and the process of obtaining them is called the decomposition or resolution of $\dfrac{f(x)}{g(x)}$ into partial fractions.

1.6 METHODS OF RESOLVE INTO PARTIAL FRACTIONS

The decomposition of $\dfrac{f(x)}{g(x)}$ into partial fractions depends on the nature of the factors of denominator i.e. g(x).

Case I : *When denominator is expressible as the product of non-repeating linear factors.*

Let $g(x) = (x - a_1)(x - a_2) \dots (x - a_n)$. Then we assume that

$$\frac{f(x)}{g(x)} = \frac{A_1}{x - a_1} + \frac{A_2}{x - a_2} + \frac{A_3}{x - a_3} + \dots + \frac{A_n}{x - a_n}$$

where, $A_1, A_2, A_3, \dots, A_n$ are constants and can be determined by comparing the coefficients of various powers of x or by substituting $x = a_1, a_2, \dots, a_n$ in the L.H.S. and R.H.S. after simplification (i.e. in numerator).

SOLVED EXAMPLES

Example 1.1 : Resolve $\frac{2x + 3}{x^2 - 2x - 3}$ into partial fractions.

Solution : Here the degree of numerator is lower than the degree of denominator, therefore the given rational fraction is a proper fraction.

Let $$\frac{2x + 3}{x^2 - 2x - 3} = \frac{2x + 3}{(x - 3)(x + 1)} \equiv \frac{A}{x - 3} + \frac{B}{x + 1}$$

OR $$2x + 3 \equiv A(x + 1) + B(x - 3)$$

Put x = 3, we get $$9 = 4A \Rightarrow A = \frac{9}{4}$$

Again, put x = – 1, we get

$$1 = -4B \Rightarrow B = -\frac{1}{4}$$

$$\therefore \quad \frac{2x + 3}{x^2 - 2x - 3} = \frac{9}{4(x - 3)} - \frac{1}{4(x + 1)}$$

Example 1.2 : Resolve $\frac{x^3 - 6x^2 + 10x - 2}{x^2 - 5x + 6}$ into partial fractions.

Solution : Here, the degree of numerator is higher than the degree of denominator, so the rational fraction is improper fraction. Therefore dividing out, we get

$$\frac{x^3 - 6x^2 + 10x - 2}{x^2 - 5x + 6} = x - 1 + \frac{(4 - x)}{x^2 - 5x + 6}$$

Now, $\frac{4 - x}{x^2 - 5x + 6}$ is a proper fraction.

So, let $$\frac{4 - x}{x^2 - 5x + 6} = \frac{4 - x}{(x - 2)(x - 3)} \equiv \frac{A}{x - 2} + \frac{B}{x - 3}$$

OR $$4 - x = A(x - 3) + B(x - 2)$$

Put x = 3, we get $$1 = B \Rightarrow B = 1$$

Again, put x = 2, we get

$$2 = -A \Rightarrow A = -2$$

$$\therefore \quad \frac{4 - x}{x^2 - 5x - 6} = -\frac{2}{x - 2} + \frac{1}{x - 3}$$

and $$\frac{x^3 - 6x^2 + 10x - 2}{x^2 - 5x - 6} = x - 1 - \frac{2}{x - 2} + \frac{1}{x - 3}$$

$$\begin{array}{r|l} & x - 1 \\ x^2 - 5x + 6 & x^3 - 6x^2 + 10x - 2 \\ & x^3 - 5x^2 + 6x \\ & - \quad + \quad - \\ \hline & -x^2 + 4x - 2 \\ & -x^2 + 5x - 6 \\ & + \quad - \quad + \\ \hline & -x + 4 \end{array}$$

Example 1.3 : Find partial fractions of $\frac{x}{(x + 1)(x - 1)(x + 2)}$.

Solution : Here the given rational function is a proper fraction.

So, let $\frac{x}{(x + 1)(x - 1)(x + 2)} \equiv \frac{A}{x + 1} + \frac{B}{x - 1} + \frac{C}{x + 2}$

OR $x \equiv A(x - 1)(x + 2) + B(x + 1)(x + 2) + C(x + 1)(x - 1)$

Put x = 1, we get $1 = 6B \Rightarrow B = \frac{1}{6}$

Put x = – 1, we get $-1 = -2A \Rightarrow A = \frac{1}{2}$

Put x = – 2, we get $-2 = 3C \Rightarrow C = -\frac{2}{3}$

$\therefore \quad \frac{x}{(x + 1)(x - 1)(x + 2)} = \frac{1}{2(x + 1)} + \frac{1}{6(x - 1)} - \frac{2}{3(x + 2)}$

Example 1.4 : Put $\frac{x^2}{(x^2 + 1)(x^2 + 4)}$ into partial fractions.

Solution : We put $x^2 = y$. Then $\frac{x^2}{(x^2 + 1)(x^2 + 4)} = \frac{y}{(y + 1)(y + 4)}$.

Let $\frac{y}{(y + 1)(y + 4)} \equiv \frac{A}{y + 1} + \frac{B}{y + 4}$ OR $y \equiv A(y + 4) + B(y + 1)$

Put y = – 1, we get $-1 = 3A \Rightarrow A = -\frac{1}{3}$

Put y = – 4, we get $-4 = -3B \Rightarrow B = \frac{4}{3}$

$\therefore \quad \frac{y}{(y + 1)(y + 4)} = -\frac{1}{3(y + 1)} + \frac{4}{3(y + 4)}$

and Hence $\frac{x^2}{(x^2 + 1)(x^2 + 4)} = -\frac{1}{3(x^2 + 1)} + \frac{4}{3(x^2 + 4)}$

Case-II : *When the denominator g(x) is expressible as the product of linear factors such that some of them are repeating.*

Let $g(x) = (x - a)^k (x - a_1)(x - a_2) \ldots (x - a_r)$

Then assume that

$$\frac{f(x)}{g(x)} \equiv \frac{A_1}{(x - a)} + \frac{A_2}{(x - a)^2} + \frac{A_3}{(x - a)^3} + \ldots \frac{A_k}{(x - a)^k} + \frac{B_1}{x - a_1} + \frac{B_2}{x - a_2} + \ldots\ldots \frac{B_r}{(x - a_r)}$$

Example 1.5 : Resolve $\frac{2x^2}{(x - 1)^3 (x + 1)}$ into partial fractions.

Solution : Let $\frac{2x^2}{(x - 1)^3 (x + 1)} \equiv \frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{(x - 1)^3} + \frac{D}{x + 1}$

OR $2x^2 \equiv A(x - 1)^2 (x + 1) + B(x - 1)(x + 1) + C(x + 1) + D(x - 1)^3$

OR $2x^2 \equiv A(x^3 - x^2 - x + 1) + B(x^2 - 1) + C(x + 1) + D(x^3 - 1 - 3x^2 + 3x)$

OR $2x^2 \equiv (A + D)x^3 + (-A + B - 3D)x^2 + (-A + C + 3D)x + A - B + C - D$