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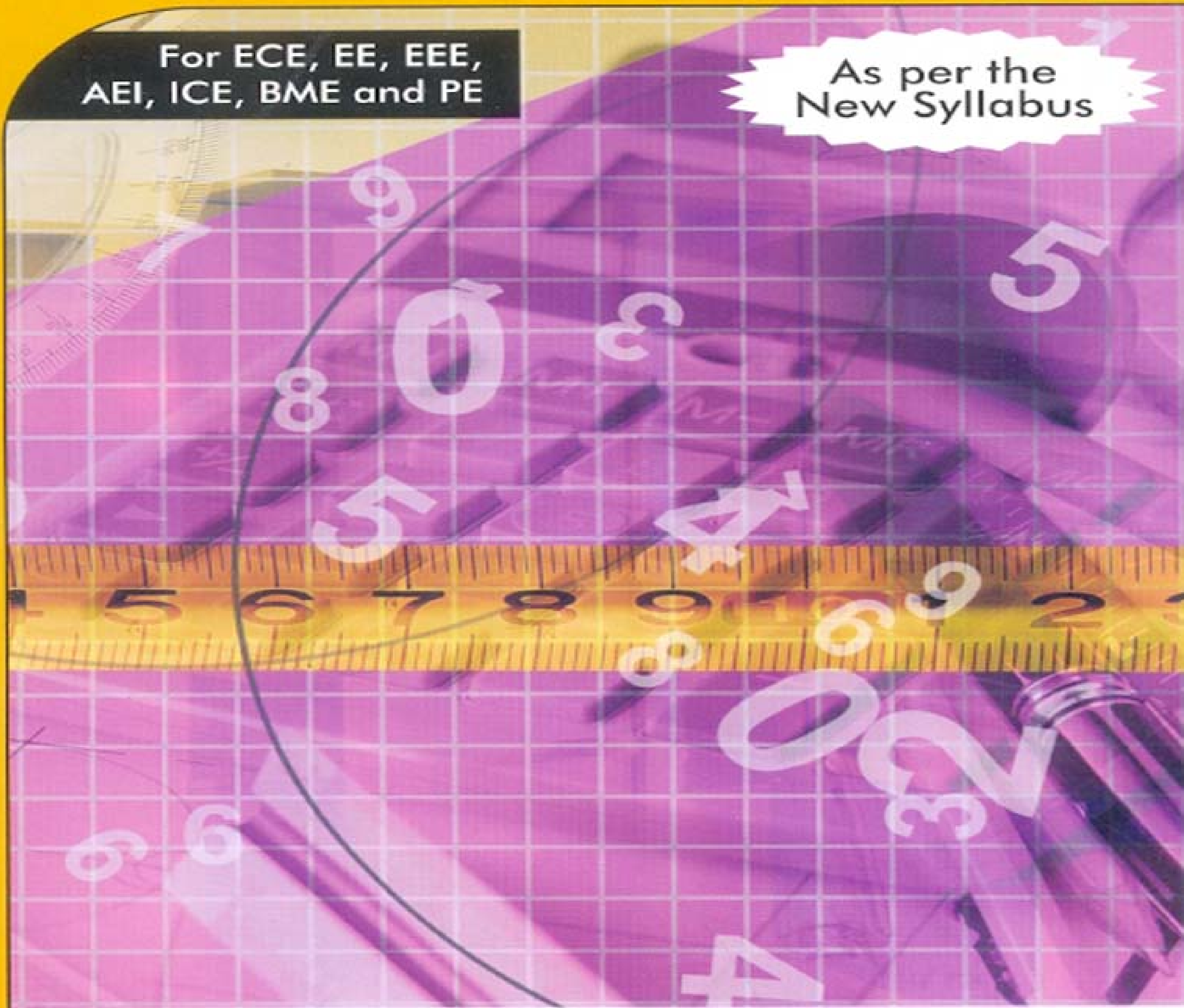

• *Bikas Chandra Bhui* • *Dipak Chatterjee*

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Dr Bikas Chandra Bhui
Head, Mathematics Department,
Meghnad Saha Institute of Technology, Kolkata

Prof D Chatterjee
Distinguished Professor,
St Xavier's College, Kolkata

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Preface

It gives us immense pleasure in presenting to our readers this third volume (for two streams 301 and 302 of the third semester) of Engineering Mathematics specially made for their courses. The incessant requests from students, guardians and teachers prompted us to work hard on these books after the grand reception received by the first two volumes. The books 3A and 3B are meant for the separate streams—301 for Computer Science and IT, and 302 for Electrical and Electronics Engineering and a few other disciplines, respectively.

Like its predecessors in this series, this duo of Volumes 3A and 3B too has the theories explained in a simple and elegant way, with numerous problems worked out so as to make concepts fully clear, and a carefully prepared set of exercises containing the choicest problems for each chapter. The number of multiple choice questions will help not only in assimilating the various mathematical concepts but also in scoring high marks in different competitive examinations.

As the university syllabus contains a number of topics some of which are not directly connected, we have written the text keeping that in mind for the benefit of the students, and insist that students do not skip those topics but study them in the same order as given in the book.

We acknowledge with deep gratitude the effort of all those professors who made suggestions for the improvement of the book, while appreciating the overall approach adopted by us.

Prof B C Bhui
Prof D Chatterjee

Contents

CHAPTER

1

Probability Theory

There is hardly any area of science where probability theory does not come into play. In computer science, the role of probability is immense. Therefore, a brief account of the theory is presented here.

Probability can be studied through three approaches, *viz.,* the classical approach developed by the French School, the empirical approach developed by the German School, and the axiomatic approach developed by the Russion school. Though the axiomatic development is by far the best and is applicable in the broadest situation, we stress on the classical one because of its simplicity and quicker appeal to intuition. Once the classical approach is understood, its generalization poses no problem as such.

To begin with, we must have a clear idea about the following notions.

1.1 SOME DEFINITIONS

The theory of probability must start with the notion of a random experiment.

Experiment: An act or activity which we perform to make critical observations about any physical or socio-economic phenomenon is called an *experiment*.

An experiment which results in many outcomes in such a way that nothing can be said in advance when the experiment is performed is called a *random experiment*.

Thus, the tossing of a coin is a random experiment as in advance nothing can be said which face will turn up.

By the same logic the tossing of two or more coins is also a random experiment. The outcomes of the tossing of two coins can be jot down as $\{HH, HT, TH, TT\}$ where H stands for the head and T stands for the tail of a coin.

The casting of a dice (a six face cube) is a random experiment with six outcomes, namely, 1, 2, 3, 4, 5 and 6.

Sample Space: The set of all outcomes of a random experiment is called the *sample space* or *event space* of the experiment. This is usually denoted by Ω or S.

Thus, Ω(Tossing a coin) = $\{H, T\}$

Ω(Tossing two coins) = $\{HH, HT, TH, TT\}$.

Ω(Tossing three coins) = $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Ω(Casting a die) = $\{1, 2, 3, 4, 5, 6\}$.

Events: The outcomes of a random experiment are called *simple* or *elementary* events of the experiment. Any combination of two or more elementary events is called a *compound* event. For example, the experiment of casting a dice has six elementary events, *viz.,* $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$ and $\{6\}$. But the event of getting a prime number is composed of the events $\{2\}$, $\{3\}$ and $\{5\}$ and hence is a compound event, described by $\{2, 3, 5\}$. Similarly, the event of getting an even number is a compound event; the event of getting a multiple of 3 is a compound event.

In the random experiment of tossing two coins, the event of one head is a compound event, represented by $\{HT, TH\}$; the event of at least one head is also a compound event written as $\{HT, TH, HH\}$.

Note: Events are nothing but subsets of the sample space. Elementary events are singleton subsets only.

Froms of Events: Events, simple or compound, are said to be *equally likely* if there is no reason to prefer any of them to others in respect of occurrence, *i.e.*, they have the same likelihood of their occurrence.

Thus, in the experiment of tossing a coin, the event of a head and the event of a tail are equally likely. In the experiment of casting an unbiased dice, all the elementary events are equally likely. Similarly, in the same experiment the event of an odd number, the event of an even number and the event of a prime number are all equally likely. Note that set theoretically, equally likely events have the same cardinality where the cardinality of the event A is the number of elementary events favourable to A. In the experiments of casting an unbiased dice, the events $\{1, 2\}$, $\{2, 3\}$, $\{3, 4\}$, $\{4, 5\}$ are equally likely, just as the events $\{2, 4, 6\}$, $\{1, 3, 5\}$ and $\{2, 3, 5\}$ are equally likely.

Events, simple or compound, are said to be *mutually exclusive* if the occurrence of any one of them precludes the occurrence of all others at the same time.

For example, all the elementary events in the experiment of casting a die are mutually exclusive since when any one of the six faces turns up, no other face can turn up at the same time. Similarly, the event of an even number, *i.e.*, $\{2, 4, 6\}$ and the event of an odd number are mutually exclusive but the events $\{1, 3, 5\}$ and $\{2, 3, 5\}$ are not mutually exclusive since when 3 turns up, both the events occur simultaneously.

Note that mutual exclusiveness implies (pairwise) disjointness in set theory.

Thus, the events {1, 3, 5} and {2, 3, 5} are not mutually exclusive as $\{1, 3, 5\} \cap \{2, 3, 5\} = \{3, 5\} \neq \phi$ where ϕ is the null set, but the events {1, 2}, {3, 4} and {5, 6} are mutually exclusive.

A family of events, simple or compound, is said to be *exhaustive* if the family includes all outcomes of the experiments. Thus, in the experiment of casting a die, the family of all elementary events is exhaustive but the family of [{1, 2}, {2, 3}, {3, 4}, {4, 5}] is not exhaustive because the latter does not include the outcome {6}.

Set theoretically, a family of events is exhaustive if their union is the sample space. Thus $A_1, A_2, ..., A_n$ form an exhaustive family if $\bigcup_{i=1}^{n} A_i = \Omega$.

An event A is said to be *favourable* to the event B if the occurrence of A entails the occurrence of B. Set theoretically, A is favourable to B if $A \subset B$.

Thus, the elementary events favourable to the event {3, 6} in the experiment of casting a die are {3} and {6}.

1.2 CLASSICAL DEFINITION OF PROBABILITY

We are now in a position to define probability as per Laplace's classical approach.

Laplace's Definition: If in a random experiment, there are n equally likely, mutually exclusive and exhaustive elementary events of which m are favourable to the occurrence of an event A, then the *probability of occurrence* of A, denoted by $P(A)$, is defined as the ratio m/n.

Thus, $$P(A) = \frac{m}{n} = \frac{\text{No. of elementary events favourable to } A}{\text{Total no. of elementary events in the experiment}}$$

$$= \frac{\text{card}(A)}{\text{card}(\Omega)}.$$

Four observations may be made from the definition:

1. $0 \leq P(A) \leq 1$.

This is clear as m and n being positive integers and $0 \leq m \leq n$, we get

$$\frac{0}{n} \leq \frac{m}{n} \leq \frac{n}{n} \quad \text{or} \quad 0 \leq P(A) \leq 1.$$

2. $P(A) = 1 - P(A^c)$ where $A^c = \Omega - A$ denotes the complementary event. This follows from the fact that card $(A) = m$ implies card $(A^c) = n - m$.

Hence, $$P(A^c) = \frac{\text{card}(A^c)}{\text{card}(\Omega)} = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

or $$P(A) = 1 - P(A^c).$$

Note: (*i*) A' or $\overline{A}$ is also complementary event of A

(*ii*) $A + B$ and AB are denote by $A \cup B$ and $A \cap B$

3. $P(\Omega) = 1$ and $P(\phi) = 0$.

4. If $A \cap B = \phi$, then $P(A \cup B) = P(A) + P(B)$

To see this, we note that

$$\text{card}(A \cup B) = \text{card}(A) + \text{card}(B) \text{ if } A \cap B = \phi.$$

Hence, $$P(A \cup B) = \frac{\text{card}(A \cup B)}{\text{card}(\Omega)} = \frac{\text{card}(A) + \text{card}(B)}{\text{card}(\Omega)}$$

$$= \frac{\text{card}(A)}{\text{card}(\Omega)} + \frac{\text{card}(B)}{\text{card}(\Omega)} = P(A) + P(B)$$

The following examples illustrate the definition above.

Example 1. From a class of 5 Punjabi, 6 Tamil, 3 Bengali students, a student is chosen at random. Find the probability that a Tamil has been chosen.

Solution: Since there are 14 students and each has an equal chance of being chosen, there are 14 equally likely, mutually exclusive and exhaustive elementary events of choosing a student. Clearly six of them are favourable to the event of getting a Tamil student.

Hence, $P(\text{a Tamil is chosen}) = \frac{6}{14} = \frac{3}{7}$.

Example 2. A die is cast. Find the probability of getting (*i*) an even number, (*ii*) an odd number, (*iii*) a prime number, (*iv*) a number less than 4, (*v*) a number more than 6, (*vi*) a positive number.

Solution: Let Ω denote the sample space. Then $\Omega = \{1, 2, 3, 4, 5, 6\}$. Let A denote the event of getting an even number, B denote the event of getting an odd number, C denote the event of getting a prime number, D denote the event of getting a number less than 4, E denote a number greater than 6 and F denote the event of getting a positive number.

Then, $A = \{2, 4, 6\}$, $B = \{1, 3, 5\}$, $C = \{2, 3, 5\}$, $D = \{1, 2, 3\}$, $E = \phi$, $F = \{1, 2, 3, 4, 5, 6\}$

Hence, $$P(\text{even no.}) = P(A) = \frac{\text{card}\,(A)}{\text{card}\,(\Omega)} = \frac{3}{6} = \frac{1}{2}$$

$$P(\text{odd no.}) = P(B) = \frac{\text{card}\,(B)}{\text{card}\,(\Omega)} = \frac{3}{6} = \frac{1}{2}$$

$$P(\text{prime no.}) = P(C) = \frac{\text{card}(C)}{\text{card}(\Omega)} = \frac{3}{6} = \frac{1}{2}$$

$$P(\text{no. less than } 4) = P(D) = \frac{\text{card}(D)}{\text{card}(\Omega)} = \frac{3}{6} = \frac{1}{2}$$

$$P(\text{no. greater than } 6) = P(E) = \frac{\text{card}(\phi)}{\text{card}(\Omega)} = \frac{0}{6} = 0$$

$$P(\text{positive no.}) = P(F) = \frac{\text{card}(F)}{\text{card}(\Omega)} = \frac{6}{6} = 1$$

Remark: An event whose probability is 0 is often referred to as an impossible event. An event whose probability is 1 is referred to as a sure event.

Example 3. A coin is tossed twice. Find the probability of getting (*i*) one head, (*ii*) at least one head, (*iii*) at most one head, (*iv*) no head.

Solution: Let Ω denote the sample space. Then $\Omega = \{HH, HT, TH, TT\}$. Let A denote the event of getting one head, B denote the event of getting at least one head, C denote the event of getting at most one head, D denote the event of getting no head.

Then clearly

$$A = \{HT, TH\}$$
$$B = \{HH, HT, TH\}$$
$$C = \{TT, HT, TH\}$$
$$D = \{TT\}.$$

Hence,

$$P(\text{one head}) = P(A) = \frac{\text{card}(A)}{\text{card}(\Omega)} = \frac{2}{4} = \frac{1}{2}$$

$$P(\text{at least one head}) = P(B) = \frac{\text{card}(B)}{\text{card}(\Omega)} = \frac{3}{4}$$

$$P(\text{at most one head}) = P(C) = \frac{\text{card}(C)}{\text{card}(\Omega)} = \frac{3}{4}$$

$$P(\text{no head}) = P(D) = \frac{\text{card}(D)}{\text{card}(\Omega)} = \frac{1}{4}$$

Example 4. A card is drawn from a full pack. Find the probability that it is (*i*) a king, (*ii*) an ace or a jack, (*iii*) a spade or a king.

Solution: Let Ω denote the sample space of a drawing a card from a full pack. Then $\Omega = \{S_A, S_2, ..., S_{10}, S_J, S_Q, S_K, C_A, C_2, ... C_{10}, C_J, C_Q, C_K, D_A, ..., D_K, H_A, ..., H_K\}$

Let A denote the event of getting a king, B denote the event of an ace or a jack, C denote the event of a spade or a king.

Then $A = \{S_K, C_K, D_K, H_K\}, B = \{S_A, C_A, D_A, H_A, S_J, C_J, D_J, H_J\}$ and
$C = \{S_A, S_2, ..., S_J, S_Q, S_K, C_K, D_K, H_K\}$

Hence, $$P(\text{king}) = P(A) = \frac{\text{card}(A)}{\text{card}(\Omega)} = \frac{4}{52}$$

$$P(\text{an ace or a jack}) = P(B) = \frac{\text{card}(B)}{\text{card}(\Omega)} = \frac{8}{52} = \frac{2}{13}$$

$$P(\text{a spade or a king}) = P(C) = \frac{\text{card}(C)}{\text{card}(\Omega)} = \frac{16}{52} = \frac{4}{13}$$

Example 5. Find the probability of getting 53 Sundays in a randomly chosen leap year.

Solution: A leap year has 366 days, *i.e.*, 52 weeks 2 days. The year may start on one of the seven days, which are equally likely, mutually exclusive and exhaustive. If a year starts on Sunday, in 52 weeks 52 Sundays will be there and the last 2 days will be Sunday and Monday, thus giving 53 Sundays altogether. If the year starts on Monday, the last two days will be Monday and Tuesday and so no extra Sunday will be there. Thus, only in two cases, *viz.*, when the year starts on Saturday and Sunday, the last two days include Sunday. Hence, out of 7 cases only two cases are favourable. Therefore, the required probability is 2/7.

1.3 COMBINATORICS IN PROBABILITY

Permutations and combinations have an important role in the computations of probabilities. The results that we shall make use of are the following:

Result 1. The number of distinct linear arrangements with n different objects, taking all together is $\lfloor n$.

Result 2. The number of distinct linear arrangements of n different objects taking r at a time is $\frac{\lfloor n}{\lfloor n-r}$, *i.e.*, ${}^nP_r$.

Result 3. The number of cyclic arrangements of n different objects taken all together is $\lfloor n-1$.

Result 4. The number of ways r objects can be chosen from n distinct objects is $\frac{\lfloor n}{\lfloor n-r \lfloor r}$, *i.e.*, ${}^nC_r$.

Result 5. The number of ways n distinct objects can be distributed to r boxes, repetitions being allowed is r^n.

Result 6. If n distinct objects, numbered 1 to n are distributed to n numbered boxes, one to each box, the number of ways all the objects are placed wrongly (such distributions are called derangements) is

$$\lfloor n \left\{ 1 - \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \dots + (-1)^n \frac{1}{\lfloor n} \right\}$$

Result 7. If n distinct objects numbered 1 to n are distributed to n numbered boxes, one to each box, the number of ways only r of them go to right places (such distributions are called r matchings) is

$${}^nC_r \lfloor n-r \left\{ 1 - \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \dots + (-1)^{n-r} \frac{1}{\lfloor n-r} \right\}$$

i.e.,

$$\frac{\lfloor n}{\lfloor r} \left\{ 1 - \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} - \dots + (-1)^{n-r} \frac{1}{\lfloor n-r} \right\}$$

We now show a few applications of the above results.

Example 1. From a class containing 40 boys and 25 girls, a committee of five is made choosing the students randomly. Find the probability, that the committee contains (*i*) exactly 2 girls, (ii) no girl.

Solution: Since five students are chosen at random from (40 + 25) i.e., 65 students, this selection can be done in ${}^{65}C_5$ ways which are equally likely, mutually exclusive and exhaustive as well.

Out of these the number of ways favourable for the occurrence of the event of having 2 girls is ${}^{25}C_2 \times {}^{40}C_3$. Note when 2 girls are selected in a committee, the remaining 3 must be boys; 2 girls can be chosen in ${}^{25}C_2$ ways and 3 boys can be chosen in ${}^{40}C_3$ ways and hence all 5 can be chosen in ${}^{25}C_2 \times {}^{40}C_3$ ways.

Therefore, the required probability = ${}^{25}C_2 \times {}^{40}C_3 / {}^{65}C_5$.

For the event of having no girl in a committee, we note that this can happen only when all the five chosen are boys, which can occur in ${}^{40}C_5$ ways.

Hence, the probability of having no girl = $\frac{{}^{40}C_5}{{}^{65}C_5}$.

Example 2. Five letters are written and their destination addresses are written on five envelopes. The letters are then put inside envelopes randomly. Find the probability that (*i*) none of the letters are placed in the right envelopes, (*ii*) only two letters are correctly placed.

Solution: We first note that five letters can be put in five envelopes, one in each, in $\underline{|5}$ ways.

The event of derangement can happen in $\underline{|5}\left\{1-\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}-\frac{1}{\underline{|3}}+\frac{1}{\underline{|4}}-\frac{1}{\underline{|5}}\right\}$ ways.

Hence, the probability of having letters placed in wrong envelopes

$$= \frac{\cancel{\underline{|5}}\left\{1-\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}-\frac{1}{\underline{|3}}+\frac{1}{\underline{|4}}-\frac{1}{\underline{|5}}\right\}}{\cancel{\underline{|5}}}$$

$$= \frac{1}{2}-\frac{1}{6}+\frac{1}{24}-\frac{1}{120}=\frac{11}{30}.$$

For the second part, we note that the number of 2-matchings is

$$\frac{\underline{|5}}{\underline{|2}}\left\{1-\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}-\frac{1}{\underline{|3}}\right\} \text{ or } \frac{\underline{|5}}{2}\left(\frac{1}{2}-\frac{1}{6}\right) \text{ or } \frac{\underline{|5}}{6}$$

Hence the probability of having letters placed wrongly except two

$$= \frac{\;\frac{\cancel{\underline{|5}}}{6}\;}{\cancel{\underline{|5}}} = \frac{1}{6}.$$

Example 3. A committee comprising of President, Secretary, Treasurer and two other members sit at a round table randomly. Find the probability that the Secretary and Treasurer sit on either side of the President.

Solution: Five members of the committee can sit at a round table in $\underline{|4}$ ways, *i.e.*, 24 ways.

Now, the event that Secretary and Treasurer sit on either side of the President can happen in $\underline{|2}\,\underline{|2}$ ways.

Hence, the required probability $= \frac{\underline{|2}\,\underline{|2}}{\underline{|4}} = \frac{1}{6}.$

Example 4. If n biscuits are distributed to N beggars at random, find the chance that a particular beggar receives $r\ (< n)$ biscuits.

Solution: Note that n biscuits can be distributed to N beggars in N^n ways, which are equally likely, mutually exclusive and exhaustive.

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Author : Bikas Chandra Bhui, Dipak Chatterjee