Revised Edition

ENGINEERING MATHEMATICS

VOLUME - II

Recommended For B. Tech. Second Year Students of JNTU, ANANTAPUR as per New Syllabus R13 (2013-2014)

[Common to Mechanical, Civil and Chemical]

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S. CHAND

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First Edition 2001 Subsequent Editions and Reprints 2004, 2005, 2007, 2010, 2011 (Twice), 2013

Sixth Revised Edition 2014

ISBN : 81-219-3658-6 **Code : 10A 478**

PRINTED IN INDIA

By Rajendra Ravindra Printers Pvt. Ltd., 7361, Ram Nagar, New Delhi -110 055 and published by S. Chand & Company Pvt. Ltd., 7361, Ram Nagar, New Delhi -110 055.

PREFACE TO THE SIXTH REVISED EDITION

It gives us great pleasure to bringout the **Sixth revised** edition of the book **"Engineering Mathematics (Volume - II)"**. The earlier editions have received positive response from the teachers and the students. This Textbook has been written strictly according to the revised syllabus **R13 (2013 - 14)** of Second Year B.Tech. students of JNTU, Anantapur.

This edition is a slight enlargement of the earlier edition, made by the inclusion of solutions to various problems appeared in the previous JNTU(A) Question Papers in the approximate places.

The treatment of all topics has been made as elementary as possible and in some instances with detailed explanation as the book is meant to be understood with a minimum effort on the part of the reader. However, as Mathematics is a subject to be understood and practiced, the students are advised to practice the exercises.

We are thankful to Sri. B.V.S.S. Sarma, author of Intermediate and Degree Text books on Mathematics for his cooperation in bringing out this book.

We are thankful to the Management Team and the Editorial Department of S. Chand & Company Pvt. Ltd., New Delhi for all help and support in the publication of this book.

We invite constructive criticism from the teachers for further improvement of the book.

AUTHORS

SYLLABUS R13

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, ANANTAPUR

B.Tech. - II Year (2014 - 15)

MATHEMATICS – II (13A54301)

(Common to Mechanical, Civil & Chemical)

UNIT – I

Rank – Echelon form, Normal form – Consistency of System of Linear equations. Linear Transformations

Complex Matrices : Hermitian, Skew-Hermitian and Unitary matrices and their properties.

Eigen Values, Eigen Vectors for both real and complex matrices. Cayley – Hamilton Theorem and its applications – Diagonolization of matrix. Calculation of powers of matrix. Quadratic forms – Reduction of Quadratic Form to canonical form and their nature.

UNIT – II

Solution of Algebraic and Transcendental Equations : Introduction - The Method of False Position– Newton-Raphson Method.

Interpolation : Introduction - Newton's forward and backward interpolation formulae – Lagrange's Interpolation formula.

Curve fitting : Fitting of a straight line – Second degree curve – Exponentional curve-Power curve by method of least squares.

UNIT – III

Numerical Differentiation and Integration – Trapezoidal rule – Simpson's 1/3 Rule – Simpson's 3/8 Rule.

Numerical solution of Ordinary Differential equations : Solution by Taylor's series-Picard's Method of successive Approximations-Euler's Method-Runge-Kutta Methods – Predictor-Corrector Method – Milne's Method.

UNIT – IV

Fourier Series: Determination of Fourier coefficients – Fourier series – Even and Odd functions – Fourier series in an arbitrary interval – Even and Odd periodic continuation – Half-range Fourier sine and Cosine expansions.

Fourier Integral Theorem – Fourier Sine and Cosine integrals. Fourier Transform – Fourier Sine and Cosine Transforms – Properties – Inverse transforms – Finite Fourier transforms.

UNIT – V

Formation of partial differential equations by elimination of arbitrary constants and arbitrary functions – Method of separation of variables – Solutions of one dimensional wave equation, Heat equation and two-dimensional Laplace's equation under initial and boundary conditions.

Reference Books :

1. Engineering Mathematics, Volume - II by
   Dr. T.K.V. Iyengar, Dr. B. Krishna Gandhi, S. Ranganatham and Dr. M.V.S.S.N. Prasad,
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CONTENTS

UNIT-I

1. Real and Complex Matrices & Linear System of Equations
2. Eigen Values & Eigen Vectors
3. Quadratic Forms

Chapter 1 Real and Complex Matrices & Linear System of Equations

1.1 REVIEW

The student is already familier with the definition and properties of matrices. Matrix is an inevitable tool in the study of many subjects like Physics, Mechanics, Statistics, Electronic circuits and Computers. Here we will briefly review some definitions and properties of matrices.

Matrix Definition: A system of mn numbers (real or complex) arranged in the form of an ordered set of m rows, each row consisting of an ordered set of n numbers between [ ] or ( ) or || || is called a matrix of order or type $m \times n$.

Each of mn numbers constituting the $m \times n$ matrix is called an element of the matrix.

Thus we write a matrix,

$$A = \begin{bmatrix} a_{11} & a_{12} \ldots\ldots\ldots\ldots a_{1n} \\ a_{21} & a_{22} \ldots\ldots\ldots\ldots a_{2n} \\ \ldots\ldots\ldots & \ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots & \ldots\ldots\ldots\ldots \\ a_{m1} & a_{m2} \ldots\ldots\ldots\ldots a_{mn} \end{bmatrix}_{m\times n} = \left[a_{ij}\right]_{m\times n}, \text{ where } 1 \le i \le m,\ 1 \le j \le n$$

In relation to a matrix, we call the numbers as scalars.

1.2 TYPES OF MATRICES

Definitions :

1. If $A = [a_{ij}]_{m \times n}$ and $m = n$, then A is called a ***Square matrix.*** A square matrix A of order $n \times n$ is sometimes called as a **n-rowed matrix** A or simply a square matrix of order n.

   e.g. $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ is 2nd order matrix.

2. A matrix which is not a square matrix is called a ***Rectangular matrix.***

   e.g. $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 4 \end{bmatrix}$ is a 2×3 matrix.

3. A matrix of order $1 \times m$ is called a ***Row matrix.***

   e.g. $[1 \quad 2 \quad 3]_{1 \times 3}$

4. A matrix of order $n \times 1$ is called a ***Column matrix.***

   e.g. $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}_{3\times 1}$

   Row and Column matrices are also called as ***Row*** and ***Column vectors*** respectively.

5. If $A = [a_{ij}]_{n\times n}$ such that $a_{ij} = 1$ for $i = j$ and $a_{ij} = 0$ for $i \neq j$, then A is called a ***Unit matrix***. It is denoted by I_n.

e.g. $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6. If $A = [a_{ij}]_{m \times n}$ such that $a_{ij} = 0 \ \forall \ i$ and j, then A is called a ***Zero matrix*** or a ***Null matrix***. It is denoted by O or more clearly $O_{m \times n}$.

e.g. $O_{2 \times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times 3}$

7. Diagonal Elements of a Square matrix and Principal Diagonal :

Definition : In a matrix $A = [a_{ij}]_{n \times n}$, the elements a_{ij} of A for which $i = j$ (*i.e.* $a_{11}, a_{22}, \ldots, a_{nn}$) are called the ***diagonal elements of A***. The line along which the diagonal elements lie is called the ***principal diagonal* of *A***.

8. A square matrix all of whose elements except those in leading diagonal are zero is called ***Diagonal matrix.*** If $d_1, d_2, \ldots, d_n$ are diagonal elements of a diagonal matrix A, then A is written as $A = diag(d_1, d_2, \ldots, d_n)$.

e.g. $A = \text{diag}(3, 1, -2) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

9. A diagonal matrix whose leading diagonal elements are equal is called a ***Scalar matrix***.

e.g. $B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

10. Equal Matrices :

Definition: Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if and only if

(*i*) A and B are of the same type (or oder) and (*ii*) $a_{ij} = b_{ij}$ for every i and j.

Algebra of Matrices :

11. Addition of two matrices :

Let $A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{m \times n}$ be two matrices. The matrix $C = [c_{ij}]_{m \times n}$ where $c_{ij} = a_{ij} + b_{ij}$, is called the sum of the matrices A and B. The sum of A and B is denoted by $A + B$.

Thus $[a_{ij}]_{m \times n} + [b_{ij}]_{m \times n} = [a_{ij} + b_{ij}]_{m \times n}$ and $[a_{ij} + b_{ij}]_{m \times n} = [a_{ij}]_{m \times n} + [b_{ij}]_{m \times n}$

12. Difference of Two Matrices :

If A, B are two matrices of the same type (order) then $A + (-B)$ is taken as $A - B$.

13. Multiplication of a Matrix by a Scalar :

Let A be a matrix. The matrix obtained by multiplying every element of A by K, a scalar, is called the product of A by K and is denoted by KA or AK.

Thus if $A = [a_{ij}]_{m \times n}$, then

$KA = [Ka_{ij}]_{m \times n}$ and $[Ka_{ij}]_{m \times n} = K\,[a_{ij}]_{m \times n} = KA$.

Properties :

(*i*) $OA = O$ (null matrix), $(-1)A = -A$, called the negative of A.

(*ii*) $K_1(K_2\, A) = (K_1K_2)\, A = K_2(K_1A)$ where K_1, K_2 are scalars.

(*iii*) $KA = O \Rightarrow A = O$ if $K \neq 0$.

(*iv*) $K_1A = K_2A$ and A is not a null matrix $\Rightarrow K_1 = K_2$.

14. Matrix Multiplication :

Let $A = [a_{ik}]_{m \times n}$ and $B = [b_{kj}]_{n \times p}$. Then the matrix $C = [c_{ij}]_{m \times p}$ where $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$ is called the **product of the matrices *A* and *B*** in that order and we write $C = AB$.

In the product AB, the matrix A is called the **pre-factor** and B the **post-factor**.

If the number of columns of A is equal to the number of rows in B then the matrices are said to be conformable for multiplication in that order.

15. Positive Integral Powers of Square Matrices :

Let A be a square matrix. Then A^2 is defined as $A.A$. Now, by the Associative law,

$A^2 A = (AA)A = A(AA) = AA^2$ so that we write

$A^2 A = AA^2 = AAA = A^3$

Similarly we have $AA^{m-1} = A^{m-1}A = A^m$, where m is a positive integer.

Further we have $A^m A^n = A^{m+n}$ and $(A^m)^n = A^{mn}$ where m, n are positive integers.

Note : $I^n = I$, $O^n = O$

Theorem 1: Matrix multiplication is associative.

i.e., if A, B, C are matrices, then $(AB)C = A(BC)$. **[JNTU 2002 (Set No. 2)]**

Proof: Let $A = [a_{ij}]_{m \times n}$, $B = [b_{jk}]_{n \times p}$ and $C = [c_{kl}]_{p \times q}$

Then $AB = [u_{ik}]_{m \times p}$, where $u_{ik} = \sum_{j=1}^{n} a_{ij}\, b_{jk}$ ... (1)

Also $BC = [v_{jl}]_{n \times q}$, where $v_{jl} = \sum_{k=1}^{p} b_{jk}\, c_{kl}$ ... (2)

Now, $A(BC)$ is an $m \times q$ matrix and $(AB)C$ is also an $m \times q$ matrix.

Let $A(BC) = [w_{il}]_{m \times q}$ where w_{il} is the $(i, j)^{th}$ element of $A(BC)$.

Then $$w_{il} = \sum_{j=1}^{n} a_{ij}\, v_{jl} = \sum_{j=1}^{n}\left[a_{ij}\left\{\sum_{k=1}^{p} b_{jk}\, c_{kl}\right\}\right] \quad \text{[by (2)]}$$

$$= \sum_{k=1}^{p}\left[\left\{\sum_{j=1}^{n} a_{ij}\, b_{jk}\right\} c_{kl}\right] \quad [\because \text{Finite summations can be interchanged}]$$

$$= \sum_{k=1}^{p} u_{ik}\, c_{kl} \quad \text{[From (1)]}$$

= The $(i, j)^{th}$ element of $(AB)C$

Hence, by the equality of two matrices, we have

$A(BC) = (AB)C$

Note : $(AB)C = A(BC) = ABC$

Theorem 2: Multiplication of matrices is distributive w.r.t. addition of matrices.

i.e., $A(B + C) = AB + AC$ and $(B + C) A = BA + CA$

Note : $A(B - C) = AB - AC$ and $(B - C) A = BA - CA$

Theorem 3: If A is a matrix of order $m \times n$, then $AI_n = I_n A = A$.

16. Trace of A Square Matrix :

Let $A = [a_{ij}]_{n \times n}$. Then trace of the square matrix A is defined as $\sum_{i=1}^{n} a_{ii}$ and is denoted by '*tr* (A)'.

Thus $$\operatorname{tr}(A) = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \ldots.. + a_{nn}$$

Properties : If A and B are square matrices of order n and λ is any scalar, then

(i) $\operatorname{tr}(\lambda A) = \lambda \operatorname{tr} A$.

(ii) $\operatorname{tr}(A + B) = \operatorname{tr} A + \operatorname{tr} B$

(iii) $\operatorname{tr}(AB) = \operatorname{tr}(BA)$

17. Triangular Matrix :

A square matrix all of whose elements below the leading diagonal are zero is called an ***Upper Triangular matrix***. A square matrix all of whose elements above the leading diagonal are zero is called a ***Lower Triangular matrix.***

e.g. $$\begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & 4 & 2 & 1 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$ is an *upper triangular matrix*

and $$\begin{bmatrix} 7 & 0 & 0 & 0 & 0 \\ 5 & 3 & 0 & 0 & 0 \\ -4 & 6 & 0 & 0 & 0 \\ 2 & 1 & -8 & 5 & 0 \\ 2 & 0 & 4 & 1 & 6 \end{bmatrix}$$ is a *lower triangular matrix.*

18. If A is a square matrix such that $A^2 = A$ then A is called **Idempotent.**

19. If A is a square matrix such that $A^m = O$ where m is a positive integer, then A is called **'Nilpotent'**. If m is least positive integer such that $A^m = O$ then A is called **'Nilpotent' of index *m*.**

20. If A is a square matrix such that $A^2 = I$ then A is called **Involutory.**

21. The Transpose of a Matrix :

Definition: The matrix obtained from any given matrix A, by inter changing its rows and columns is called *the* ***Transpose of A***. It is denoted by A' or A^T.

If $A = [a_{ij}]_{m \times n}$, then the transpose of A is $A' = [b_{ji}]_{n \times m}$, where $b_{ji} = a_{ij}$

Also $(A')' = A$

Note : If A' and B' be the transposes of A and B respectively, then

(i) $(A')' = A$

(ii) $(A + B)' = A' + B'$, A and B being of the same order.