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CLASS-X

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For Class X

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We are committed to serve students with best of our knowledge and resources. We have taken utmost care and paid much attention while editing and printing this book but we would beg to state that Authors and Publishers should not be held responsible for unintentional mistakes that might have crept in. However, errors brought to our notice shall be gratefully acknowledged and attended to.

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Preface

Preface

We feel pleasure in bringing out our book "Modern's abc + of Mathematics" for class X students. The book has been prepared strictly according to the latest syllabus and guidelines laid down by C.B.S.E.

The textual material of the book has been presented in a systematic and sequential manner, easily comprehensible by the learners to make it learner-friendly. In addition to full coverage of the content, each chapter of the book includes Illustrative Investigation (Activity), Solved NCERT Textbook Examples, Exercises, Solved NCERT Exemplar Problems and Additional Exercises (with Answers and Hints for selected questions) for practice, Summary (Content Revision) and Project.

Key features of this book are:

- Solved examples and unsolved problems have been selected very carefully and graded properly.
- Keeping in view with the latest trends, the exercises have been divided into three categories viz., 'Short Answer Type (Slab - I) Questions', 'Short Answer Type (Slab - II) Questions' and 'Long Answer Type Questions'.
- Value Based Questions **(VBQ)** to enthuse ethical skills have also been added in the text.
- **HOTS** (Higher Order Thinking Skills) questions based on analytical skills have also been included.
- **NTSE** (National Talent Search Examination) questions have also been added in the text.
- **Chapter Test** is given at the end of each chapter.

We are really very grateful to our dynamic publisher Sh. Balwant Sharma, Executive Director, Mr. Manik Juneja, A.I. Content Manager and Sh. B.S. Rawat (General Manager Publication) and other members of the staff for making the project successful.
We are grateful to Mr. Vinay Sharma, Editor, who has made the project successful. We are also grateful to Mr. Rohit Kumar, Mr. H K Thakur and the associated team for updating the book according to Modern Technology.

We hope that the book in its present form would prove to be more interesting and stimulating and would succeed in luring the students on a deeper interest of the art how to tackle with the problems.

Suggestions for further improvements from the readers will be thankfully received and will be duly incorporated.

J. P. Mohindru
Bharat Mohindru

SYLLABUS

Mathematics

Class-X

Marks : 80

Units	Unit Name	Marks
I.	Number Systems	06
II.	Algebra	20
III.	Coordinate Geometry	06
IV.	Geometry	15
V.	Trigonometry	12
VI.	Mensuration	10
VII.	Statistics and Probability	11
	Total	80

UNIT I : NUMBER SYSTEMS

1. **REAL NUMBERS** **(15) Periods**

   Euclid's division lemma, Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of $\sqrt{2}, \sqrt{3}, \sqrt{5}$ Decimal representation of rational numbers in terms of terminating/non-terminating recurring decimals.

UNIT II : ALGEBRA

1. **POLYNOMIALS** **(7) Periods**

   Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials. Statement and simple problems on division algorithm for polynomials with real coefficients.

2. **PAIR OF LINEAR EQUATIONS IN TWO VARIABLES** **(15) Periods**

   Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency.

   Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically—by substitution, by elimination and by cross multiplication method. Simple situational problems. Simple problems on equations reducible to linear equations.

3. **QUADRATIC EQUATIONS** **(15) Periods**

   Standard form of a quadratic equation $ax^2 + bx + c = 0$, (a ≠ 0). Solutions of the quadratic equations (only real roots) by factorization, by completing the square and by using quadratic formula. Relationship between discriminant and nature of roots.

   Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. **ARITHMETIC PROGRESSIONS** **(8) Periods**

   Motivation for studying Arithmetic Progression Derivation of the n^{th} term and sum of the first n terms of A.P. and their application in solving daily life problems.

UNIT III : COORDINATE GEOMETRY

1. **LINES (In two-dimensions)** **(14) Periods**

   Review: Concepts of coordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division). Area of a triangle.

UNIT IV : GEOMETRY

1. TRIANGLES **(15) Periods**

Definitions, examples, counter examples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.
5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.
6. (Motivate) If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.
7. (Prove) The ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides.
8. (Prove) In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
9. (Prove) In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angles opposite to the first side is a right angle.

2. CIRCLES **(8) Periods**

Tangent to a circle at a point of contact.

1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
2. (Prove) The lengths of tangents drawn from an external point to circle are equal.

3. CONSTRUCTIONS **(8) Periods**

1. Division of a line segment in a given ratio (internally).
2. Tangent to a circle from a point outside it.
3. Construction of a triangle similar to a given triangle.

UNIT V : TRIGONOMETRY

1. INTRODUCTION TO TRIGONOMETRY **(10) Periods**

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios, whichever are defined at 0° and 90°. Values (with proofs) of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES **(15) Periods**

Proof and applications of the identity $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given. Trigonometric ratios of complementary angles.

3. HEIGHTS AND DISTANCES : Angle of elevation, Angle of Depression. **(8) Periods**

Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30°, 45°, 60°.

UNIT VI : MENSURATION

1. AREAS RELATED TO CIRCLES **(12) Periods**

Motivate the area of a circle; area of sectors and segments of a circle. Problems based on areas and perimeter/circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only. Plane figures involving triangles, simple quadrilaterals and circle should be taken.)

2. SURFACE AREAS AND VOLUMES **(12) Periods**

(i) Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones. Frustum of a cone.

(ii) Problems involving converting one type of metallic solid into another and other mixed problems. (Problems with combination of not more than two different solids be taken.)

UNIT VII : STATISTICS AND PROBABILITY

1. STATISTICS **(18) Periods**

Mean, median and mode of grouped data (bimodal situation to be avoided). Cumulative frequency graph.

2. PROBABILITY **(10) Periods**

Classical definition of probability. Simple problems on single events (not using set notation).

CONTENTS

CHAPTER

1

REAL NUMBERS

1.0 INTRODUCTION

In the previous class, we explored the real numbers in two categories — rational and irrational numbers. In this chapter, we shall continue our discussion on real numbers. We shall begin with two important properties of positive integers namely Euclid's Division Algorithm and Fundamental Theorem of Arithmetic.

Euclid's division lemma is connected with the divisibility of integers. This lemma states that any positive 'a' can be divided by any other positive integer 'b' such that it leaves 'r', which is less than b. This is nothing but **Long Division Process**. In fact, the lemma gives us the step-wise procedure to find the HCF of two positive integers. This step-wise procedure is known as Euclid's Algorithm.

Fundamental Theorem of Arithmetic is concerned with the methods of expressing positive integers as the product of prime integers. This theorem is used to determine the nature of decimal expansion of rational numbers.

CONTENTS...

In This Chapter...

1.1 DIVISIBILITY

A non-zero integer 'a' is said to divide another positive integer 'b' if there exists an integer 'c' such that b = ac.

Here integer 'b' is called the **dividend**, integer 'a' is called the **divisor** and integer 'c' is called the **quotient**.

For example: (I) *4 divides 36.*

Because there is an integer 9 such that 36 = 4 × 9.

(II) 4 does not divide 35.

Because there does not exist an integer c such that 35 = 4 × c.

When a non-zero integer 'a' divides another integer 'b', then write $a \mid b$, *which is read as 'a divides b'* or' *b is divisible by a'* or *'a is a* factor of *b*, or *'b is a multiple of a'* or *'a is a divisor of b'*.

When b is not divisible by a, then we write $a \nmid b$.

1.2 EUCLID'S DIVISION LEMMA

Theorem 1.1 Euclid's Division Lemma

Given two positive integers a and b, there exist unique whole numbers q and r such that

$$a = bq + r, \text{ where } 0 \leq r < b.$$

In Words:

Dividend = (Divisor) (Quotient) + Remainder.

An **algorithm** is a series of well defined steps which gives a procedure for solving a type of problem.

The word *algorithm* comes from the name of the 9th century Persian mathematician **al-Khwarizmi**. In fact, even the word 'algebra' is derived from his book, called **Hisab al-jabr w'al-muqabala.**

A **lemma** is a *proven statement used for proving another statement*.

Euclid's Division Algorithm

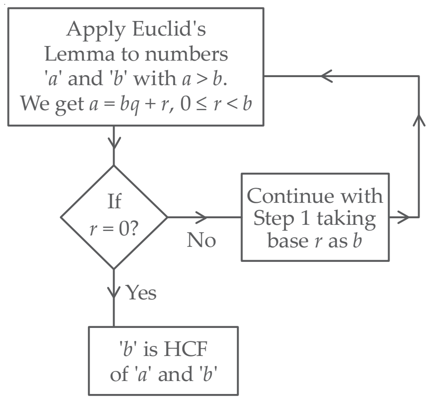
To obtain the HCF of two positive integers say a and b, with a > b, the following steps are followed.

Step 1: Apply Euclid's Lemma to a and b, we find whole numbers, q and r such that:

$$a = bq + r; \text{ where, } 0 \le r < b.$$

Step 2: When $r = 0$, then b is the HCF of a and b.
When $r \neq 0$, then we again apply the division Lemma to b and r.

Step 3: Continue the process till the remainder is zero *i.e.*, repeat the step 2 again and again until we get remainder (r) zero. Then, the divisor at this stage will be the required HCF.



Since, 96 > 56,

∴ we take $a = 96$ and $b = 56$.

Important Remarks

1. Euclid's division algorithm is not only useful for calculating the HCF of very large numbers, but also because it is one of the earliest examples of an algorithm that a computer had been programmed to carry out.
2. Euclid's division lemma and algorithm are so closely interlinked that people often call former as the division algorithm also.
3. Although Euclid's Division/Algorithm is stated for only positive integers, it can be extended for all integers except zero, *i.e.*, $b \neq 0$.
4. Let c and d be two positive integers such that $c = dq + r$; where $0 \le r < d$, then HCF (c, d) = HCF (d, r).
5. For any integer m, HCF $(m, 0) = m$.

Illustrative Examples

Example 1: Use Euclid's division algorithm to find the HCF of 56 and 88.

Solution: The given numbers are 56 and 88.

Since 88 > 56,

∴ we take $a = 88$ and $b = 56$.

Applying *Euclid's division lemma* to a and b, we get:

$$88 = 56 \times 1 + 32$$

Here, remainder $(r) = 32 \neq 0$.

So, we take $a = 56$ and $b = 32$.

Applying *Euclid's division lemma,* to new 'a' and 'b', we get:

$$56 = 32 \times 1 + 24$$

Here, remainder $(r) = 24 \neq 0$.

So, we take $a = 32$ and $b = 24$.

Applying *Euclid's division lemma* to new a and b, we get:

56) 88 (1
56
32

32) 56 (1
32
24

24) 32 (1
24
8

8) 24 (3
24
0

$$32 = 24 \times 1 + 8$$

Here, remainder $(r) = 8 \neq 0$.

So, we take $a = 24$ and $b = 8$.

Applying *Euclid's division lemma* to a and b, we get:

$$24 = 8 \times 3 + 0.$$

Here remainder $(r) = 0$.

Hence, we stop to conclude that HCF (56, 88) = 8.

Example 2: Find the HCF of 90 and 144 and express it as linear combination of 90 and 144.

[*Linear combination of 'a' and 'b' is ax + by, where x and y are integers*]

Solution: First of all, we find HCF of 144 and 90.

Take $a = 144$ and $b = 90$.

Applying *Euclid's division lemma,* we get:

$$144 = 90 \times 1 + 54 \quad \text{...(1)}$$

Here, remainder $(r) = 54 \neq 0$.

So, we take $a = 90$ and $b = 54$.

Applying *Euclid's division lemma*,

we get: $90 = 54 \times 1 + 36$ ...(2)

Here, remainder $(r) = 36 \neq 0$.

So, we take $a = 54$ and $b = 36$.

Applying *Euclid's division lemma*, we get:

$$54 = 36 \times 1 + 18 \quad ...(3)$$

Here, remainder $(r) = 18 \neq 0$.

So, we take $a = 36$ and $b = 18$.

Applying *Euclid's division lemma*, we get:

$$36 = 18 \times 2 + 0 \quad ...(4)$$

90) 144 (1
90
54) 90 (1
54
36) 54 (1
36
18) 36 (2
36
0

Here, remainder $r = 0$.

Thus, HCF (144, 90) = 18.

Now to represent HCF as linear combination of 144 and 90, we start from last but one step *i.e.*, (3) and successively eliminate the remainder of previous step as below:

From (3), $18 = 54 - 36 \times 1$

$= 54 - (90 - 54 \times 1) \times 1$

[*From (2), 36 = 90 – 54 × 1*]

$= 54 - 90 + 54 \times 1 = 54 \times 2 - 90$

$= (144 - 90 \times 1) \times 2 - 90$

[*From (1), 54 = 144 – 90 × 1*]

$= 144 \times 2 - 90 \times (2 + 1)$

$= 144 \times 2 - 90 \times 3$

$= 144 \times 2 + 90 \times (-3)$.

Hence, $18 = 90x + 144y$, where $x = -3$ and $y = 2$.

Example 3: Let '*k*' be HCF of 609 and 957. Determine *x* and *y* such that $k = 609x + 957y$. And establish that *x* and *y* are not unique.

Solution: Here 957 > 609.

So, we take $a = 957$ and $b = 609$.

Applying *Euclid's division algorithm*, we get:

$957 = 609 \times 1 + 348$ ...(1)

$609 = 348 \times 1 + 261$ ...(2)

$348 = 261 \times 1 + 87$ ...(3)

$261 = 87 \times 3 + 0$

$0 =$ remainder (r) ...(4)

609) 957 (1
609
348) 609 (1
348
261) 348 (1
261
87) 261 (3
261
0

Thus, HCF (609, 957) = 87.

From (3), $87 = 348 - 261 \times 1$

$= 348 - (609 - 348 \times 1) \times 1$

[∵ *From (2), 261 = 609 – 348 × 1*]

$= 348 - 609 \times 1 + 348 \times 1$

$= 348 \times 2 - 609 \times 1$

$= (957 - 609 \times 1) \times 2 - 609 \times 1$

[∵ *Find (1), 348 = 957-109 × 1*]

$= 957 \times 2 - 609 \times 2 - 609 \times 1$

$= 957 \times 2 + 609 (-2 - 1)$

$= 957 \times 2 + 609 \times (-3)$ ...(5)

Thus, $87 = 957x + 609y$, where $x = 2$ and $y = -3$.

Adding and subtracting 957×609 in (4), we get:

$87 = 957 \times 2 + 609 \times (-3) + 957 \times 609 - 957 \times 609$

$= 957 \times 2 + 957 \times 609 + 609 \times (-3) + 609 \times (-957)$

$= 957 \times (2 + 609) + 609 \times (-3 - 957)$

$\Rightarrow$ $87 = 957 \times (611) + 609 \times (-960)$ ...(6)

$= 957x + 609y$, where $x = 611$, $y = -960$.

Hence, (6) represents another linear combination.

And *x* and *y* are not unique.

Example 4: Find the HCF of 72, 126 and 150, using Euclid's division algorithm.

Solution: First of all, we find HCF of 72 and 126.

Take $a = 126$ and $b = 72$ and applying *Euclid's division algorithm* we get:

$126 = 72 \times 1 + 54$

$72 = 54 \times 1 + 18$

$54 = 18 \times 3 + \mathbf{0}$

↑

Zero Remainder (Stop!)

72) 126 (1
72
54) 72 (1
54
18) 54 (3
54
0

Thus, HCF (126, 72) = 18.

Now we find HCF of 18 and 150.

Take $a = 150$ and $b = 18$.

Applying *division algorithm*, we get:

$150 = 18 \times 8 + 6$

$18 = 6 \times 3 + \mathbf{0}$ ← Zero Remainder (Stop!)

18) 150 (8
144
6) 18 (3
18
0

Thus, HCF (18, 150) = 6.

Hence, H.C.F. (72, 126, 150) = 6.

Example 5: What is the greatest number which divides 442, 569, 697 leaving remainder 1, 2 and 4 respectively.

Solution: When we divide 442, 569, 697 by the required number, remainders are 1, 2, 4 respectively.

∴ 442 – 1, 569 – 2, 697 – 4 *i.e.*, 441, 567, 693 are exactly divisible by the required number.

∴ Required number = HCF (441, 567, 693)

Firstly find HCF of 441 and 567.

Take $a = 567$ and $b = 441$ and applying *Euclid's division algorithm successively*, we get :

$567 = 441 \times 1 + 126$

$441 = 126 \times 3 + 63$

$126 = 63 \times 2 + \mathbf{0}$

↑

Zero Remainder (Stop!)

441) 567 (1
441
126) 441 (3
378
63) 126 (2
126
0

Thus, HCF (567, 441) = 63

Now we find HCF of 63 and 697.

Take $a = 697$ and $b = 63$ and applying *Euclid's division algorithm*, we get:

$693 = 63 \times 11 + \boxed{0}$
↑
Zero Remainder (Stop!)

63) 693 (11
63
63
63
0

Thus HCF (63, 693) = 63

⇒ HCF (441, 567, 693) = 63.

Hence, 63 divides 442, 569, 696 leaving remainders 1, 2 and 3 respectively.

Example 6: Verify by using Euclid's division algorithm that the numbers 350 and 849 are coprime.

Solution: We find HCF of 350 and 849, by applying *Euclid's division algorithm* as below:

$849 = 350 \times 2 + 149$

$350 = 149 \times 2 + 52$

$149 = 52 \times 2 + 45$

$52 = 45 \times 1 + 7$

$45 = 7 \times 6 + 3$

$7 = 3 \times 2 + 1$

$3 = 1 \times 3 + \mathbf{0}$
↑
Zero Remainder

350) 849 (2
700
149) 350 (2
298
52) 149 (2
104
45) 52 (1
45
7) 45 (6
42
3) 7 (2
6
1) 3 (3
3
0

Thus, HCF (350, 849) = 1.

Hence, 350 and 849 are co-prime.

Example 7: Two reservoirs contain 960 litres and 640 litres of water respectively. Obtain the maximum capacity of the container which can measure the water of reservoir in exact number of times.

Solution: The value of maximum capacity of the container should divide both 960 litres and 640 litres.

∴ Maximum capacity of measuring container
= HCF (640, 960).

Applying *Euclid's division algorithm* to $a = 960$ and $b = 640$, we get:

$960 = 640 \times 1 + 320$

$640 = 320 \times 2 + \mathbf{0}$
↑
Zero Remainder

640) 960 (1
640
320) 640 (2
640
0

Thus, HCF (960, 640) = 320.

Hence, the maximum capacity of measuring container is 320 litres.

EXERCISE 1(*a*)

Short Answer Type Questions (SLAB–I)

1. Use Euclid's division algorithm to find HCF of each of the following pairs of numbers:
   (*i*) 18, 24 (*ii*) 70, 30
   (*iii*) 714, 924 (*iv*) 100, 190 (**CBSE 2009**)
   (*v*) 105, 120 (*vi*) 155, 1385
   (*vii*) 4641, 4095 (*viii*) 9367, 3451.
2. Using Euclid's division algorithm find HCF of the following numbers:
   (*i*) 296, 999, 925
   (*ii*) 480, 704, 3680
   (*iii*) 1215, 513, 1134.
3. Find the largest positive integer which divides 615 and 963 leaving remainder 6 in each case.
4. Determine the greatest number which will divide 445, 572, 699 leaving remainder 4, 5, 6 respectively.
5. Using Euclid's division algorithm state whether the numbers 47 and 149 are coprimes or not.
6. Using Euclid's division algorithm, find which of the following pairs of numbers are co-primes:
   (*i*) 272, 1032 (*ii*) 847, 2160
   (*iii*) 867, 255 (*iv*) 616, 309
7. Find the HCF of 135 and 225 and express it as a linear combination of 135 and 255.
8. Find HCF of 693 and 567 and express it as a linear combination of them. Is this combination unique? If not, then write one more linear combination.

Short Answer Type Questions (SLAB–II)

9. The length and breadth of a field are 16 m 17 cm and 22 m 77 cm respectively. Obtain the maximum length of the rope which can measure the dimensions of the field in exact number of times.
10. Two oil tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of the container which can measure the petrol of either tank in exact number of times.
11. The dimensions of a room are 8 m 25 cm, 6 m 75 cm and 4 m 30 cm. Find the length of longest rod which can measure the three dimensions of the room exactly.
12. 144 cartons of coke cans and 90 cartons of pepsi cans are to be stacked in a store. If each stack is of same height and is to contain cartons of same drink, then find least number of stacks required.

Long Answer Type Questions

13. The length, breadth and height of a room are 8 m, 25 m 75 cm and 4 m 50 cm, respectively. Find the largest rod that can measure the three dimension of the room exactly.
14. 375 male athletes and 105 female athletes of a country are to be divided into different groups such that in a group either all male or all female athletes should be there. Also, the number of athletes in each group should be same and no one is left without a group. What should be the maximum number of athletes in a group so that minimum number of groups are formed? Also find how many groups are formed?
15. A stockist has 120 cartons of anacin, 180 cartons of dispirin and 240 cartons of bruffen. He wants to sell the three kinds of medicines to retailer in containers of equal capacity. What should be the greatest capacity of each container in terms of cartons if each container has cartons of same medicine.

ANSWERS

1. (*i*) 6 (*ii*) 10 (*iii*) 42
   (*iv*) 10 (*v*) 15 (*vi*) 5
   (*vii*) 273 (*viii*) 493
2. (*i*) 37 (*ii*) 32 (*iii*) 27.
3. 87 **4.** 63 **5.** coprimes
6. (*i*) No (*ii*) Yes (*iii*) No
   (*iv*) Yes
7. 45 ; $45 = 135x + 225y$; where $x = 2$, $y = -1$
8. 63 ; $63 = 567 \times 5 + 693 \times (-4)$; No,
   $63 = 567 \times 698 + 693 \times (-571)$
9. 33 cm **10.** 170 litres **11.** 75 cm
12. 18 **13.** 75 cm
14. 32 groups of 15 athletes
15. 60.

1.2.1 Application of Euclid's Division Lemma

Positive numbers exhibit certain interesting patterns so as to satisfy various properties, which are proved using Euclid's division lemma.

ILLUSTRATIVE EXAMPLES

Example 1: Show that any positive integer is of the form: $3q$, $3q + 1$ or $3q + 2$ for some integer q.

Solution: Let a be any positive integer and $b = 3$.

Applying *Euclid's division lemma,* we get :

$a = 3q + r$, where q is some integer and $0 \leq r < 3$ *i.e.*, $r = 0, 1, 2$.

$\therefore a = 3q$ or $3q + 1$ or $3q + 2$.

Hence, any positive integer is of the form $3q$ or $3q + 1$ or $3q + 2$.

Example 2: Show that one of every three consecutive positive integers is divisible by 3.

Solution: Let n, $n + 1$ and $n + 2$ be three consecutive positive integers.

Since, we know that n is of the form $3q$, $3q + 1$ or $3q + 2$. [*Ex. 1*]

$\therefore$ three cases arise.

Case I: When $n = 3q$.

Here n is divisible by 3 while $(n + 1)$ and $(n + 2)$ are not divisible by 3.

Case II: When $n = 3q + 1$.

Here $n + 2 = (3q + 1) + 2 = 3q + 3 = 3(q + 1)$ is divisible by 3 while n and $(n + 1)$ are not divisible by 3.

Case III: When $n = 3q + 2$.

Here $n + 1 = (3q + 2) + 1 = 3q + 3 = 3(q + 1)$ is divisible by 3 while n and $(n + 2)$ are not divisible by 3.

Hence, one of n, $n + 1$ and $n + 2$ is divisible by 3

Example 3: Prove that if x and y are odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

Solution: We know that any positive integer is of the form $2q + 1$, where q is some integer.

Let $x = 2m + 1$ and $y = 2n + 1$, for some integers m and n.

$$\therefore \quad x^2 + y^2 = (2m + 1)^2 + (2n + 1)^2$$
$$= (4m^2 + 4m + 1) + (4n^2 + 4n + 1)$$
$$= 4\{(m^2 + n^2) + (m + n)\} + 2$$
$$= 4q + 2. \text{ where } q = (m^2 + n^2) + (m + n)$$

$\Rightarrow x^2 + y^2$ is even and leaves remainder 2 when divided by 4.

Hence, $x^2 + y^2$ is even but not divisible by 4.

Example 4: Show that $n^2 - 1$ is divisible by 8, if n is an odd positive integer.

Solution: We know that any positive odd integer is of the form: $4q + 1$ or $4q + 3$, for some integer q.

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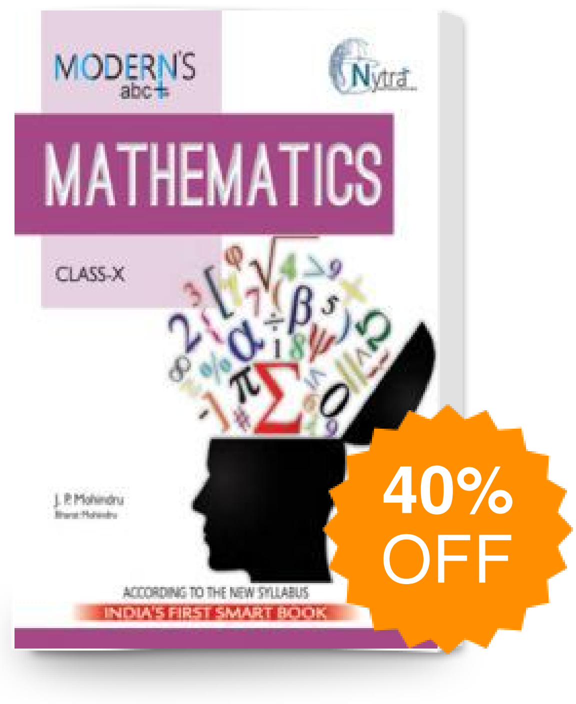

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