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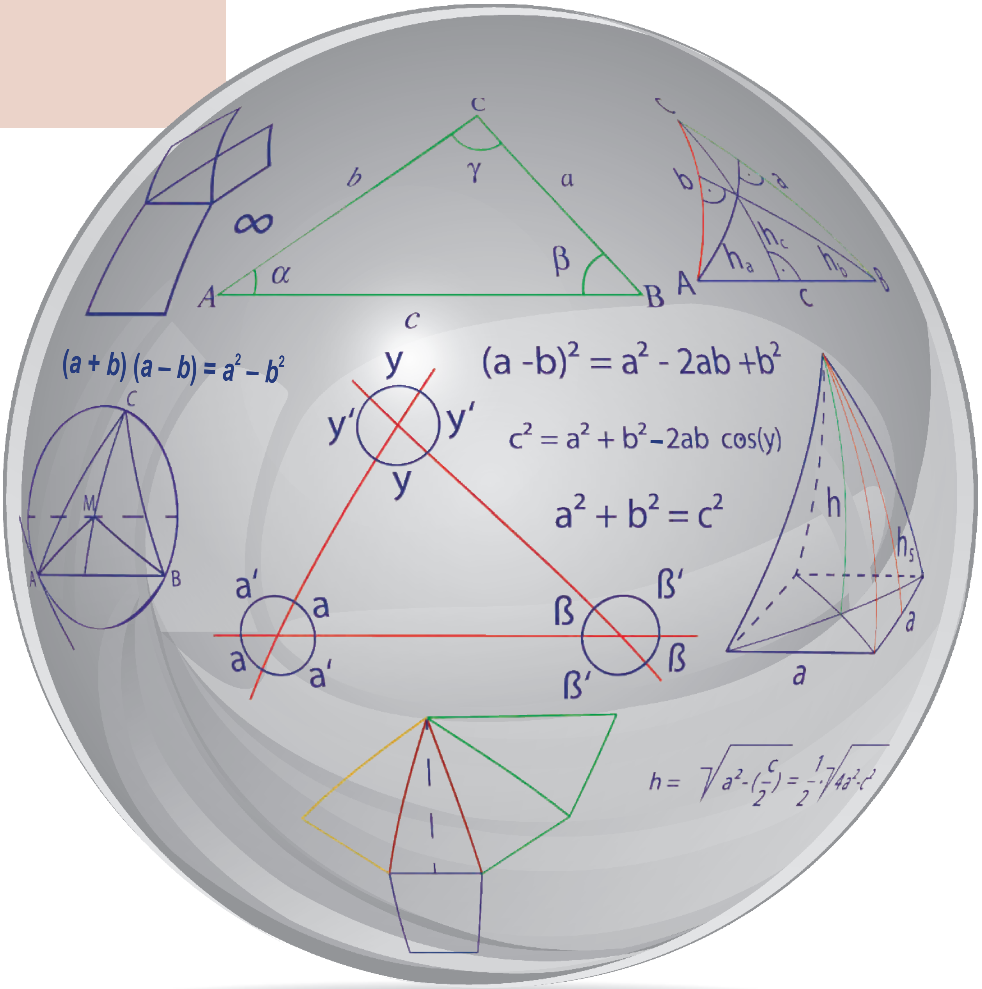

J. P. Mohindru
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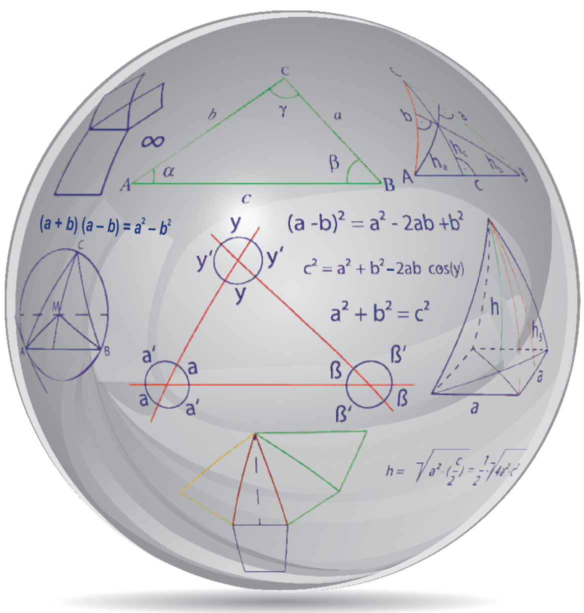

By

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We are committed to serve students with best of our knowledge and resources. We have taken utmost care and paid much attention while editing and printing this book but we would beg to state that Authors and Publishers should not be held responsible for unintentional mistakes that might have crept in. However, errors brought to our notice shall be gratefully acknowledged and attended to.

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Preface

We feel pleasure in bringing out our book "Modern's abc + of Mathematics" for class IX students. The book has been prepared strictly according to the latest syllabus and guidelines laid down by C.B.S.E.

The textual material of the book has been presented in a systematic and sequential manner, easily comprehensible by the learners to make it learner-friendly. In addition to full coverage of the content, each chapter of the book includes Illustrative Investigation (Activity), Solved NCERT Textbook Examples, Exercises, Solved NCERT Exemplar Problems and Additional Exercises (with Answers and Hints for selected questions) for practice, Summary (Content Revision) and Project.

Key features of this book are:

- Solved examples and unsolved problems have been selected very carefully and graded properly.
- Keeping in view with the latest trends, the exercises have been divided into three categories viz., 'Short Answer Type (Slab - I) Questions', 'Short Answer Type (Slab - II) Questions' and 'Long Answer Type Questions'.
- Value Based Questions **(VBQ)** to enthuse ethical skills have also been added in the text.
- **HOTS** (Higher Order Thinking Skills) questions based on analytical skills have also been included.
- **NTSE** (National Talent Search Examination) questions have also been added in the text.
- **Chapter Test** is given at the end of each chapter.
- **Solution of this book is available separately.**

We are really very grateful to our dynamic publisher Sh. Balwant Sharma, Executive Director, Mr. Manik Juneja, Director-Content & Production and Sh. B.S. Rawat (General Manager Publication) and other members of the staff for making the project successful.

We are grateful to Mr. Vinay Sharma, Editor, who has made the project successful. We are also grateful to Mr. Rohit Kumar, Mr. H K Thakur and the associated team for updating the book according to Modern Technology.

We hope that the book in its present form would prove to be more interesting and stimulating and would succeed in luring the students on a deeper interest of the art how to tackle with the problems.

Suggestions for further improvements from the readers will be thankfully received and will be duly incorporated.

J. P. Mohindru

Bharat Mohindru

SYLLABUS

Mathematics

Class-IX

Marks : 80

Units	Unit Name	Marks
I.	Number Systems	08
II.	Algebra	17
III.	Coordinate Geometry	04
IV.	Geometry	28
V.	Mensuration	13
VI.	Statistics and Probability	10
	Total	80

UNIT-I : NUMBER SYSTEMS

1. REAL NUMBERS **(18) Periods**

1. Review of representation of natural numbers, integers, rational numbers on the number line. Representation of terminating / non-terminating recurring decimals on the number line through successive magnification. Rational numbers as recurring/ terminating decimals. Operations on real numbers.
2. Examples of non-recurring/non-terminating decimals. Existence of non-rational numbers (irrational numbers) such as $\sqrt{2}, \sqrt{3}$ and their representation on the number line. Explaining that every real number is represented by a unique point on the number line and conversely, viz. every point on the number line represents a unique real number.
3. Definition of n^{th} root of a real number.
4. Existence of $\sqrt{x}$ for a given positive real number x and its representation on the number line with geometric proof.
5. Rationalization (with precise meaning) of real numbers of the type $\frac{1}{a+b\sqrt{x}}$ and $\frac{1}{\sqrt{x}+\sqrt{y}}$ (and their combinations) where x and y are natural number and a and b are integers.
6. Recall of laws of exponents with integral powers. Rational exponents with positive real bases (to be done by particular cases, allowing learner to arrive at the general laws.)

UNI-II : ALGEBRA

1. POLYNOMIALS **(23) Periods**

Definition of a polynomial in one variable, with examples and counter examples. Coefficients of a polynomial, terms of a polynomial and zero polynomial. Degree of a polynomial. Constant, linear, quadratic and cubic polynomials. Monomials, binomials, trinomials. Factors and multiples. Zeros of a polynomial. Motivate and State the Remainder Theorem with examples. Statement and proof of the Factor Theorem. Factorization of $ax^2+bx+c, a \neq 0$ where a, b and c are real numbers, and of cubic polynomials using the Factor Theorem.

Recall of algebraic expressions and identities. Verification of identities:

$(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

$(x \pm y)^3 = x^3 \pm y^3 \pm 3xy\,(x \pm y)$

$x^3 \pm y^3 = (x \pm y)\,(x^2 \mp xy + y^2)$

$x^3+y^3+z^3-3xyz = (x+y+z)\,(x^2+y^2+z^2-xy-yz-zx)$ and their use in factorization of polynomials.

2. LINEAR EQUATIONS IN TWO VARIABLES **(14) Periods**

Recall of linear equations in one variable. Introduction to the equation in two variables. Focus on linear equations of the type $ax + by + c = 0$. Prove that a linear equation in two variables has infinitely many solutions and justify their being written as ordered pairs of real numbers, plotting them and showing that they lie on a line. Graph of linear equations in two variables. Examples, problems from real life, including problems on Ratio and Proportion and with algebraic and graphical solutions being done simultaneously.

UNIT -III : COORDINATE GEOMETRY

1. COORDINATE GEOMETRY **(6) Periods**

The Cartesian plane, coordinates of a point, names and terms associated with the coordinate plane, notations, plotting points in the plane.

UNIT -IV : GEOMETRY

1. INTRODUCTION TO EUCLID'S GEOMETRY **(6) Periods**

History - Geometry in India and Euclid' s geometry. Euclid's method of formalizing observed phenomenon into rigorous Mathematics with definitions, common/obvious notions, axioms/postulates and theorems. The five postulates of Euclid. Equivalent versions of the fifth postulate. Showing the relationship between axiom and theorem, for example:

(Axiom) 1. Given two distinct points, there exists one and only one line through them.

(Theorem) 2. (Prove) Two distinct lines cannot have more than one point in common.

2. LINES AND ANGLES **(13) Periods**

1. (Motivate) If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and the converse.
2. (Prove) If two lines intersect, vertically opposite angles are equal.
3. (Motivate) Results on corresponding angles, alternate angles, interior angles when a transversal intersects two parallel lines.
4. (Motivate) Lines which are parallel to a given line are parallel.
5. (Prove) The sum of the angles of a triangle is 180°.
6. (Motivate) If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

3. TRIANGLES **(20) Periods**

1. (Motivate) Two triangles are congruent if any two sides and the included angle of one triangle is equal to any two sides and the included angle of the other triangle (SAS Congruence).
2. (Prove) Two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle (ASA Congruence).
3. (Motivate) Two triangles are congruent if the three sides of one triangle are equal to three sides of the other triangle (SSS Congruence).
4. (Motivate) Two right triangles are congruent if the hypotenuse and a side of one triangle are equal (respectively) to the hypotenuse and a side of the other triangle. (RHS Congruence)
5. (Prove) The angles opposite to equal sides of a triangle are equal.
6. (Motivate) The sides opposite to equal angles of a triangle are equal.
7. (Motivate) Triangle inequalities and relation between 'angle and facing side' inequalities in triangles.

4. QUADRILATERALS **(10) Periods**

1. (Prove) The diagonal divides a parallelogram into two congruent triangles.
2. (Motivate) In a parallelogram opposite sides are equal, and conversely.
3. (Motivate) In a parallelogram opposite angles are equal, and conversely.
4. (Motivate) A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal.
5. (Motivate) In a parallelogram, the diagonals bisect each other and conversely.
6. (Motivate) In a triangle, the line segment joining the mid points of any two sides is parallel to the third side and in half of it and (motivate) its converse.

5. AREA **(7) Periods**

Review concept of area, recall area of a rectangle.

1. (Prove) Parallelograms on the same base and between the same parallels have the same area.
2. (Motivate) Triangles on the same (or equal base) base and between the same parallels are equal in area.

6. CIRCLES **(15) Periods**

Through examples, arrive at definition of circle and related concepts-radius, circumference, diameter, chord, arc, secant, sector, segment, subtended angle.

1. (Prove) Equal chords of a circle subtend equal angles at the center and (motivate) its converse.

2. (Motivate) The perpendicular from the center of a circle to a chord bisects the chord and conversely, the line drawn through the center of a circle to bisect a chord is perpendicular to the chord.
3. (Motivate) There is one and only one circle passing through three given non-collinear points.
4. (Motivate) Equal chords of a circle (or of congruent circles) are equidistant from the center (or their respective centers) and conversely.
5. (Prove) The angle subtended by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle.
6. (Motivate) Angles in the same segment of a circle are equal.
7. (Motivate) If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, the four points lie on a circle.
8. (Motivate) The sum of either of the pair of the opposite angles of a cyclic quadrilateral is 180° and its converse.

7. CONSTRUCTIONS **(10) Periods**

1. Construction of bisectors of line segments and angles of measure 60°, 90°, 45° etc., equilateral triangles.
2. Construction of a triangle given its base, sum/difference of the other two sides and one base angle.
3. Construction of a triangle of given perimeter and base angles.

UNIT -V : MENSURATION

1. AREAS **(4) Periods**

Area of a triangle using Heron's formula (without proof) and its application in finding the area of a quadrilateral.

2. SURFACE AREAS AND VOLUMES **(12) Periods**

Surface areas and volumes of cubes, cuboids, spheres (including hemispheres) and right circular cylinders/cones.

UNIT -VI : STATISTICS and PROBABILITY

1. STATISTICS **(13) Periods**

Introduction to Statistics: Collection of data, presentation of data — tabular form, ungrouped/grouped, bar graphs, histograms (with varying base lengths), frequency polygons. Mean, median and mode of ungrouped data.

2. PROBABILITY **(9) Periods**

History, Repeated experiments and observed frequency approach to probability.

Focus is on empirical probability. (A large amount of time to be devoted to group and to individual activities to motivate the concept; the experiments to be drawn from real-life situations, and from examples used in the chapter on statistics).

CONTENTS

CHAPTER

1

NUMBER SYSTEMS

CONTENTS...

In This Chapter...

1.0 INTRODUCTION

We have learnt about the number line and the representation of natural numbers, integers and rational numbers on the number line in earlier classes.

In this chapter, we shall study rational numbers as terminating/recurring. We shall also study decimals irrational (non-rational) numbers *viz.* $\sqrt{2}, \sqrt{3}$ and their representation on the number line.

HISTORY

Irrational numbers were introduced by **Pythagoras** in Greece in 400 B.C. **Archimedes (476–550 A.D.)** computed digits in the decimal expansion of π. He proved that π lies between 3.140845 and 3.142857. **Ramanujan (1887–1920 A.D.)** calculated the value of π, correct to millions of decimal places.

Let us, first of all, review the representation of various types of numbers on the number line.

1.1 REVIEW OF NUMBERS

We call the counting numbers as natural numbers:

1, 2, 3, ...

(a) Natural Numbers. The collection of natural numbers is denoted by **N** and is written as:

N = {1, 2, 3, ...}.

Key Points

(*i*) 1 is the first natural number

(*ii*) There is no last natural number.

(b) Whole Numbers. If we include '0' in the collection of natural numbers, we get:

0, 1, 2, ...

These are called whole numbers and the new collection is denoted by **W** and is written as **W** = {0, 1, 2, 3 ....}.

Key Points

(*i*) Every natural number is a whole number

(*ii*) 0 is a whole number but not a natural number.

(c) Integers. The collection of all natural numbers, 0 and negatives of natural numbers form the collection of integers. This new collection is denoted by **Z** and is written as:

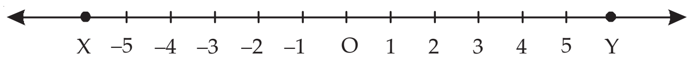
$$\mathbf{Z} = \{..........., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}.$$

Key Points

(*i*) Every natural number is an integer
(*ii*) Every whole number is an integer
(*iii*) Every integer is not a natural number
(*iv*) Every integer is not a whole number.

Representation on Number Line

Draw a line XY, which extends endlessly in both directions as shown in the figure as below:



Take any point O on the line, which represents the integer zero (0). Let us take a fixed length, called unit length. Now cut off equal distances on both sides of O.

Key Point

We can represent each integer by some point on the number line.

On the right of O, the points at distances of 1 unit, 2 units, 3 units; .... from O represent the integers 1, 2, 3, ... respectively.

On the left of O, the points at distances of 1 unit, 2 units, 3 units; .... from O represent the integers –1, –2, –3, ... respectively.

The line is called **number** line.

(d) Rational Numbers: The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are known as rational numbers. This collection is denoted by **Q** and is written as:

$$\mathbf{Q} = \left\{\frac{p}{q} : q \neq 0, p, q \in \mathbf{Z}\right\}.$$

Key Points

(*i*) Every natural number is a rational number
(*ii*) Every integer is a rational number
(*iii*) 0 is a rational number.

(I) Equivalent Rational Numbers:

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = ... = \frac{15}{45} = \frac{16}{48} = ...$$

These are called *equivalent rational numbers.*

For Example: *Write three rational numbers, which are equivalent to* $\frac{3}{7}$.

Here, $\frac{3}{7} = \frac{3 \times 2}{7 \times 2} = \frac{3 \times 3}{7 \times 3} = \frac{3 \times 4}{7 \times 4}$

$\therefore$ $\frac{3}{7} = \frac{6}{14} = \frac{9}{21} = \frac{12}{28}$.

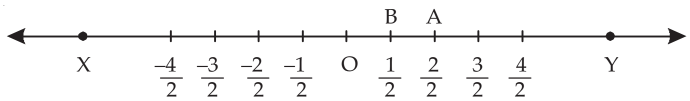
Hence, three equivalent rational numbers are $\frac{6}{14}, \frac{9}{21}$ and $\frac{12}{28}$.

(II) Simplest Form: A rational number $\frac{p}{q}$ is said to be in the simplest form if p, q are integers, having no common factor other than 1 and $q \neq 0$.

For Example: Simplest form of each of:

$$\frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}; \text{ ..... is } \frac{1}{3}.$$

(III) Representation on Real Line: Draw a line XY, which extends endlessly in both directions as shown in the figure as below:



Take any point O on the line, which represents the integer zero (0). Mark OA = 1 unit.

Mid-point B of OA denotes the rational number $\frac{1}{2}$. From O, set equal distances (each having = OB = $\frac{1}{2}$ unit).

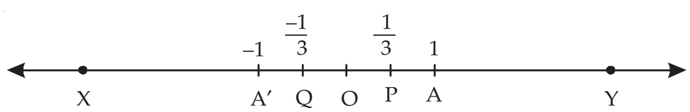
On the right of O, the points at distances equal to OB, 2OB, 3OB, ... denote the rational numbers $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, ...$ respectively.

Similarly, on the left of O, the points at distances equal to OB, 2OB, 3OB, ... denote the rational numbers $-\frac{1}{2}, -\frac{2}{2}, -\frac{3}{2}, ...$ respectively.

Hence, each rational number having 2 as its denominator can be represented on the number line by some point.

Similarly, we can represent the rational numbers $\frac{1}{3}, \frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$ by dividing the segments OA and OA′ into three equal parts.

Then P and Q represent $\frac{1}{3}$ and $-\frac{1}{3}$ respectively when



A and A′ represent 1 and − 1 respectively.

IMPORTANT RESULTS

If x and y are two rational numbers, where $x < y$, then:

(*i*) $\frac{x+y}{2}$ is a rational number, which lies between x and y

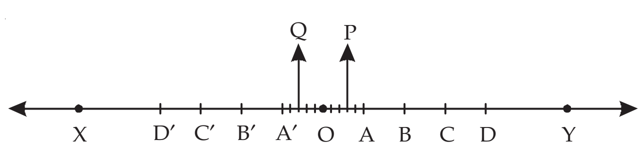
(*ii*) n rational numbers, lying between x and y, are: $(x + d)$, $(x + 2d)$, ..., $(x + nd)$, where $d = \frac{y-x}{n+1}$.

ILLUSTRATIVE EXAMPLES

Example 1: Represent $\frac{3}{5}$ and $-\frac{3}{5}$ on the number line.

Solution: Draw a line XY, which extends endlessly in both directions as shown in the following figure as below:

Take a fixed length as unit length in order to represent integers on this line.



(*i*) On the right of O, take OA = 1 unit. Divide the unit OA into 5 equal parts. OP represents $\frac{3}{5}$ of a unit.

Hence, P represents $\frac{3}{5}$.

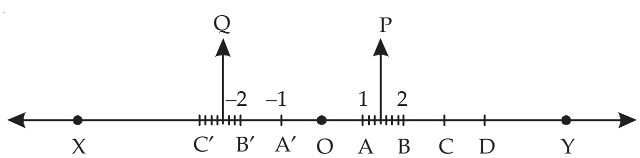
(*ii*) On the left of O, take OA′ = 1 unit. Divide the unit OA′ into 5 equal parts. OQ represents $\frac{3}{5}$ of a unit.

Hence, Q represents $-\frac{3}{5}$.

Example 2: **Represent (*i*) $1\frac{3}{7}$ and (*ii*) $-2\frac{3}{7}$ on the number line.**

Solution: Draw a line XY, which extends endlessly in both directions as shown in the figure as below:

Take fixed length as unit length in order to represent integers on this line.



(*i*) On the right of O, take OA = 1 unit. Divide the 2nd unit AB into 7 equal parts. AP represents $\frac{3}{7}$ of a unit.

Hence, P represents $1\frac{3}{7}$.

(*ii*) On the left of O, take OA′ = –1, OB′ = –2. Divide the 3rd unit B′C′ into 7 equal parts. BQ represents $\frac{3}{7}$ of a unit. Hence, Q represents $-2\frac{3}{7}$.

Example 3: **Find a rational number between – 4 and 8.**

Solution: We know that a rational number lying between x and y ($x < y$) is $\frac{x+y}{2}$.

Thus, $x < \frac{x+y}{2} < y$.

Here, $x = -4$ and $y = 8$.

∴ Rational number between – 4 and 8

$$= \frac{1}{2}(x+y)$$

$$= \frac{1}{2}(-4+8) = \frac{1}{2}(4) = 2.$$

Example 4: **Find five rational numbers between 2 and 3.**

Solution: Here, $x = 2$, $y = 3$ and $n = 5$.

$$\therefore \quad d = \frac{y-x}{n+1} = \frac{3-2}{5+1} = \frac{1}{6}.$$

Thus, five rational numbers between 2 and 3 are:

$(x + d)$, $(x + 2d)$, $(x + 3d)$, $(x + 4d)$ and $(x + 5d)$

i.e., $\left(2+\frac{1}{6}\right), \left(2+\frac{2}{6}\right), \left(2+\frac{3}{6}\right), \left(2+\frac{4}{6}\right)$ and $\left(2+\frac{5}{6}\right)$

i.e., $2\frac{1}{6}, 2\frac{2}{6}, 2\frac{3}{6}, 2\frac{4}{6}$ and $2\frac{5}{6}$

i.e., $\frac{13}{6}, \frac{14}{6}, \frac{15}{6}, \frac{16}{6}$ and $\frac{17}{6}$.

Hence, the required five rational numbers between 2 and 3 are

$$\frac{13}{6}, \frac{14}{6}, \frac{15}{6}, \frac{16}{6} \text{ and } \frac{17}{6}.$$

Example 5: **Find nine rational numbers between 0 and 0.1.**

Solution: Here, $x = 0$, $y = 0.1$ and $n = 9$.

$$\therefore \quad d = \frac{y-x}{n+1} = \frac{0.1-0}{9+1} = \frac{0.1}{10} = 0.01.$$

Thus, nine rational numbers between 0 and 0.1 are:

$(x + d)$, $(x + 2d)$, $(x + 3d)$, ..., $(x + 8d)$, $(x + 9d)$

i.e., 0.01, 0.02, 0.03, ..., 0.08, 0.09

i.e., $\frac{1}{100}, \frac{2}{100}, \frac{3}{100}, \ldots, \frac{8}{100}, \frac{9}{100}$

i.e., $\frac{1}{100}, \frac{1}{50}, \frac{3}{100}, \frac{1}{25}, \frac{1}{20}, \frac{3}{50}, \frac{7}{100}, \frac{2}{25}$ and $\frac{9}{100}$.

Hence, the required nine rational numbers between 0 and 0.1 are:

$$\frac{1}{100}, \frac{1}{50}, \frac{3}{100}, \frac{1}{25}, \frac{1}{20}, \frac{3}{50}, \frac{7}{100}, \frac{2}{25} \text{ and } \frac{9}{100}.$$

Example 6: **Insert 100 rational numbers between $-\frac{5}{13}$ and $\frac{8}{13}$.**

Solution: Here, $-\frac{5}{13} = \frac{-5\times10}{13\times10} = -\frac{50}{130}$

and $\frac{8}{13} = \frac{8\times10}{13\times10} = \frac{80}{130}$.

Since, $-50 < -49 < -48 < \ldots < -1 < 0 < 1 < 2 < \ldots < 50$,

$$\therefore \quad -\frac{50}{130} < -\frac{49}{130} < -\frac{48}{130} < \ldots$$

$$< -\frac{1}{130} < \frac{0}{130} < \frac{1}{130} < \frac{2}{130} < \ldots < \frac{50}{130}.$$

Hence, 100 rational numbers between $-\frac{5}{13} = -\frac{50}{130}$ and $\frac{8}{13} = \frac{80}{130}$ are:

$$-\frac{49}{130}, \ldots, -\frac{1}{130}, \frac{0}{130}, \frac{1}{130}, \ldots, \frac{50}{130}.$$

EXERCISE 1(*a*)

Short Answer Type Questions (Slab–I)

1. Represent each of the following rational numbers on the number line:

   (*i*) 3 (*ii*) 5 (*iii*) – 3 (*iv*) – 7.

2. Find a rational number between:

   (*i*) –2 and 6 (*ii*) $\frac{1}{4}$ and $\frac{1}{3}$

   (*iii*) $-\frac{3}{4}$ and $-\frac{2}{5}$ (*iv*) 0.75 and 1.2.

3. Find three rational numbers between:

   (*i*) – 2 and 5 (*ii*) $\frac{1}{5}$ and $\frac{1}{4}$.

Short Answer Type Questions (Slab–II)

4. Represent each of the following rational numbers on the number line:

   (*i*) $\frac{3}{7}$ (*ii*) $\frac{7}{3}$ (*iii*) 1.5 (*iv*) $\frac{25}{6}$.

5. Find five rational numbers between $\frac{2}{5}$ and $\frac{3}{4}$.
6. Find six rational numbers between 3 and 4.
7. Insert 16 rational numbers between 2.1 and 2.2.

ANSWERS

2. (*i*) 2 (*ii*) $\frac{7}{24}$ (*iii*) $-\frac{23}{40}$ (*iv*) 0.975.
3. (*i*) $-\frac{1}{4}, \frac{3}{2}$ and $\frac{13}{4}$ (*ii*) $\frac{17}{80}, \frac{18}{80}, \frac{19}{80}$.
5. $\frac{55}{120}, \frac{62}{120}, \frac{69}{120}, \frac{76}{120}, \frac{83}{120}$.
6. $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$.
7. 2.105, 2.11, 2.115, 2.12, 2.125, 2.13, 2.135, 2.14, 2.145, 2.15, 2.155, 2.16, 2.165, 2.17, 2.175, 2.18.

HINTS TO SELECTED QUESTIONS

7. Take n = 16 so that n + 1 = 17. $\therefore\ d = \frac{y-x}{n+1} = \frac{2.2-2.1}{16+1} = \frac{0.1}{17} = 0.006.$

1.2 DECIMAL REPRESENTATION OF RATIONAL NUMBERS

We know that a rational number is a number which can be expressed in the form:

$$\frac{p}{q}, \text{ where } p, q \in \mathbf{Z} \text{ and } q \neq 0.$$

A rational number is said to be in lowest terms, if

$q \in$ **N** and p, q have no common factor except 1.

For Examples: $\frac{3}{5}, \frac{4}{7}, \frac{5}{12}$; etc. are all rational numbers in lowest terms.

So far, we have learnt about the conversion of fractions into decimal numbers and vice-versa. Like all fractions, a rational number can be expressed as a decimal number.

Illustration 1: *Express* $\frac{9}{8}$ *in the decimal form, by Long Division.*

```
8 ) 9.000 ( 1.125
    8
    --
    10
     8
    --
     20
     16
    ---
      40
      40
     ---
       0
     ---
```

Hence, $\frac{9}{8}$ = 0.125.

Illustration 2: *Express* $-\frac{17}{8}$ *in the decimal form, by Long Division.*

First of all, we express $\frac{17}{8}$ in the decimal form.

$$\begin{array}{r|l} 8 & 17.000\ (2.125 \\ & \underline{16} \\ & \ \ 10 \\ & \ \ \underline{\ 8} \\ & \ \ \ 20 \\ & \ \ \ \underline{16} \\ & \ \ \ \ 40 \\ & \ \ \ \ \underline{40} \\ & \ \ \ \ \ 0 \end{array}$$

Hence, decimal form of $-\frac{17}{8} = -2.125$.

In the above illustrations, we have observed that division process stops after a finite number of steps. Thus there are finite number of digits in the decimal part of the given rational numbers. Such decimals are called **finite** or **terminating decimals**. We have the proper definition:

Definition: *Rational numbers having finite decimal part (or long division terminates after a finite number of steps) are called finite or terminating decimals.*

Observation: *A fraction* $\frac{p}{q}$ *is a terminating decimal only, if the prime factors of q are 2 and 5 only.*

For Examples: Each of the following fractions:

$$\frac{1}{2}, \frac{3}{4}, \frac{13}{25}$$

is a terminating decimal. [∵ *Denominator of each has no prime factor other than* 2 *and* 5]

There are rational numbers in whose case the division process never comes to an end. In such cases the remainder starts repeating after a certain number of steps. Here a digit or block of digits repeat itself.

For Examples: 0.444..., 0.1777..., 0.123412341234..., 1.3672408672408672408... ; etc.

These decimals are called **non-terminating repeating decimals** or **non-terminating recurring decimals**.

These decimal numbers are represented by putting a bar over the first block of repeating part and omitting the other repeating blocks.

Thus, we write,

$0.444... = 0.\overline{4}$, $0.1777... = 0.1\overline{7}$

$0.123412341234... = 0.\overline{1234}$, $1.3672408672408672408... = 1.3\overline{672408}$.

ILLUSTRATIVE EXAMPLES

Example 1: Without actual division, find which of the following rational numbers are terminating decimal fractions:

(*i*) $\frac{5}{64}$ **(*ii*)** $\frac{13}{24}$ **(*iii*)** $\frac{23}{80}$.

Solution: (*i*) $\frac{5}{64}$ has denominator 64.

And $64 = 2^6$

⇒ 64 has no prime factor other than 2.

Hence, $\frac{5}{64}$ is a terminating decimal.

(*ii*) $\frac{13}{24}$ has denominator 24.

And $24 = 2^3 \times 3$

⇒ 24 has a prime factor 3, other than 2 and 5.

Hence, $\frac{13}{24}$ is not a terminating decimal.

(*iii*) $\frac{23}{80}$ has denominator 80.

And $80 = 2^4 \times 5$

⇒ 80 has no prime factor other than 2 and 5.

Hence, $\frac{23}{80}$ is a terminating decimal.

Example 2: Find the decimal representation of $\frac{2}{3}$.

Solution: By *Long Division*, we have:

$$\begin{array}{r|l} 3 & 2.0\ (0.666 \\ & \underline{1\ 8} \\ & \ \ 20 \\ & \ \ \underline{18} \\ & \ \ \ 20 \\ & \ \ \ \underline{18} \\ & \ \ \ \ 2 \end{array}$$

Hence, $\frac{2}{3} = 0.666 .... = 0.\overline{6}$.

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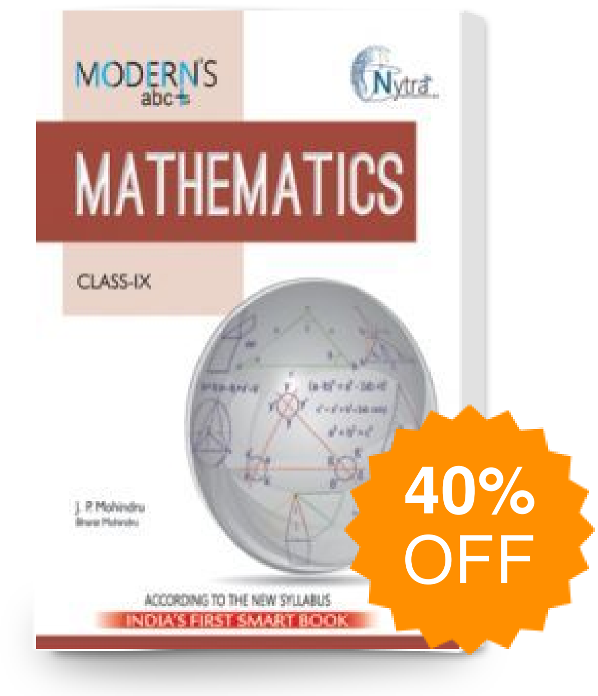

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