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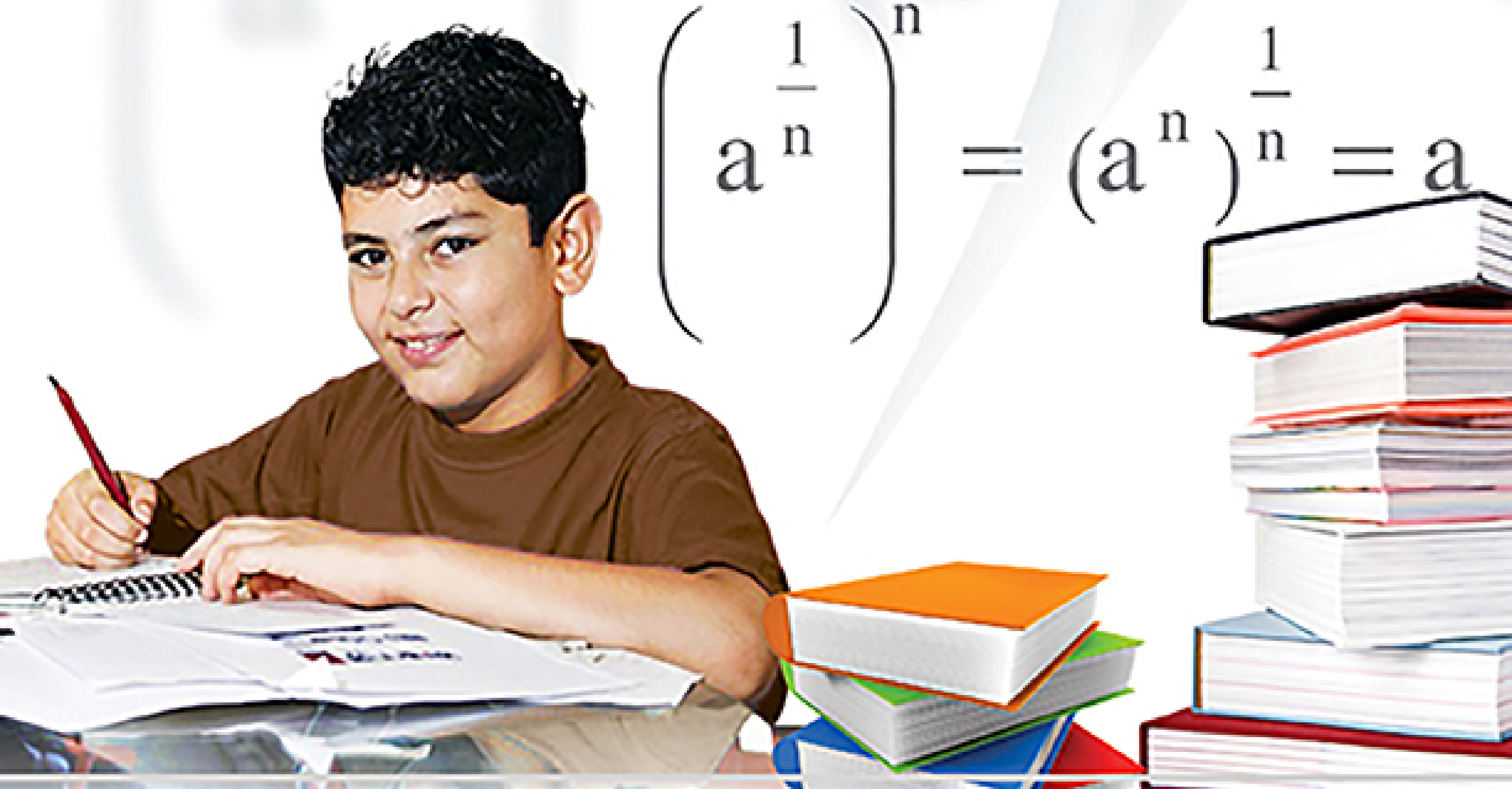
# IIT Foundation Mathematics

## Class – IX

$$\left(a^{\frac{1}{n}}\right)^{n} = \left(a^{n}\right)^{\frac{1}{n}} = a$$




S.K. GUPTA

ANUBHUTI GANGAL



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# PREFACE AND A NOTE FOR THE STUDENTS

***ARE YOU ASPIRING TO BECOME AN ENGINEER AND AN IIT SCHOLAR ?***

Here is the book especially designed to motivate you, to sharpen your intellect, to develop the right attitude and aptitude, and to lay a solid foundation for your success in various entrance examinations like **IIT, EAMCET, WBJEE, MPPET, SCRA, J&K CET, Kerala PET, OJEE, Rajasthan PET, AMU, BITSAT,** etc.

**SALIENT FEATURES**

1. Content based on the curriculum of the classes for ***CBSE, ICSE***, ***Andhra Pradesh*** and ***Boards of School Education of Other States.***
2. Full and comprehensive coverage of all the topics.
3. Detailed synopsis of each chapter at the beginning in the form of ***'Key Concepts'***. This will not only facilitate thorough ***'Revision'*** and ***'Recall'*** of every topic but also greatly help the students in understanding and mastering the concepts besides providing a ***back-up*** to classroom teaching.
4. The books are enriched with an exhaustive range of hundreds of thought provoking objective questions in the form of solved examples and practice questions in practice sheets which not only offer a great variety and reflect the modern trends but also invite, explore, develop and put to test the ***thinking, analysing*** and ***problem-solving skills of the students***.
5. **Answers, Hints** and **Solutions** have been provided to boost up the morale and increase the confidence level.
6. **Self Assessment Sheets** have been given at the end of each chapter to help the students to assess and evaluate their understanding of the concepts and learn to attack the problems independently.

We hope this book will be able to fulfil its aims and objectives and will be found immensely useful by the students aspiring to become top class engineers.

Suggestions for improvement and also the feedback received from various sources would be most welcome and gratefully acknowledged.

**AUTHORS**

# CONTENTS

# 1 Logarithms

## KEY FACTS

**1. Definition:** If $a$ and $n$ are positive real numbers such that $a \neq 1$ and $x$ is real, then $a^x = n \Rightarrow x = \log_a n$.
Here $x$ is said to be the logarithm of the number $n$ to the base $a$.

**Ex.** $4^3 = 64 \Rightarrow \log_4 64 = 3$, $10^{-1} = \frac{1}{10} = 0.1 \Rightarrow \log_{10} 0.1 = -1$, $5^x = 4 \Rightarrow x = \log_5 4$,

$a^0 = 1 \Rightarrow \log_a 1 = 0$, $a^1 = a \Rightarrow \log_a a = 1$.

**2. Some Important Facts about Logarithms**

- $\log_a n$ is real if $n > 0$
- $\log_a n$ is imaginary if $n < 0$
- $\log_a n$ is not defined if $n = 0$
- The logarithm of 1 to any base ***a***, $a > 0$ and $a \neq 1$ is zero. $\boxed{\log_a 1 = 0}$
- The logarithm of any number ***a***, $a > 0$ and $a \neq 1$, to the same base is 1. $\boxed{\log_a a = 1}$
- If ***a*** and ***x*** are positive real numbers, where $a \neq 1$, then $\boxed{a^{\log_a x} = x}$

**Proof.** Let $\log_a x = p$. Then, $x = a^p$ (By def.) $\Rightarrow x = a^{\log_a x}$ (Substituting the value of $p$)

**Ex.** $3^{\log_3 7} = 7$, $2^{\log_2 9} = 9$, $5^{\log_5 x} = x$

- For $a > 0$, $a \neq 1$, $\mathbf{\log_a x_1 = \log_a x_2 \Rightarrow x_1 = x_2\ (x_1, x_2 > 0)}$
- If $a > 1$ and $x > y$, then $\log_a x > \log_a y$.
- If $0 < a < 1$ and $x > y$, then $\log_a x < \log_a y$

**3. Laws of Logarithms**

For $x > 0$, $y > 0$ and $a > 0$ and $a \neq 1$, any real number $n$

- $\mathbf{\log_a xy = \log_a x + \log_a y}$ **Ex.** $\log_2(15) = \log_2(5 \times 3) = \log_2 5 + \log_2 3$
- $\mathbf{\log_a(x/y) = \log_a x - \log_a y}$ **Ex.** $\log_2\left(\frac{3}{7}\right) = \log_2 3 - \log_2 7$
- $\mathbf{\log_a(x)^n = n \log_a x}$ **Ex.** $\log (2)^5 = 5 \log 2$,

  $\log\left(\frac{a^3}{b^3}\right) = \log a^3 - \log b^3 = 3 \log a - 3 \log b$
- $\mathbf{\log_a x = \frac{1}{\log_x a}}$ **Ex.** $\log_5 2 = \frac{1}{\log_2 5}$
- $\mathbf{\log_{a^n} x = \frac{1}{n} \log_a x}$ **Ex.** $\log_8 7 = \log_{2^3}(7) = \frac{1}{3} \log_2 7$, $\log_{\sqrt{5}} 3 = \log_{(5)^{\frac{1}{2}}}(3) = \log_{5^{1/2}}(3) = \frac{1}{1/2} \log_5 3 = 2 \log_5 3$
- $\mathbf{\log_{a^n} x^m = \frac{m}{n} \log_a x}$ **Ex.** $\log_{2^5} 5^4 = \frac{4}{5} \log_2 5$

**Base changing formula**

- $\log_a x = \log_b x . \log_a b$ **Ex.** $\log_{12}32 = \log_{16}32.\ \log_{12}16.$ (The base has been changed from 12 to 16)

  ↑ Old base ↑ New base

- $x^{\log_a y} = y^{\log_a x}$ **Ex.** $3^{\log 7} = 7^{\log 3}$ (It being understood that base is same)

[**Proof**. $x^{\log_a y} \to = x^{\log_x y \,.\, \log_a x}$ (Base changing formula)

$= \left(x^{\log_x y}\right)^{\log_a x}$ (Using $n \log_a x = (\log_a x)^n$)

$= y^{\log_a x}$ (Using $x^{\log_x y} = y$.)

- $\log_a b = \dfrac{\log b}{\log a}$ (It being understood that base is same)
- If $\log_a b = x$ for all $a > 0$, $a \neq 1$, $b > 0$ and $x \in R$, then $\log_{1/a} b = -x$, $\log_a 1/b = -x$ and $\log_{1/a} 1/b = x$

## 4. Some Important Properties of Logarithms

- $a, b, c$ are in G.P. $\Leftrightarrow \log_a x, \log_b x, \log_c x$ are in H.P.
- $a, b, c$ are in G.P. $\Leftrightarrow \log_x a, \log_x b, \log_x c$ are in A.P.

## 5. Natural or Naperian logarithm **is denoted by $\log_e N$, where the base is $e$.**

**Ex.** $\log_e 7$, $\log_e\left(\dfrac{1}{64}\right)$, $\log_e b$, etc.

- **Common or Brigg's logarithm** is denoted by $\mathbf{\log_{10} N}$, where the base is **10.**

  **Ex.** $\log_{10}5$, $\log_{10}\left(\dfrac{1}{81}\right)$, etc.

- $\log_a x$ is a decreasing function if $0 < a < 1$
- $\log_a x$ is an increasing function if $a > 1$.

## 6. Characteristic and Mantissa

- **Characteristic:** The integral part of the logarithm is called characteristic.

(*i*) If the number is greater than unity and there are $n$ digits in integral part, then its characteristic = $(n - 1)$

(*ii*) When the number is less than 1, the characteristic is one more than the number of *zeroes* between the decimal point and the first significant digit of the number and is negative. It is written as $\overline{(n+1)}$ or Bar $(n + 1)$.

**Ex.**

| Number | Characteristic | Number | Characteristic |
|---|---|---|---|
| 4.1456 | 0 | 0.823 | $\bar{1}$ |
| 24.8920 | 1 | 0.0234 | $\bar{2}$ |
| 238.1008 | 2 | 0.000423 | $\bar{4}$ |

## 7. Arithmetic Progression. **A sequence** $a_1, a_2, a_3, \ldots\ldots, a_n$ is said to be in arithmetic progression, when $a_2 - a_1 = a_3 - a_2 = \ldots\ldots\ldots = a_n - a_{n-1}$, *i.e.*, when the terms in the sequence increase or decrease by a constant quantity called the ***common difference***.

**Ex.** 1, 3, 5, 7, 9, .... 6, 11, 17, 23, .... – 5, –2, 1, 4, 7, ....

- **Sum of first '$n$' terms of an Arithmetic Progression**

  $$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + l],$$

  where $a$ = first term, $n$ = number of terms, $d$ = common difference, $l$ = last term.

- **Sum of first "$n$" natural numbers.**

  $$S_n = 1 + 2 + 3 + \ldots\ldots\ldots\ldots + n = \frac{n(n+1)}{2}.$$

Also, written as $\Sigma n = \dfrac{n(n+1)}{2}$

- Also, **if *a, b, c* are in A.P.** then $2b = a + c$

**8. Geometric Progression :** A sequence $a_1, a_2, a_3, ..........., a_n$ is said to be in Geometric Progression when,

$$\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} = ........... = \frac{a_n}{a_{n-1}} = r(\text{say})$$

where $a_1, a_2, a_3, ..........$ are all non zero numbers and $r$ is called the **common ratio.**

**Ex.** 3, 6, 12, 24, ................. $r = 2;$

64, 16, 4, 1, $\frac{1}{4}, \frac{1}{16}. \frac{1}{64}$, ........... $r = \frac{1}{4}$

- **Sum of first *n* terms of a G.P.** $S_n = \dfrac{a(r^n - 1)}{(r-1)}$ if $r > 1$ $= \dfrac{a(1-r^n)}{(1-r)}$ if $r < 1$ $= \dfrac{lr - a}{r-1}$

  where, $a$ = first term, $r$ = common ratio, $l$ = last term

- **Sum of an infinite G.P.** $S_\infty = \dfrac{a}{1-r}$, where $a$ = first term, $r$ = common ratio.

- For three terms $a, b, c$ to be in G.P., $b^2 = ac$

**9. Harmonic Progression :** A series of quantities $a_1, a_2, a_3, ..........., a_n$ are said to be in H.P. when their reciprocals $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, .........., \frac{1}{a_n}$ are in A.P.

- When three quantities $a, b, c$ are in H.P., then, $b = \dfrac{2ac}{a+c}$.

## SOLVED EXAMPLES

**Ex. 1. If $\log_a 5 + \log_a 25 + \log_a 125 + \log_a 625 = 10$, then find the value of *a*.**

**Sol.** $\log_a 5 + \log_a 25 + \log_a 125 + \log_a 625 = 10$

$\Rightarrow \log_a (5 \times 25 \times 125 \times 625) = 10$

$\Rightarrow \log_a (5^1 \times 5^2 \times 5^3 \times 5^4) = 10$

$\Rightarrow \log_a 5^{10} = 10 \Rightarrow a^{10} = 5^{10} \Rightarrow \mathbf{a = 5.}$ [Using $\log_a x = n \Rightarrow x = a^n$]

**Ex. 2. Solve for *x* : $\log_{10} [\log_2 (\log_3 9)] = x$.**

**Sol.** $\log_{10} [\log_2 (\log_3 9)] = x$

$\Rightarrow \log_2 (\log_3 9) = 10^x$

$\Rightarrow \log_2 (\log_3 3^2) = 10^x$

$\Rightarrow \log_2 (2 \log_3 3) = 10^x$

$\Rightarrow \log_2 2 = 10^x \Rightarrow 10^x = 1 = 10^0 \Rightarrow \mathbf{x = 0}.$

**Ex. 3. Find the value of $\log_x x + \log_x x^3 + \log_x x^5 + ........ + \log_x x^{2n-1}$.**

**Sol.** $\log_x x + \log_x x^3 + \log_x x^5 + ........ + \log_x x^{2n-1} = \log_x x + 3\log_x x + 5\log_x x + ......... + (2n-1)\log_x x$

$= 1 + 3 + 5 + ........... + (2n-1) = \frac{n}{2}[1 + (2n-1)] = \mathbf{n^2}$ $\left[\text{Using } \log_x x = 1 \text{ and for A.P. } S_n = \frac{n}{2}(a + l)\right]$

**Ex. 4. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, show that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$.**

**Sol.** $f\left(\frac{2x}{1+x^2}\right) = \log\left[\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right] = \log\left[\frac{1+x^2+2x}{1+x^2-2x}\right] = \log\left[\frac{(1+x)^2}{(1-x)^2}\right] = 2\log\left[\frac{1+x}{1-x}\right] = \mathbf{2f(x)}.$

**Ex. 5. If $a = \log_{24}12$, $b = \log_{36}24$, $c = \log_{48}36$, then prove that $1 + abc = 2bc$.**

**Sol.** $1 + abc = 1 + \log_{24}12.\ \log_{36}24.\ \log_{48}36 = 1 + \log_{36}12.\ \log_{48}36$

$= 1 + \log_{48}12 = \log_{48}48 + \log_{48}12$ $[\because \log_a x.\ \log_b a = \log_b x]$

$= \log_{48}(48 \times 12) = \log_{48}(24 \times 24)$

$= \log_{48}(24)^2 = 2\log_{48}24.$ ...(*i*)

Also, $2bc = 2\log_{36}24.\ \log_{48}36 = 2\log_{48}24$ ...(*ii*)

From (*i*) and (*ii*), we have RHS = LHS.

**Ex. 6. Solve $\log_{2x+3}(6x^2 + 23x + 21) = 4 - \log_{3x+7}(4x^2 + 12x + 9)$.**

**Sol.** Given, $\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$

$\Rightarrow \log_{(2x+3)}(2x+3)(3x+7) = 4 - \log_{(3x+7)}(2x+3)^2$

$\Rightarrow \log_{(2x+3)}(2x+3) + \log_{(2x+3)}(3x+7) = 4 - 2\log_{(3x+7)}(2x+3)$

$\Rightarrow \log_{(2x+3)}(3x+7) + 2\log_{(3x+7)}(2x+3) = 4 - 1 = 3$ [Since $\log_{2x+3}(2x+3) = 1$]

$\Rightarrow \log_{(2x+3)}(3x+7) + \frac{2}{\log_{(2x+3)}(3x+7)} = 3$ $\left[\text{Using } \log_a x = \frac{1}{\log_x a}\right]$

Let $\log_{(2x+3)}(3x+7) = t$. Then,

$t + \frac{2}{t} = 3 \Rightarrow t^2 - 3t + 2 = 0 \Rightarrow (t-1)(t-2) = 0 \Rightarrow t = 1, 2$

$t = 1 \Rightarrow \log_{(2x+3)}(3x+7) = 1 \Rightarrow \log_{(2x+3)}(3x+7) = \log_{(2x+3)}(2x+3)$ [Replacing 1 by $\log_{(2x+3)}(2x+3)$]

$\Rightarrow 3x + 7 = 2x + 3 \Rightarrow x = -4.$

$t = 2 \Rightarrow \log_{(2x+3)}(3x+7) = 2 \Rightarrow \log_{(2x+3)}(3x+7) = \log_{(2x+3)}(2x+3)^2$

$\Rightarrow (3x+7) = (2x+3)^2 \Rightarrow 4x^2 + 9x + 2 = 0 \Rightarrow (4x+1)(x+2) = 0 \Rightarrow x = -1/4, -2$

But $x = -4$ and $-2$ are extraneous solutions, so $x = -\mathbf{\frac{1}{4}}$.

**Ex. 7. If $\log_x(a-b) - \log_x(a+b) = \log_x(b/a)$, find $\frac{a^2}{b^2} + \frac{b^2}{a^2}$.** ***(CAT 2012)***

**Sol.** Given, $\log_x(a-b) - \log_x(a+b) = \log_x(b/a) \Rightarrow \log_x\left[\frac{(a-b)}{(a+b)}\right] = \log_x\left(\frac{b}{a}\right)$

$\Rightarrow a(a-b) = b(a+b) \Rightarrow a^2 - ab = ab + b^2$

$\Rightarrow a^2 - b^2 = 2ab \Rightarrow a^2 - 2ab - b^2 = 0 \Rightarrow \left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right) - 1 = 0$

This is a quadratic equation in $\frac{a}{b}$ and the product of the roots is $-1$ *i.e*, if $a/b$ is a root, then $\left(-\frac{b}{a}\right)$ is the other root. Also, sum of its roots = 2

$\therefore \left(\frac{a}{b}\right)^2 + \left(\frac{b}{a}\right)^2 = \frac{a^2}{b^2} + \frac{b^2}{a^2} = \left[\frac{a}{b} + \left(-\frac{b}{a}\right)\right]^2 + 2 = 2^2 + 2 = \mathbf{6}.$

**Ex. 8. If $\log_e 2.\log_b 625 = \log_{10} 16.\log_e 10$, then find the value of $b$.**

**Sol.** Given, $\log_e 2.\log_b 625 = \log_{10}16.\log_e 10 \Rightarrow \log_e 2.\log_b 5^4 = \log_{10}2^4.\log_e 10$

$\Rightarrow \log_e 2.\ 4\log_b 5 = 4\log_{10}2.\log_e 10$

$\Rightarrow \log_b 5. = \dfrac{\log_{10}2.\log_e 10}{\log_e 2} = \dfrac{\log_e 2}{\log_e 2} = 1 \Rightarrow b^1 = 5 \Rightarrow b = \mathbf{5}.$ $\quad [\because \log_a x.\log_x b = \log_a b]$

**Ex. 9. If $(x^4 - 2x^2y^2 + y^2)^{a-1} = (x-y)^{2a}(x+y)^{-2}$, then the value of $a$ is**

(*a*) $x^2 - y^2$ (*b*) $\log(xy)$ (*c*) $\dfrac{\log(x-y)}{\log(x+y)}$ (*d*) $\log(x-y)$

**Sol.** Given, $(x^4 - 2x^2y^2 + y^2)^{a-1} = (x-y)^{2a}(x+y)^{-2}$

$\Rightarrow [(x^2-y^2)^2]^{a-1} = (x-y)^{2a}(x+y)^{-2}$

$\Rightarrow (x-y)^{2(a-1)}(x+y)^{2(a-1)} = (x-y)^{2a}(x+y)^{-2}$

$\Rightarrow \dfrac{(x-y)^{2(a-1)}}{(x-y)^{2a}}.\dfrac{(x+y)^{2(a-1)}}{(x+y)^{-2}} = 1 \Rightarrow (x-y)^{-2}(x+y)^{2a} = 1$

$\Rightarrow \log[(x-y)^{-2}(x+y)^{2a}] = \log 1 \Rightarrow -2\log(x-y) + 2a\log(x+y) = \log 1$

$\Rightarrow 2a\log(x+y) = 2\log(x-y) \Rightarrow a = \mathbf{\dfrac{\log(x-y)}{\log(x+y)}}.$ [Since $\log 1 = 0$]

**Ex. 10. If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in GP, then $x$ is**

(*a*) $\log_a(\log_b a)$ (*b*) $\log_a(\log_e a) + \log_a(\log_e b)$

(*c*) $-\log_a(\log_a b)$ (*d*) $\log_a(\log_e b) - \log_a(\log_e a)$

**Sol.** If $\log_x a$, $a^{x/2}$ and $\log_b x$ are in GP, then $\left(a^{x/2}\right)^2 = (\log_b x) \times (\log_x a)$

$\Rightarrow a^x = \log_b a \Rightarrow \log a^x = \log(\log_b a) \Rightarrow x\log a = \log(\log_b a) \Rightarrow x\log_a a = \log_a(\log_b a)$

$\Rightarrow$ **$x = \log_a(\log_b a)$.**

**Ex. 11. What is the least value of the expression $2\log_{10}x - \log_x(1/100)$ for $x > 1$ ?**

**Sol.** $2\log_{10}x - \log_x \dfrac{1}{100} = 2\log_{10}x - \dfrac{\log_{10}10^{-2}}{\log_{10}x}$ $\quad \left[\text{Using } \log_a b = \dfrac{\log_x b}{\log_x a}\right]$

$= 2\log_{10}x + \dfrac{2}{\log_{10}x} = 2\left(\log_{10}x + \dfrac{1}{\log_{10}x}\right)$

Given, $x > 1 \Rightarrow \log_{10}x > 0$

But since AM $\geq$ GM

$\therefore \left[\dfrac{\log_{10}x + \dfrac{1}{\log_{10}x}}{2}\right] \geq \sqrt{\log_{10}x \times \dfrac{1}{\log_{10}x}}$

$\Rightarrow \log_{10}x + \dfrac{1}{\log_{10}x} \geq 2 \Rightarrow 2\left[\log_{10}x + \dfrac{1}{\log_{10}x}\right] \geq 4$

For $x = 10$, $2[\log_{10}x + \log_{10}x] \geq 4$

Hence, the least value of $\left[\log_{10}x - \log_x \dfrac{1}{100}\right]$ is 4.

**Ex. 12. If $\log_3 2$, $\log_3(2^x - 5)$ and $\log_3(2^x - 7/2)$ are in A.P., then what is the value of $x$ ?**

**Sol.** Given, $\log_3 2$, $\log_3(2^x - 5)$ and $\log_3(2^x - 7/2)$ are in A.P.

$\Rightarrow 2\left[\log_3(2^x - 5)\right] = \log_3 2 + \log_3\left(2^x - \dfrac{7}{2}\right)$

$\Rightarrow$ $\log_3 (2^x - 5)^2 = \log_3 [2 \times (2^x - 7/2)]$

$\Rightarrow$ $(2^x - 5)^2 = (2^{x+1} - 7) \Rightarrow 2^{2x} - 10.2^x + 25 = 2.2^x - 7$

$\Rightarrow$ $2^{2x} - 12.2^x + 32 = 0 \Rightarrow y^2 - 12y + 32 = 0$ [Let $y = 2^x$]

$\Rightarrow$ $(y - 8)(y - 4) = 0 \Rightarrow y = 8$ or $4 \Rightarrow 2^x = 8$ or $2^x = 4 \Rightarrow x =$ **3 or 2.**

**Ex. 13. Let $u = (\log_2 x)^2 - 6(\log_2 x) + 12$, where $x$ is a real number. Then the equation $x^u = 256$ has :**

**(*a*) No solution for $x$** **(*b*) Exactly one solution for $x$**

**(*c*) Exactly two distinct solutions for $x$** **(*d*) Exactly three distinct solutions for $x$** ***(CAT 2004)***

**Sol.** Given, $u = (\log_2 x)^2 - 6(\log_2 x) + 12 = p^2 - 6p + 12$ (where $p = \log_2 x$) ...(*i*)

Also, given, $x^u = 256$

Taking log to the base 2 of both the sides, we have

$u \log_2 x = \log_2 256 = \log_2 2^8 = 8 \log_2 2 \Rightarrow u \log_2 x = 8 \Rightarrow u = \dfrac{8}{\log_2 x} = 8/p$ ...(*ii*)

From (*i*) and (*ii*) $\dfrac{8}{p} = p^2 - 6p + 12$

$\Rightarrow$ $8 = p^3 - 6p^2 + 12 \Rightarrow p^3 - 6p^2 + 12p - 8 = 0$

$\Rightarrow$ $(p - 2)^3 = 0 \Rightarrow p = 2.$ [Using $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$]

$\therefore$ $\log_2 x = 2 \Rightarrow x = 2^2 = 4$

Hence the equation $u^4 = 256$ has **exactly one solution.**

**Ex. 14. If $\log_y x = (a.\ \log_z y) = (b.\ \log_x z) = ab$, then which of the following pairs of values for $(a, b)$ is not possible ?**

**(*a*) $\left(-2, \dfrac{1}{2}\right)$** **(*b*) (1, 1)** **(*c*) (0.4, 2.5)** **(*d*) $\left(\pi, \dfrac{1}{\pi}\right)$** **(*e*) (2, 2)**

***(CAT 2004)***

**Sol.** Given, $\log_y x = (a.\ \log_z y) = (b \log_x z) = ab$

$\Rightarrow a = \dfrac{\log_y x}{\log_z y}$ and $b = \dfrac{\log_y x}{\log_x z}$

$$\therefore a \times b = \frac{\log_y x}{\log_z y} \times \frac{\log_y x}{\log_x z} = \frac{\left[\frac{\log x}{\log y}\right]}{\left[\frac{\log y}{\log z}\right]} \times \frac{\left[\frac{\log x}{\log y}\right]}{\left[\frac{\log z}{\log x}\right]} = \left(\frac{\log x}{\log y}\right)^3 = (\log_y x)^3 = (ab)^3$$

$\Rightarrow ab - a^3b^3 = 0 \Rightarrow ab(1 - a^2b^2) = 0 \Rightarrow ab = \pm 1$

$\therefore$ Only option (*e*) does not satisfy the condition, sin(2, 2) is not a possible value of $(a, b)$.

## PRACTICE SHEET

### LEVEL–1

**1.** (*i*) The solution of $\log_\pi (\log_2 (\log_7 x)) = 0$ is

(*a*) 2 (*b*) $\pi^2$ (*c*) $7^2$ (*d*) None of these

**(*WBJEE 2008*)**

**Similar questions**

(*ii*) $\log_{27} (\log_3 x) = \dfrac{1}{3} \Rightarrow x =$

(*a*) 3 (*b*) 6 (*c*) 9 (*d*) 27

**(*EAMCET 2004*)**

(*iii*) The solution of $\log_{99} (\log_2 (\log_3 x)) = 0$

(*a*) 4 (*b*) 9 (*c*) 44 (*d*) 99

**(*BCECE 2006*)**

**2.** If $x = \log_b a$, $y = \log_c b$, $z = \log_a c$, then $xyz$ is

(*a*) 0 (*b*) 1 (*c*) $abc$ (*d*) $a + b + c$

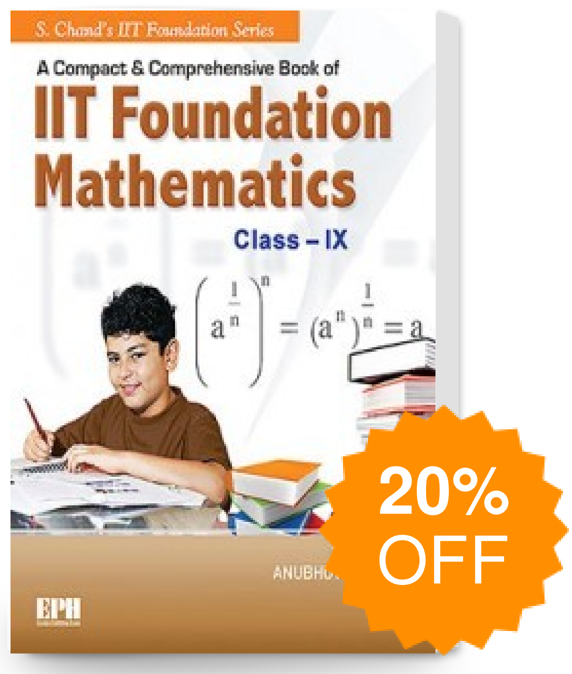
**(*UPSEE 2003*)**

**3.** (*i*) $7^{2 \log_7 5}$ is equal to

(*a*) 5 (*b*) 25 (*c*) $\log_7 25$ (*d*) $\log_7 35$

**(*KCET 2007*)**

# IIT Foundation Mathematics For Class IX

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