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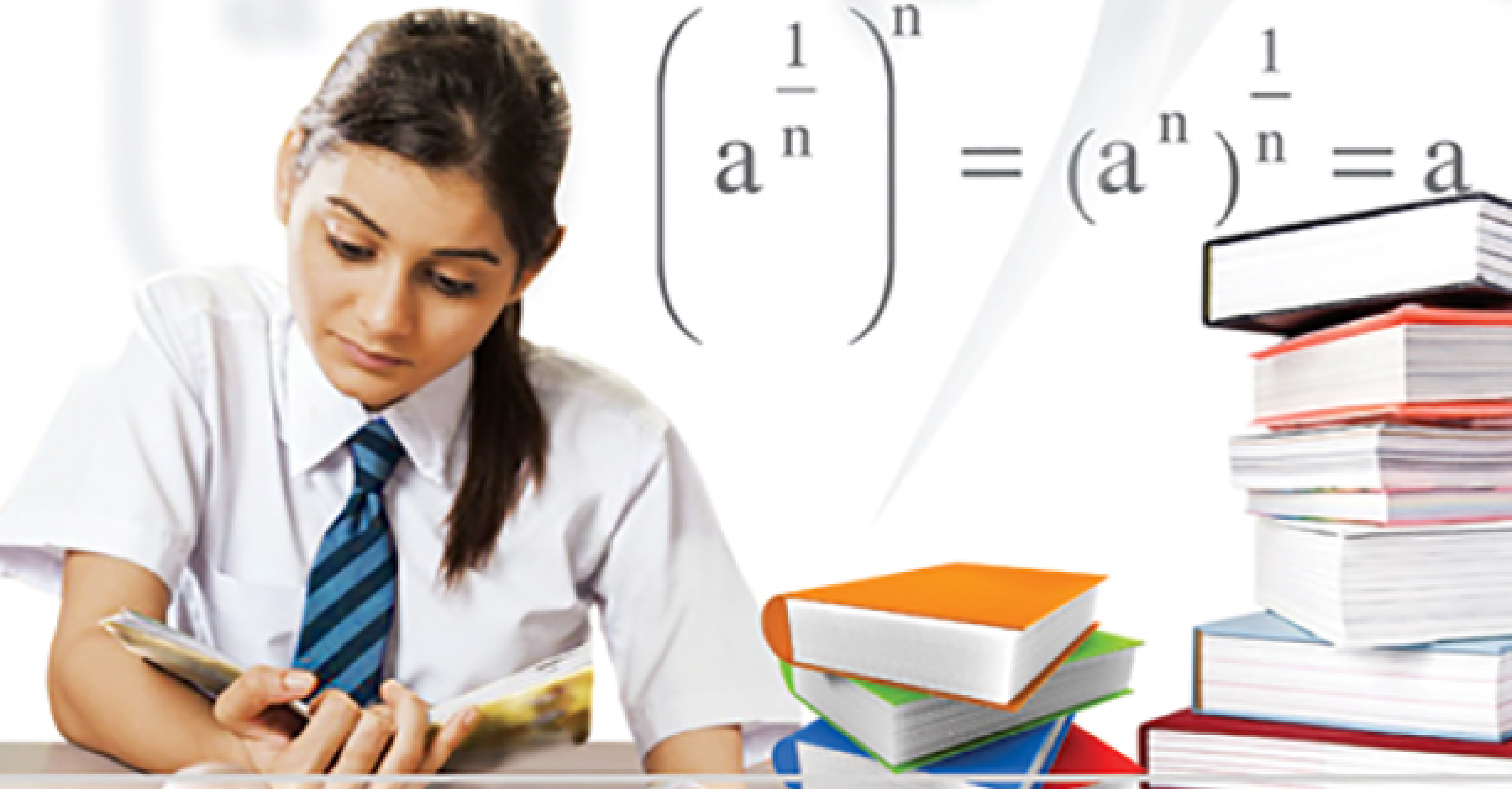
Class – X

$$\left(a^{\frac{1}{n}}\right)^n = (a^n)^{\frac{1}{n}} = a$$



S.K. GUPTA
ANUBHUTI GANGAL

EPH

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PREFACE AND A NOTE FOR THE STUDENTS

ARE YOU ASPIRING TO BECOME AN ENGINEER AND AN IIT SCHOLAR ?

Here is the book especially designed to motivate you, to sharpen your intellect, to develop the right attitude and aptitude, and to lay a solid foundation for your success in various entrance examinations like **IIT, EAMCET, WBJEE, MPPET, SCRA, J&K CET, Kerala PET, OJEE, Rajasthan PET, AMU, BITSAT,** etc.

SALIENT FEATURES

1. Content based on the curriculum of the classes for ***CBSE, ICSE***, ***Andhra Pradesh*** and ***Boards of School Education of Other States.***
2. Full and comprehensive coverage of all the topics.
3. Detailed synopsis of each chapter at the beginning in the form of ***'Key Facts'***. This will not only facilitate thorough ***'Revision'*** and ***'Recall'*** of every topic but also greatly help the students in understanding and mastering the concepts besides providing a ***back-up*** to classroom teaching.
4. The books are enriched with an exhaustive range of hundreds of thought provoking objective questions in the form of solved examples and practice questions in practice sheets which not only offer a great variety and reflect the modern trends but also invite, explore, develop and put to test the ***thinking, analysing*** and ***problem-solving skills of the students***.
5. **Answers, Hints** and **Solutions** have been provided to boost up the morale and increase the confidence level.
6. **Self-Assessment Sheets** have been given at the end of each chapter to help the students to assess and evaluate their understanding of the concepts and learn to attack the problems independently.

We hope this book will be able to fulfil its aims and objectives and will be found immensely useful by the students aspiring to become top class engineers.

Suggestions for improvement and also the feedback received from various sources would be most welcome and gratefully acknowledged.

AUTHORS

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1 Sequence and Series

ARITHMATIC PROGRESSION (A.P.)

KEY FACTS

1. A **sequence** is a set of numbers specified in a definite order by some assigned rule or law.

Ex. 2, 7, 12, 17.... (Each succeeding term is obtained by adding 5 to the preceding term)

1, 2, 4, 8, 16.... (Each succeeding term is obtained by multiplying the preceding term by 2)

A **finite sequence** is that which ends or has a last term.

Ex. 5, 9, 13, 17, 21.

An **infinite sequence** is one which has no last term.

Ex. 3, 6, 12, 24, 48....

In general, $\boldsymbol{a_n}$ or $\boldsymbol{T_n}$ denotes the ***n*th term** of a sequence.

2. An expression consisting of the term of a sequence, alternating with the symbol '+' is called a **series.**

Ex. The sequence $\frac{2}{3}, \frac{4}{5}, \frac{6}{7},$ .... expressed as a series is $\frac{2}{3} + \frac{4}{5} + \frac{6}{7} + ...$

3. **Arithmetic Progression (A.P.):** A sequence is called an arithmetic progression if its terms continually increase or decrease by the same number. The fixed number by which they increase or decrease is called the **common difference.** Three quantities *a, b, c* will be in A.P. if $b - a = c - b$, *i.e.*, $\boldsymbol{2b = a + c}$.

(*a*) ***n*th term of an A.P.:** The *n*th term of an A.P. $a, a + d, a + 2d, a + 3d, ...$ is

$$\boldsymbol{T_n = a + (n - 1)d}$$

where $\boldsymbol{T_n}$ denotes *n*th term, $\boldsymbol{a}$ the first term, $\boldsymbol{n}$ the number of terms and $\boldsymbol{d}$ the common difference.

Also, common difference $\boldsymbol{d = T_n - T_{n-1}}$.

Ex. The 9th term of the A.P.: 2, 5, 8.... is

$T_9 = 2 + (9 - 1) \times 3 = 2 + 24 = \mathbf{26}.$

Note: Here $a = 2$, $d = 3$.

(*b*) **Sum of *n* terms of an A.P.**

Let the A.P. be $a, a + d, a + 2d, ...$ Let l be the last term and S the required sum. Then,

$$\boldsymbol{S = \frac{n}{2}(a + l)} = \frac{\text{Number of terms}}{2}(\text{First term} + \text{Last term})$$

$$= \frac{n}{2}(a + a + (n - 1)\, d) = \frac{n}{2}(2a + (n - 1)d),$$

where n is the number of terms, a is first term and d is common difference.

Also, nth term = Sum of n terms – Sum of $(n - 1)$ terms

i.e., $\boldsymbol{T_n = S_n - S_{n-1}}$.

Ex. The sum of 20 terms of the the A.P. 1, 3, 5, 7, 9.... is

$$S_{20} = \frac{20}{2}(2\times1+(20-1)\times2) \qquad (\because a = 1, d = 2, n = 20)$$
$$= 10 \times 40 = \mathbf{400.}$$

(*c*) **Arithmetic Mean:** The Arithmetic Mean between two numbers is the number which when placed between them forms an arithmetic progression with them. Thus if x is the arithmetic mean of two given numbers a and b, then a, x, b form an A.P.

$\therefore \quad x - a = b - x \Rightarrow \boldsymbol{x = \frac{a+b}{2}}$

Ex. (*a*) The arithmetic mean between – 4 and 6 is $\frac{6+(-4)}{2} = \mathbf{1.}$

(*b*) **Find 4 arithmetic means between 3 and 23.**

Let A_1, A_2, A_3, A_4, be the four arithmetic means between 3 and 23.

Then, 3, A_1, A_2, A_3, A_4, 23 form an A.P.

Here, First term = a = 3

Sixth term = T_6 = 23

Number of terms = n = 6

Common difference = d = ?

$T_6 = a + (n - 1) \times d$

$\Rightarrow 23 = 3 + (6 - 1) \times d$

$\Rightarrow 23 = 3 + 5d \Rightarrow 5d = 20 \Rightarrow d = \mathbf{4.}$

$\therefore \quad A_1 = 3 + 4 = 7, A_2 = 7 + 4 = 11, A_3 = 11 + 4 = 15, A_4 = 15 + 4 = 19.$

(*d*) **Some useful facts about an A.P.**

I. ***If each term of a given A.P. is increased or decreased or multiplied or divided by the same number, the resulting progression is also an A.P.***

II. ***If a, b, c are in A.P., then*** $\boldsymbol{\frac{a-b}{b-c} = 1}$, ***i.e.,*** $\frac{\textbf{First term – Second term}}{\textbf{Second term – Third term}} = \mathbf{1.}$

(*e*) If we have to find an odd number of terms in A.P. whose sum is given, it is convenient to take a as the middle term and d as the common difference. Thus, three terms may by taken as $\boldsymbol{a - d, a, a + d}$ and five terms as $\boldsymbol{a - 2d, a - d, a, a + d, a + 2d}$. **[Solved Ex. 13, 14].** If we have to find even number of terms, we take $a - d, a + d$ as the middle terms and $2d$ as the common difference. Then, four terms are taken as $\boldsymbol{a - 3d, a - d, a + d, a + 3d}$.

SOLVED EXAMPLES

Ex. 1. The 8th term of a series in A.P. is 23 and the 102th term is 305. Find the series.

Sol. Let a be the first term and d be the common difference.

Then, $T_8 = a + (8 - 1)\, d \Rightarrow 23 = a + 7d$ ...(*i*)

$T_{102} = a + (102 - 1)\, d \Rightarrow 305 = a + 101d$ ...(*ii*)

$\therefore$ Eqn (*ii*) – Eqn (*i*)

$\Rightarrow 94d = 282 \Rightarrow d = 3$

Now substituting $d = 3$ in (*i*), we get $23 = a + 21 \Rightarrow a = 2$.

$\therefore \quad a = 2, d = 3 \Rightarrow$ Series is 2 + 5 + 8 + 11 + ...

Ex. 2. If *a*, *b* and *c* be respectively the *p*th, *q*th and *r*th terms of an A.P., prove that $\boldsymbol{a(q - r) + b(r - p) + c(p - q) = 0}$.

Sol. Let A be the first term and D the common difference of the given A.P.

Then, $T_p = A + (p - 1)\, D = a$ ...(*i*)

$T_q = A + (q - 1)\, D = b$ ...(*ii*)

$T_r = A + (r - 1)\, D = c$ ...(*iii*)

$\therefore$ $(i) \times (q - r) + (ii) \times (r - p) + (iii) \times (p - q)$

$\Rightarrow A\,[(q - r) + (r - p) + (p - q)] + D\,[(p - 1)(q - r) + (q - 1)(r - p) + (r - 1)(p - q)]$
$= a\,(q - r) + b\,(r - p) + c\,(p - q)$

$\Rightarrow A\,[0] + D\,[pq - q - pr + r + qr - r - pq + p + pr - p - qr + q] = a\,(q - r) + b\,(r - p) + c\,(p - q)$

$\Rightarrow a\,(q - r) + b\,(r - p) + c\,(p - q) = A \times 0 + D \times 0 =$ **0.**

Ex. 3. Which term of the progression 19, $18\frac{1}{5}, 17\frac{2}{5}$, ... is the first negative term?

Sol. Here $a = 19$, $d = 18\frac{1}{5} - 19 = -\frac{4}{5}$

Let the nth term be the first negative term. Then,

$$T_n < 0 \Rightarrow a + (n - 1)\,d < 0$$
$$\Rightarrow 19 + (n - 1)\,(-4/5) < 0$$
$$\Rightarrow 19 - \frac{4}{5}n + \frac{4}{5} < 0$$
$$\Rightarrow \frac{99}{5} - \frac{4}{5}n < 0 \quad \Rightarrow n > \frac{99}{5} \times \frac{5}{4} \Rightarrow n > \frac{99}{4} = 24\frac{3}{4}$$
$$\Rightarrow n = \mathbf{25.}$$

Ex. 4. Find the sum of the series 101 + 99 + 97 + ... + 47.

Sol. In this case, we have to first find the number of terms.

Here $a = 101$, $l = T_n = 47$, $d = 99 - 101 = -2$

$\therefore$ $47 = 101 + (n - 1) \times (-2)$, where n = number of terms

$\Rightarrow 47 = 101 - 2n + 2$

$\Rightarrow 2n = 103 - 47 = 56 \Rightarrow n = 28$

$\therefore$ $S_n = \frac{n}{2}(a + l)$

$\Rightarrow S_{28} = \frac{28}{2}(101 + 47) = 14 \times 148 =$ **2072.**

Ex. 5. The sums of n terms of two arithmetic series are in the ratio $2n + 1 : 2n - 1$. Find the ratio of their 10th terms.

Sol. Let the two arithmetic series be $a, a + d, a + 2d$ ..... and $A, A + D, A + 2D$, ...

Given that, $\frac{n/2[2a + (n-1)\,d]}{n/2[2A + (n-1)\,D]} = \frac{2n+1}{2n-1}$

$$\Rightarrow \frac{2a + (n-1)\,d}{2A + (n-1)\,D} = \frac{2n+1}{2n-1} \qquad ...(i)$$

Ratio of the 10th terms of these series $= \frac{t_{10}}{T_{10}} = \frac{a + 9d}{A + 9D} = \frac{2a + 18d}{2A + 18D}$

$\therefore$ Putting $n = 19$ in (i), we have $\frac{t_{10}}{T_{10}} = \frac{2a + 18d}{2A + 18D} = \frac{2 \times 19 + 1}{2 \times 19 - 1} = \mathbf{\frac{39}{37}}$.

Ex. 6. The sum of the first n terms of the arithmetical progression 3, $5\frac{1}{2}$, 8, ... is equal to the 2nth term of the A.P. $16\frac{1}{2}, 28\frac{1}{2}, 40\frac{1}{2}$, ... Calculate the value of n.

Sol. For the A.P. : 3, $5\frac{1}{2}$, 8, ... $a = 3, d = 2\frac{1}{2}$, number of terms = n

$\Rightarrow \quad S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}\left(6 + (n-1)\times\frac{5}{2}\right)$ ...(*i*)

For the A.P.: $16\frac{1}{2}, 28\frac{1}{2}, 40\frac{1}{2}$ ... $a = 16\frac{1}{2}, d = 12.$

$\therefore \quad T_{2n} = a + (2n-1)d = 16\frac{1}{2} + (2n-1)\times 12$ ...(*ii*)

Given, $S_n = T_{2n} \Rightarrow \frac{n}{2}\left(6 + (n-1)\frac{5}{2}\right) = \frac{33}{2} + (2n-1)\times 12$ (From (*i*) and (*ii*))

$\Rightarrow \frac{6n}{2} + \frac{5n^2}{4} - \frac{5n}{4} = \frac{33}{2} + 24n - 12$ $\quad \Rightarrow \frac{7n}{4} + \frac{5n^2}{4} = \frac{9}{2} + 24n$

$\Rightarrow \frac{5n^2}{4} - \frac{89n}{4} - \frac{9}{2} = 0$ $\quad \Rightarrow 5n^2 - 89n - 18 = 0$

$\Rightarrow 5n^2 - 90n + n - 18 = 0$ $\quad \Rightarrow 5n(n-18) + 1(n-18) = 0$

$\Rightarrow (n-18)(5n+1) = 0$ $\quad \Rightarrow n = 18 \text{ or } -\frac{1}{5}$

$\Rightarrow$ Neglecting – *ve* value, we have n = **18.**

Ex. 7. Let a_1, a_2, a_3, ... be the terms of an A.P. If $\frac{a_1 + a_2 + a_3 + ... + a_p}{a_1 + a_2 + a_3 + ... + a_q} = \frac{p^2}{q^2}$ ($p \neq q$), then find $\frac{a_6}{a_{21}}$.

(*AIEEE 2006*)

Sol. Let d be the common difference for the A.P.; a_1, a_2, a_3, ...

Then, $\frac{S_p}{S_q} = \frac{p/2\,[2a_1 + (p-1)d]}{q/2\,[2a_1 + (q-1)d]} = \frac{p^2}{q^2}$

$\Rightarrow \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$ $\quad \Rightarrow q[2a_1 + (p-1)d] = p[2a_1 + (q-1)d]$

$\Rightarrow 2a_1q + pqd - qd = 2a_1p + pqd - pd$ $\quad \Rightarrow pd - qd = 2a_1p - 2a_1q$

$\Rightarrow d(p-q) = 2a_1(p-q)$ $\quad \Rightarrow d = 2a_1$

Now $\frac{\text{Term 6}}{\text{Term 21}} = \frac{a_6}{a_{21}} = \frac{a_1 + (6-1)d}{a_1 + (21-1)d} = \frac{a_1 + 5\times 2a_1}{a_1 + 20\times 2a_1} = \frac{11a_1}{41a_1} = \mathbf{\frac{11}{41}}$.

Ex. 8. If the number of terms of an A.P. is (2*n* + 1), then what is the ratio of the sum of the odd terms to the sum of even terms? (*NDA/NA 2008*)

Sol. Let the A.P. be $a, a+d, a+2d, ..., a+(2n-1)d, a+2nd$

Then, the progression of odd terms is $a, a+2d, a+4d, ..., a+2nd.$

This progression has $(n+1)$ terms.

Its sum = $\frac{n+1}{2}[a + a + 2nd] = \frac{n+1}{2}[2a + 2nd] = (n+1)(a+nd)$ ...(*i*)

The progression of even terms is $a+d, a+3d, ... a+(2n-1)d.$

This progression has n terms.

Its sum $= \frac{n}{2}\left[(a+d)+(a+(2n-1)\,d\right] = \frac{n}{2}[2a+2nd] = n\,(a+nd)$ ...(*ii*)

$\therefore \quad \frac{\text{Sum of odd terms}}{\text{Sum of even terms}} = \frac{(n+1)\,(a+nd)}{n\,(a+nd)} = \frac{\boldsymbol{n+1}}{\boldsymbol{n}}$.

Ex. 9. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then show that ab^2, ca^2, bc^2 are in A.P. **(*DCE*)**

Sol. Let α, β be the roots of the equation $ax^2 + bx + c = 0$.

Then $\alpha + \beta = -b/a$, $\alpha\beta = c/a$

Given, Sum of roots = Sum of squares of reciprocals of roots

$\Rightarrow \quad \alpha+\beta = \frac{1}{\alpha^2}+\frac{1}{\beta^2} \qquad \Rightarrow \quad \alpha+\beta = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2} \qquad \Rightarrow \quad \alpha+\beta = \frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$

$\Rightarrow \quad -b/a = \frac{(-b/a)^2 - \frac{2c}{a}}{(c/a)^2} \qquad \Rightarrow \quad -\frac{b}{a} = \frac{b^2-2ca}{c^2} \qquad \Rightarrow \quad -bc^2 = ab^2 - 2ca^2$

$\Rightarrow \quad 2ca^2 = ab^2 + bc^2 \qquad \Rightarrow \quad ab^2, ca^2, bc^2$ are in A.P. $\qquad (\because a,b,c \text{ in A.P.} \Rightarrow 2b = a + c)$

Ex. 10. Find the value of n, if $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the arithmetic mean between a and b. **(*WBJEE 2009*)**

Sol. The arithmetic mean between a and b is $\frac{a+b}{2}$.

Given, $\frac{a+b}{2} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$

$\Rightarrow (a^n + b^n)\,(a + b) = 2(a^{n+1} + b^{n+1}) \qquad \Rightarrow a^{n+1} + a^nb + b^na + b^{n+1} = 2a^{n+1} + 2b^{n+1}$

$\Rightarrow a^{n+1} + b^{n+1} = a^nb + b^na \qquad \Rightarrow (a^{n+1} - ba^n) - (b^na - b^{n+1}) = 0$

$\Rightarrow a^n\,(a - b) - b^n\,(a - b) = 0 \qquad \Rightarrow (a^n - b^n)\,(a - b) = 0$

$\Rightarrow (a^n - b^n) = 0$ or $(a - b) = 0$

But $a \neq b \Rightarrow a - b \neq 0$

$\therefore \; a^n - b^n = 0 \Rightarrow a^n = b^n \Rightarrow \left(\frac{a}{b}\right)^n = 1 \;\Rightarrow\; \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^0 \;\Rightarrow\; \boldsymbol{n = 0.}$

Ex. 11. If $\log_{10}2$, $\log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$ be three consecutive terms of an A.P., then find the value of x. **(*AMU 2012*)**

Sol. Given, $\log_{10}2$, $\log_{10}(2^x - 1)$, $\log_{10}(2^x + 3)$ are in A.P. Then,

$\log_{10}(2^x - 1) - \log_{10}2 = \log_{10}(2^x + 3) - \log_{10}(2^x - 1)$

$\Rightarrow \quad \log_{10}\left(\frac{2^x-1}{2}\right) = \log_{10}\left(\frac{2^x+3}{2^x-1}\right)$

$\Rightarrow \quad \frac{2^x-1}{2} = \frac{2^x+3}{2^x-1} \qquad \Rightarrow \quad (2^x - 1)^2 = 2\,(2^x + 3)$

$\Rightarrow \quad 2^{2x} - 2.2^x + 1 = 2.2^x + 6 \qquad \Rightarrow \quad 2^{2x} - 4.2^x - 5 = 0$

$\Rightarrow 2^{2x} - 5.2^x + 2^x - 5 = 0 \qquad \Rightarrow \quad 2^x\,(2^x - 5) + 1\,(2^x - 5) = 0$

$\Rightarrow (2^x - 5)\,(2^x + 1) = 0 \qquad \Rightarrow \quad 2^x = 5 \qquad (\because 2^x \neq -1)$

$\Rightarrow x = \boldsymbol{\log_2 5}$.

Ex. 12. If $a_1, a_2, a_3, \ldots a_n$ be an A.P. of non-zero terms, then find the sum: $\frac{1}{a_1 a_2}+\frac{1}{a_2 a_3}+\ldots+\frac{1}{a_{n-1}a_n}$. (*AMU 2009*)

Sol. Let $a_2 - a_1 = a_3 - a_2 = \ldots\ldots = a_n - a_{n-1} = d$ (common difference)

Then, $\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\ldots\ldots+\frac{1}{a_{n-1}a_n} = \frac{1}{d}\left[\frac{d}{a_1a_2}+\frac{d}{a_2a_3}+\ldots\ldots+\frac{d}{a_{n-1}a_n}\right]$

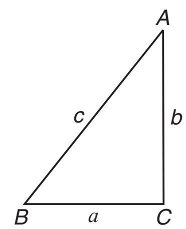
$= \frac{1}{d}\left[\frac{a_2-a_1}{a_1a_2}+\frac{a_3-a_2}{a_2a_3}+\ldots\ldots+\frac{a_n-a_{n-1}}{a_{n-1}a_n}\right] = \frac{1}{d}\left[\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+\ldots\ldots+\frac{1}{a_{n-1}}-\frac{1}{a_n}\right]$

$= \frac{1}{d}\left[\frac{1}{a_1}-\frac{1}{a_n}\right] = \frac{1}{d}\left[\frac{a_n-a_1}{a_1a_n}\right] = \frac{1}{d}\left[\frac{(a_1+(n-1)d)-a_1}{a_1a_n}\right] = \boldsymbol{\frac{n-1}{a_1a_n}}.$

Ex. 13. If the sides of a right angled triangle form an A.P., then find the sines of the acute angles. (*VITEE 2008*)

Sol. Let the ΔABC be right angled at C.

Then, $AB = c, BC = a, AC = b$



Given, the sides of the right angled Δ are in A.P. $\Rightarrow a, b, c$ are in A.P.

Now, let $a = x - d, b = x, c = x + d$

(***d*** being a +ve quantity as ***c*** being the hypotenuse is the greatest side)

$\therefore$ $c^2 = a^2 + b^2$ (Pythagoras' Theorem)

$\Rightarrow (x+d)^2 = (x-d)^2 + x^2$

$\Rightarrow x^2 + 2xd + d^2 = x^2 - 2xd + d^2 + x^2$

$\Rightarrow 4xd = x^2 \quad \Rightarrow \quad d = \frac{x}{4}.$

$\therefore$ $a = x - d = x - \frac{x}{4} = \frac{3x}{4}, b = x, c = x + d = x + x/4 = \frac{5x}{4}$

C being the right angle, A and B are the acute angles.

$\therefore$ $\sin A = \frac{a}{c} = \frac{3x/4}{5x/4} = \frac{3}{5}$ and $\sin B = \frac{b}{c} = \frac{x/5}{x/4} = \frac{x}{\frac{5x}{4}} = \frac{4}{5}$

$\therefore$ The sines of the acute angles are $\boldsymbol{\frac{3}{5}, \frac{4}{5}}$.

Ex. 14. a_1, a_2, a_3, a_4, a_5 are the first five terms of an A.P. such that $a_1 + a_3 + a_5 = -12$ and $a_1.a_2.a_3 = 8$. Find the first term and common difference.

Sol. Let $a_1 = a_3 - 2d, a_2 = a_3 - d, a_3 = a_3, a_4 = a_3 + d, a_5 = a_3 + 2d$

Then $a_1 + a_3 + a_5 = -12$, (given) $\Rightarrow a_3 - 2d + a_3 + a_3 + 2d = -12 \Rightarrow 3a_3 = -12 \Rightarrow a_3 = -4$

Also, $a_1 . a_2 . a_3 = 8$ (given)

$\Rightarrow a_1 . a_2 = -2$ ($\because a_3 = -4$) $\Rightarrow$ $(a_3 - 2d)(a_3 - d) = -2$

$\Rightarrow (-4 - 2d)(-4 - d) = -2$ $\Rightarrow$ $(2 + d)(4 + d) = 1$

$\Rightarrow d^2 + 6d + 9 = 0$ $\Rightarrow$ $(d + 3)^2 = 0 \Rightarrow d = \mathbf{-3}$

$\therefore$ $a_1 = a_3 - 2d = -4 - 2(-3) = \mathbf{2}.$

Ex. 15. If *a, b, c* are in A.P. show that $\frac{a(b+c)}{bc}, \frac{b(c+a)}{ca}, \frac{c(a+b)}{ab}$ are in A.P.

Sol. *a, b, c* are in A.P.

$\Rightarrow \frac{a}{abc}, \frac{b}{abc}, \frac{c}{abc}$ are in A.P. (Dividing each term by abc)

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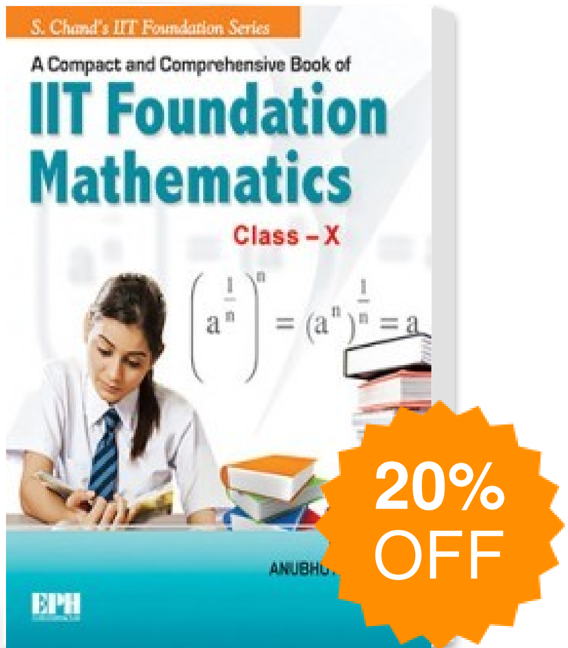

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