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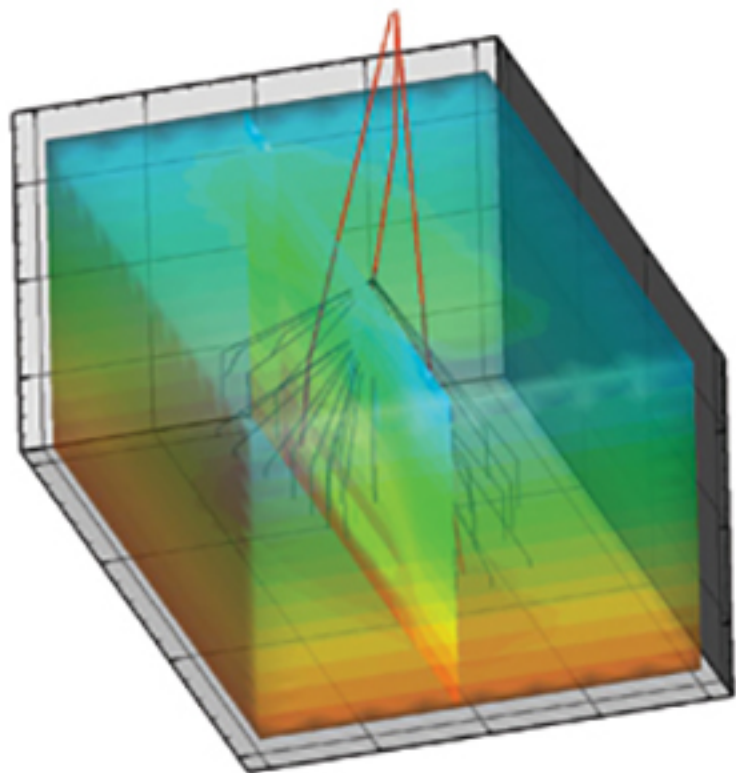
# Mathematics

**FOR CLASS XII**

**VOLUME-II**

**H.K. DASS**  
**Dr. RAMA VERMA**

$$QT = k \times \sqrt{RR}$$



*This book has been written according to the new NCERT syllabus prescribed  
by the Central Board of Secondary Education (CBSE) for Class XII*

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*S. Chand's*  
**MATHEMATICS**

**FOR CLASS XII  
VOLUME - II**

**AS PER NEW SYLLABUS**



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# PREFACE TO THE SEVENTH REVISED EDITION

The book has been revised and updated strictly in accordance with the new syllabus for CBSE examination.

We have included the following:

- (i) Solved and Unsolved Value Based Problems**
- (ii) Hints to the selected questions of the exercises**
- (iii) Very short answer type questions after each chapter**
- (iv) Multiple Choice questions after each chapter**
- (v) Solved Question papers 2014, 2015**
- (vi) Exemplar Exercises**

We are thankful to the Management Team and the Editorial Department of S. Chand & Company Pvt. Ltd. for all help and support in the publication of this book.

Awaiting feedback, comments and suggestions from the learned teachers and students.

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# CLASS XII

## SYLLABUS

One Paper

Three Hours

Marks : 100

Units		Marks
I	Relations and Functions	10
II	Algebra	13
III	Calculus	44
IV	Vectors and Three Dimensional Geometry	17
V	Linear Programming	06
VI	Probability	10
	<b>Total</b>	<b>100</b>

### UNIT-I: RELATIONS AND FUNCTIONS

- 1. Relations and Functions:** (15) Periods  
Types of relations: reflexive, symmetric, transitive and equivalence relations. Functions: one to one and onto functions, composite functions, inverse of a function. Binary operations.
- 2. Inverse Trigonometric Functions:** (15) Periods  
Definition, range, domain, principal value branches. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

### UNIT-II: ALGEBRA

- 1. Matrices:** (25) Periods  
Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).
- 2. Determinants:** (25) Periods  
Determinant of a square matrix (up to  $3 \times 3$  matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

### UNIT-III: CALCULUS

- 1. Continuity and Differentiability:** (20) Periods  
Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit function. Concept of exponential and logarithmic functions. Derivative of logarithmic and exponential functions. Logarithmic differentiation. Derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretations.
- 2. Applications of Derivatives:** (10) Periods  
Applications of derivatives: rate of change, increasing/decreasing functions, tangents and normals, use of derivatives in approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

**3. Integrals: (20) Periods**

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts. Evaluation of simple integrals of the following type and problems based on them:

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}},$$
$$\int \frac{(px + q)}{ax^2 + bx + c} dx, \int \frac{(px + q)}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2} dx, \int \sqrt{x^2 - a^2} dx$$
$$\int \sqrt{ax^2 + bx + c} dx, \int (px + q) \sqrt{ax^2 + bx + c} dx.$$

Definite integrals as a limit of a sum. Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals

**4. Applications of the Integrals: (15) Periods**

Applications in finding the area under simple curves, especially lines, arcs of circles/parabolas/ellipses (in standard form only), area between the two above said curves (the region should be clearly identifiable).

**5. Differential Equations: (15) Periods**

Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the form:

$$\frac{dy}{dx} + py = q, \text{ where } p \text{ and } q \text{ are functions of } x \text{ or constants.}$$

$$\frac{dx}{dy} + px = q, \text{ where } p \text{ and } q \text{ are functions of } y \text{ or constants.}$$

**UNIT- IV: VECTORS AND THREE - DIMENSIONAL GEOMETRY**

**1. Vectors: (15) Periods**

Vectors and scalars, magnitude and direction of a vector. Direction cosines/ratios of vectors. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio.

Definition, geometrical interpretation, properties and applications of scalar (dot) product of vectors, projection of a vector on a line. Vector (cross) product of vectors.

**2. Three-dimensional Geometry: (15) Periods**

Direction cosines/ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes, (iii) a line and a plane. Distance of a point from a plane.

**UNIT-V : LINEAR PROGRAMMING**

**1. Linear Programming: (20) Periods**

Introduction, definition of related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

**UNIT-VI: PROBABILITY**

**1. Probability: (30) Periods**

Multiplication theorem on probability. Conditional probability, independent events, total probability, Baye's theorem. Random variable and its probability distribution, mean and variance of haphazard variable. Repeated independent (Bernoulli) trials and Binomial distribution.

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**Rest of the topics are given in Mathematics for Class XII, Volume-I as details given below:**

**UNIT — I (RELATIONS AND FUNCTIONS)**

- 1. Relations**
- 2. Functions**
- 3. Binary Operations**
- 4. Inverse Trigonometric Functions**

**UNIT — II (ALGEBRA)**

- 5. Determinants**
- 6. Matrices**
- 7. Multiplication of Matrices**
- 8. Adjoint and Inverse of a Matrix**
- 9. Solutions of Simultaneous Linear Equations**

**UNIT — III (CALCULUS)**

- 10. Continuity**
  - 11. Differentiability**
  - 12. Differentiation**
  - 13. Second Order Derivatives**
  - 14. Rolle's Theorem and Mean Value Theorem**
  - 15. Rate of Change of Quantities**
  - 16. Tangents and Normals**
  - 17. Differentials, Errors and Approximations**
  - 18. Increasing And Decreasing Functions**
  - 19. Maxima and Minima**
- Table of Important Solutions of NCERT Book**

# UNIT-III (CALCULUS)

## CHAPTER 20

### INDEFINITE INTEGRALS

#### SYLLABUS

Integration as inverse of differentiation; properties of integral.

#### IMPORTANT FACTS

From the knowledge of differential calculus we can give the following table of results, which may be taken as the standard formulae of integral calculus.

- |   |  |
|---|--|
| 1. $\int x^n dx = \frac{x^{n+1}}{n+1}, n \neq -1$                           | 2. $\int \frac{1}{x} dx = \log x$  |
| 3. $\int e^x dx = e^x$  | 4. $\int a^x dx = \frac{a^x}{\log_e a}$  |
| 5. $\int \sin x dx = -\cos x$   | 6. $\int \cos x dx = \sin x$   |
| 7. $\int \sec^2 x dx = \tan x$  | 8. $\int \operatorname{cosec}^2 x dx = -\cot x$  |
| 9. $\int \sec x \tan x dx = \sec x$   | 10. $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x$                        |
| 11. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$              | 12. $-\int \frac{dx}{\sqrt{a^2 - x^2}} = \cos^{-1} \frac{x}{a}$                              |
| 13. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$         | 14. $-\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \cot^{-1} \frac{x}{a}$                         |
| 15. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a}$ | 16. $-\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{cosec}^{-1} \frac{x}{a}$ |

**Note .** (i) Result (1) may be stated in words as follows :

To obtain the integral of  $x^n$ , increase the index of  $x$  by unity and divide by the new index.

(ii) If  $x$  is replaced by  $(bx + c)$  in the above integrands, then the result is divided by  $b$ .

# INDEFINITE INTEGRALS

---

## 20.1. INTRODUCTION

The students have already learnt to find out the differential coefficient of a given function. Now, we have to find out the function whenever its derivative is given. The required function is known as antiderivative (or primitive) of the given derivative. The antiderivatives are called the indefinite integral.

Integration may be regarded as a process which is the inverse of differentiation.

In other words if  $f(x)$  be given, we have to find out a function  $\phi(x)$  whose derivative is  $f(x)$ . The function to be found is called the **integral** of the given function. The given function is known as **integrand**. The process of finding out integral of a given function is known as **integration**.

There are two forms of integral, indefinite and definite integral. Definite integral is the tool for science and engineering. Definite integral is also used to find out the area bounded by the graph of a function under certain conditions.

In this chapter we shall confine ourself to the study of simple formulae of indefinite integral.

## 20.2. DEFINITION

Let  $f(x)$  be a given function of  $x$ . If we can find a function  $\phi(x)$  such that

$$\frac{d}{dx}\phi(x) = f(x)$$

then we say that  $\phi(x)$  is an integral of  $f(x)$ . We write

$$\int f(x) dx = \phi(x)$$

Here, the sign  $\int$  is called the integral sign. The symbol  $dx$  indicates that the integration is to be performed with respect to the variable  $x$ .

For example,  $\frac{d}{dx}(\sin x) = \cos x \quad \Rightarrow \quad \int \cos x dx = \sin x$

Similarly,  $\frac{d}{dx}(x^2) = 2x \quad \Rightarrow \quad \int 2x dx = x^2$

$$\frac{d}{dx} \left( \frac{x^{n+1}}{n+1} \right) = \frac{(n+1)x^n}{(n+1)} = x^n \quad \Rightarrow \quad \int x^n dx = \frac{x^{n+1}}{n+1}$$

**Note :** (1) The symbol  $\int$  is an elongated *S* which is the first letter of sum, as the process of integration originated from summation of infinite series.

(2) The symbols  $\int$  and  $dx$ , separately have no meaning. These two symbols may be regarded as something like a pair of brackets in which the function is to be integrated.

### Technical words and their meanings :

Notation/Technical terms	Meaning in simple language
$\int f(x) dx$	Integral of $f(x)$ with respect to $x$ .
$f(x)$ in $\int f(x) dx$	Integrand.
$x$ in $\int f(x) dx$	Variable of integration.
Integrate	Evaluate the integral.
An integral of $f(x)$	$\phi(x)$ such that $\phi'(x) = f(x)$ .
Integration	Process of evaluation of integral
Arbitrary constant.	Constant of integration.

### 20.3. CONSTANT OF INTEGRATION

$$\text{Let } \frac{d}{dx} \phi(x) = f(x)$$

$$\text{By definition } \int f(x) dx = \phi(x) \quad \dots (1)$$

We know that differential coefficient of a constant  $C$  is zero.

$$\therefore \frac{d}{dx} [\phi(x) + C] = f(x)$$

$$\text{By definition } \int f(x) dx = \phi(x) + C \quad \dots (2)$$

Here  $C$  is known as the constant of integration.  $\phi(x) + C$  is called the indefinite integral of  $f(x)$  due to indefiniteness of the constant  $C$ .  $\phi(x)$  is known as the *principal primitive*.

From (1) and (2) it follows that two integrals of the same function differ by a constant.

$$\text{For example, } \frac{d}{dx} (x^4 + 2) = 4x^3 \quad \therefore \int 4x^3 dx = x^4 + 2$$

$$\frac{d}{dx} (x^4 + 5) = 4x^3 \quad \therefore \int 4x^3 dx = x^4 + 5$$

$x^4 + 2$  and  $x^4 + 5$  are the two different integrals of the given function and differ by a constant.

For the sake of convenience we omit the constant of integration from the result but the existence of constant in the result is always to be understood.

#### 20.4. GEOMETRICAL REPRESENTATION OF INDEFINITE INTEGRAL

Let  $f(x) = x$ , then

$$\int f(x) dx = \int x dx$$

$$= \frac{x^2}{2} + C \quad [C \text{ is constant of integration}]$$

For different values of  $C$  we will get a family of curves,  $y = \frac{x^2}{2} + C$ ,

By assigning different values of  $C$  we get different members of the family

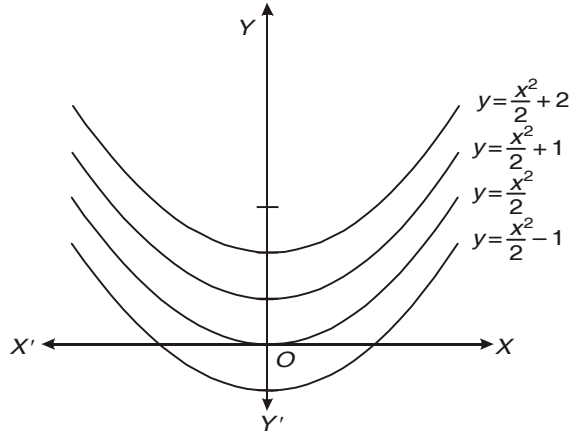
$$\text{If } C = 0, y = \frac{x^2}{2}$$

$$\text{If } C = 1, y = \frac{x^2}{2} + 1$$

$$\text{If } C = 2, y = \frac{x^2}{2} + 2$$

$$\text{If } C = 3, y = \frac{x^2}{2} + 3$$

$$\text{If } C = 4, y = \frac{x^2}{2} + 4, \text{ and so on.}$$



$\frac{x^2}{2} + C$  is known as the indefinite integral.

Let us draw the curve,  $y = \frac{x^2}{2} + C$

The Indefinite integral of a function represents geometrically a family of curves.

#### 20.5. STANDARD RESULTS

$$(i) \frac{d}{dx} \left( \frac{x^{n+1}}{n+1} \right) = x^n, n \neq -1 \quad \Rightarrow \quad \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$(ii) \frac{d}{dx} (x) = 1 \quad \Rightarrow \quad \int dx = x + C$$

$$(iii) \frac{d}{dx} (\log x) = \frac{1}{x} \quad \Rightarrow \quad \int \frac{1}{x} dx = \log |x| + C$$

$$(iv) \frac{d}{dx} (e^x) = e^x \quad \Rightarrow \quad \int e^x dx = e^x + C$$

$$(v) \frac{d}{dx} \left( \frac{a^x}{\log_e a} \right) = a^x, a > 0, a \neq 1 \quad \Rightarrow \quad \int a^x dx = \frac{a^x}{\log_e a} + C$$

$$\begin{aligned}
\text{(vi)} \quad \frac{d}{dx}(-\cos x) &= \sin x & \Rightarrow & \int \sin x \, dx = -\cos x + C \\
\text{(vii)} \quad \frac{d}{dx}(\sin x) &= \cos x & \Rightarrow & \int \cos x \, dx = \sin x + C \\
\text{(viii)} \quad \frac{d}{dx}(\tan x) &= \sec^2 x & \Rightarrow & \int \sec^2 x \, dx = \tan x + C \\
\text{(ix)} \quad \frac{d}{dx}(-\cot x) &= \operatorname{cosec}^2 x & \Rightarrow & \int \operatorname{cosec}^2 x \, dx = -\cot x + C \\
\text{(x)} \quad \frac{d}{dx}(\sec x) &= \sec x \tan x & \Rightarrow & \int \sec x \tan x \, dx = \sec x + C \\
\text{(xi)} \quad \frac{d}{dx}(-\operatorname{cosec} x) &= \operatorname{cosec} x \cot x & \Rightarrow & \int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C \\
\text{(xii)} \quad \frac{d}{dx}(\log \sin x) &= \cot x & \Rightarrow & \int \cot x \, dx = \log |\sin x| + C \\
\text{(xiii)} \quad \frac{d}{dx}(-\log \cos x) &= \tan x & \Rightarrow & \int \tan x \, dx = -\log |\cos x| + C \\
\text{(xiv)} \quad \frac{d}{dx}[\log(\sec x + \tan x)] &= \sec x & \Rightarrow & \int \sec x \, dx = \log |\sec x + \tan x| + C \\
\text{(xv)} \quad \frac{d}{dx}[\log(\operatorname{cosec} x - \cot x)] &= \operatorname{cosec} x & \Rightarrow & \int \operatorname{cosec} x \, dx = \log |\operatorname{cosec} x - \cot x| + C \\
\text{(xvi)} \quad \frac{d}{dx}\left(\sin^{-1} \frac{x}{a}\right) &= \frac{1}{\sqrt{a^2 - x^2}} & \Rightarrow & \int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C \\
\text{(xvii)} \quad \frac{d}{dx}\left(\cos^{-1} \frac{x}{a}\right) &= -\frac{1}{\sqrt{a^2 - x^2}} & \Rightarrow & \int -\frac{1}{\sqrt{a^2 - x^2}} \, dx = \cos^{-1}\left(\frac{x}{a}\right) + C \\
\text{(xviii)} \quad \frac{d}{dx}\left(\frac{1}{a} \tan^{-1} \frac{x}{a}\right) &= \frac{1}{a^2 + x^2} & \Rightarrow & \int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \\
\text{(xix)} \quad \frac{d}{dx}\left(\frac{1}{a} \cot^{-1} \frac{x}{a}\right) &= -\frac{1}{a^2 + x^2} & \Rightarrow & \int -\frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \cot^{-1}\left(\frac{x}{a}\right) + C \\
\text{(xx)} \quad \frac{d}{dx}\left(\frac{1}{a} \sec^{-1} \frac{x}{a}\right) &= \frac{1}{x\sqrt{x^2 - a^2}} & \Rightarrow & \int \frac{1}{x\sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C \\
\text{(xxi)} \quad \frac{d}{dx}\left(\frac{1}{a} \operatorname{cosec}^{-1} \frac{x}{a}\right) &= -\frac{1}{x\sqrt{x^2 - a^2}} & \Rightarrow & \int -\frac{1}{x\sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \operatorname{cosec}^{-1}\left(\frac{x}{a}\right) + C
\end{aligned}$$

With the help of the above formulae, we can evaluate the following integrals.

## 20.6. INTEGRAL OF THE PRODUCT OF A CONSTANT AND A FUNCTION

Let  $C$  be a constant then

$$\int C f(x) \, dx = C \int f(x) \, dx$$

The integral of the product of a constant and a function is equal to the product of the constant and the integral of the function.

$$\text{i.e.} \quad \int 8x^2 dx = 8 \int x^2 dx$$

### 20.7. INTEGRAL OF THE ALGEBRAIC SUM OF A NUMBER OF FUNCTIONS

$$\int [f_1(x) + f_2(x) + \dots] dx = \int f_1(x) dx + \int f_2(x) dx + \dots$$

$$\text{For example} \quad \int (\sin x + e^x) dx = \int \sin x dx + \int e^x dx$$

The integral of the sum of a number of functions is equal to the sum of the integrals of the functions.

### 20.8. PROPERTIES OF THE INDEFINITE INTEGRAL

In this sub section, we shall derive some properties of indefinite integrals.

**Property (i).** The process of differentiation and integration are inverse of each other in the sense of the following results :

$$\frac{d}{dx} \int f(x) dx = f(x)$$

and  $\int f'(x) dx = f(x) + C$ , where  $C$  is any arbitrary constant.

**Proof.** Let  $F$  be any antiderivative of  $f$ .

$$\Rightarrow \frac{d}{dx} F(x) = f(x) \quad \dots (1) \quad \Rightarrow \quad F(x) + C = \int f(x) dx$$

$$\text{Then} \quad \int f(x) dx = F(x) + C$$

On differentiating both sides, we get

$$\frac{d}{dx} \int f(x) dx = \frac{d}{dx} [F(x) + C]$$

$$\Rightarrow \frac{d}{dx} \int f(x) dx = \frac{d}{dx} F(x) = f(x) \quad \text{[Using (1)]}$$

$$\Rightarrow \frac{d}{dx} \int f(x) dx = f(x). \quad \text{Proved.}$$

Similarly,

$$\int f'(x) dx = f(x) + C \quad \left[ f'(x) = \frac{d}{dx} f(x) \right]$$

where  $C$  is arbitrary constant called constant of integration.

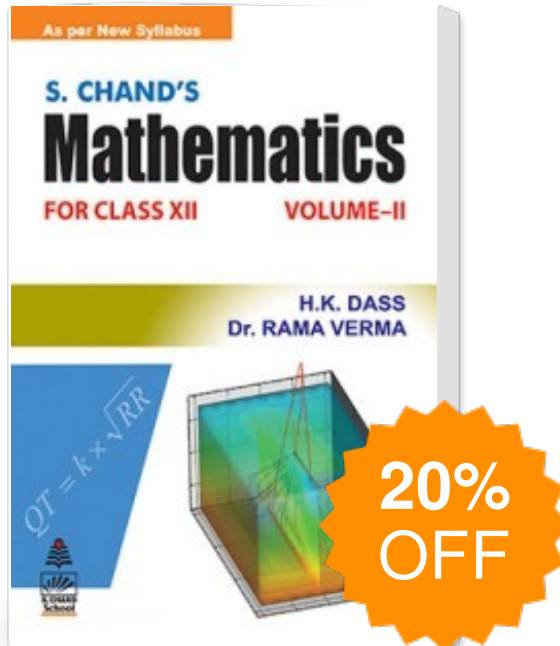
**Property (ii).** If  $\frac{d}{dx} \int f(x) dx = \frac{d}{dx} \int g(x) dx$ ,

$$\text{Then} \quad \int f(x) dx + C_1 = \int g(x) dx + C_2$$

**Proof.** We have,

$$\frac{d}{dx} \int f(x) dx = \frac{d}{dx} \int g(x) dx \quad \Rightarrow \quad \frac{d}{dx} \int f(x) dx - \frac{d}{dx} \int g(x) dx = 0$$

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