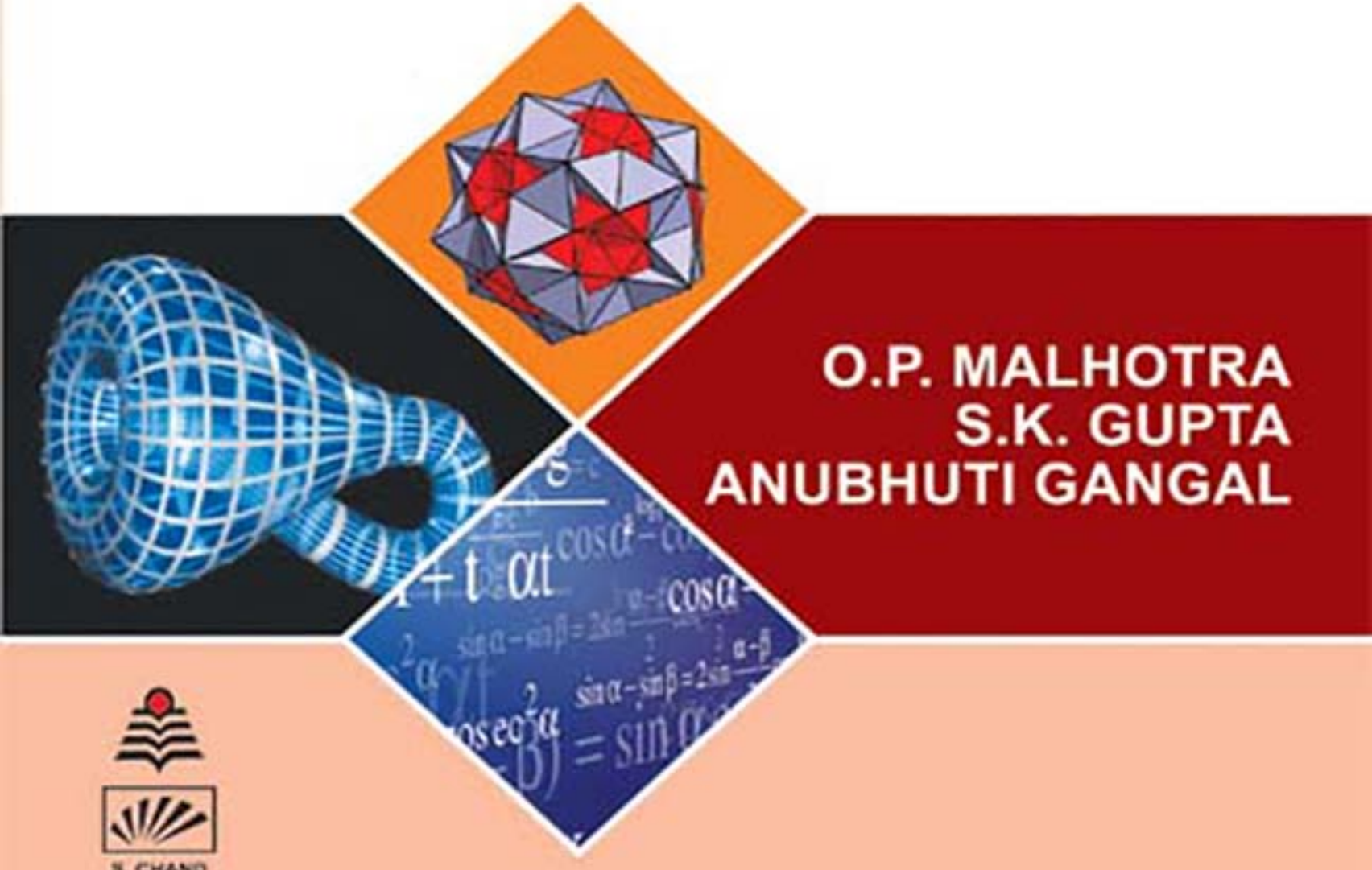
Revised Edition

ISC MATHEMATICS

BOOK II
FOR CLASS XII



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SCHOOL

Strictly according to the latest Syllabus for ISC (XII Class) prescribed by the Council for Indian School Certificate Examination, New Delhi for the Examination to be held in 2016

ISC MATHEMATICS

BOOK II

FOR CLASS XII

AS PER NEW SYLLABUS FOR 2016

Including ISC 2014 Question Paper

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S. CHAND SCHOOL BOOKS

S. CHAND SCHOOL BOOKS
(An imprint of S. Chand Publishing)
A Division of S. Chand & Co. Pvt. Ltd.
7361, Ram Nagar, Qutab Road, New Delhi-110055
Phone: 23672080-81-82, 9899107446, 9911310888; Fax: 91-11-23677446
www.schandpublishing.com; e-mail : helpdesk@schandpublishing.com

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First Published in 1965 ('A Complete Course in Additional Mathematics and Statistics')

Revised Edition 2014

Revised Edition 2015

Subsequent Editions according to new pattern and Reprints 1992, 93, 95, 96 (Twice), 97, 98, 99 (Twice), 2000, 2002 (Twice), 2003, 2004 (Twice), 2005, 2006, 2007 (Twice), 2008 (Twice), 2009 (Twice), 2010, 2011, 2012, 2013

ISBN : 978-81-219-0556-7 Code : 1014H 317

PRINTED IN INDIA
By Vikas Publishing House Pvt. Ltd., Plot 20/4, Site-IV, Industrial Area Sahibabad, Ghaziabad-201010
and Published by S. Chand & Company Pvt. Ltd., 7361, Ram Nagar, New Delhi-110055.

2015 EDITION

1. The **ISC 2014 question paper** has been included.
2. The book has once again been thoroughly checked for misprints.

S.K. GUPTA

ABOUT THE 2014 EDITION

1. The **ISC 2013 question paper** has been given at the end with Answers and Hints/Solutions. This has been done with a view to familiarise the students with the pattern of the paper.
2. The Authors are highly grateful to **Mr. S. Ganguly**, Senior Maths Teacher, St. Thomas College, Dehradun for taking immense pains and putting in lot of hard work in going minutely through each and every word of this book. He has also provided us short and elegant alternative solutions of a large number of questions solved in the book. These have been duly incorporated at relevant places in the present edition.
3. Our deep thanks are due to **Mr. C.S. Pundir**, HOD Maths, Touchwood School, Dehradun also for giving as valuable feedback from time to time. His comments on '***Integration by parts***' have been duly attended to in this edition. This topic has been re-written.
4. We are also indebted to **Mrs. Aleyamma Chacko** of Trivandrum for her kind support and the suggestions she keeps on sending for the improvement of the book.
5. The authors remain open to suggestions and new ideas and would in fact welcome and be thankful for any input received from the learned faculty for improvement of this highly popular book which has been in use for almost 60 years now in India and abroad.

S.K. GUPTA

PREFACE TO THE 2011 EDITION

We feel happy to be able to present the thoroughly revised and updated new edition of this most popular book for the ISC students which both the teachers and students have been fondly using for more than 45 years. The salient features of this edition are:

1. The entire content has been thoroughly revised, rehashed and re-cast. A lot of subject matter has been re-written making the treatment more systematic and easier to understand and learn. Special mention may be made of the chapters on **Conics, Indeterminate Forms of Limits, Mean Value Theorems, Maxima and Minima, Correlation and Regression Analysis, Complex Numbers, Vectors, 3-Dimensional Geometry, Discount and Bill of Exchange and Annuities.**
2. The following chapters **have been merged** to make the content more precise.

   (*i*) Chapters 4, 5 and 6 on Parabola, Ellipse and Hyperbola.

   (*ii*) Chapters 22, 23 and 24 on Scalar and Vector Products and Scalar Triple Product.
3. Further effort has been made to prune the exercises and leave out whatever was felt to be redundant and irrelevant and not required for this course. Care has, however, been taken to ensure that the book does not suffer from any defic iency and fully meets the students' requirements not only for the class 12 final examination but also the various competitive examinations. To ensure this, relevant questions from various entrance examinations have been interspersed throughout the text.
4. **Hints to selected questions** were given in exercises alongwith the questions in the previous edition. These at the suggestion of the teachers have now been given at the end of the exercises after the answers.
5. **ISC questions up to 2010 have been collected on every topic and given at one place after each chapter. These replace the revision exercises given in earlier editions.**

This, it is hoped will immensely help the students not only to assess and re-inforce their understanding of concepts but will also help them to familiarise themselves with the trends of questions asked at the Council's examination and revise them and prepare before the final examinations.

We are deeply grateful to those teachers who have spared their valuable time in sending feedback and useful suggestions from time to time thus giving us an opportunity to carry out continuous evaluation and improvement of the book.

AUTHORS

SYLLABUS

(*For the examination to be held in 2016*)

*There will be one paper of **three** hours duration of 100 marks. The syllabus is divided into **three** sections A, B and C. Section A is compulsory for all candidates. Candidates will have a choice of attempting questions from **either** Section B or Section C.*

***Section A (80 marks)** will consist of 9 questions. Candidates will be required to answer **Question 1 (compulsory)** and **five** out of the rest of the eight questions.*

***Section B/C (20 marks)** Candidates will be required to answer **two** questions out of **three** from either Section B **or** Section C.*

SECTION – A

1. Determinants and Matrices

(*i*) Determinants

- Order.
- Minors.
- Cofactors.
- Expansion.
- Properties of determinants.
- Simple problems using properties of determinants, *e.g.*, evaluate $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ etc.
- **Cramer's Rule**
  - Solving simultaneous equations in 2 or 3 variables,

$$x = \frac{D_x}{D},\ y = \frac{D_y}{D},\ z = \frac{D_z}{D}.$$

  - Consistency, inconsistency.
  - Dependent or independent.

NOTE: The consistency condition for three equations in two variables is required to be covered.

(*ii*) Matrices

- Types of matrices ($m \times n$; $m, n \leq 3$); Order; Identity matrix, Diagonal matrix.
- Symmetric, Skew symmetric.
- Operation : addition, subtraction, multiplication of a matrix with scalar, multiplication of two matrices (the compatibility).
- *e.g.* $\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = AB$ (*say*). But BA is not possible.
- Singular and non singular matrices.
- Existence of two non-zero matrices whose product is a zero matrix.
- Inverse (2×2, 3×3), $A^{-1} = \dfrac{\text{Adj } A}{|A|}$.
- **Martin's Rule (*i.e.*, using matrices)**

$a_1x + b_1y + c_1z = d_1.$
$a_2x + b_2y + c_2z = d_2.$
$a_3x + b_3y + c_3z = d_3.$

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix},\ B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

$$AX = B \quad \Rightarrow \quad X = A^{-1} B.$$

Simple problems based on above.

NOTE: The conditions for consistency of equations in two and three variables, using matrices, are to be covered.

2. Boolean Algebra

- Boolean algebra as an algebraic structure, principle of duality, Boolean function. Switching circuits, application of Boolean algebra to switching circuits.

3. Conics

- As a section of a cone.
- Definition of Foci, Directrix, Latus Rectum.
- $PS = ePL$, where P is a point on the conics, S is the focus, PL is the perpendicular distance of the point from the directrix.

(*i*) Parabola

- $e = 1$, $y^2 = 4ax$, $x^2 = 4ay$, $y^2 = -4ax$, $x^2 = -4ay$, $(y - \beta)^2 = 4a\ (x - \alpha)$, $(x - \alpha)^2 = 4a\ (y - \beta)$.
- Rough sketch of the above.
- The latus rectum, quadrants they lie in, coordinates of focus and vertex, and equations of directrix and the axis.
- Finding equation of parabola when foci and directrix are given.
- Simple and direct questions based on the above.

(*ii*) Ellipse

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ e < 1,\ b^2 = a^2\ (1 - e^2)$
- Cases when $a > b$ and $a < b$.
- Rough sketch of the above.
- Major axis, minor axis, latus rectum; coordinates of vertices, focus and centre; and equations of directrices and the axes.
- Finding equation of ellipse when focus and directrix are given.
- Simple and direct questions based on the above.
- Focal property, *i.e.,* $SP + SP' = 2a$.

(*iii*) Hyperbola

- $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,\ e < 1,\ b^2 = a^2\ (e^2 - 1)$
- Cases when coefficient of y^2 is negative and coefficient of x^2 is negative.
- Rough sketch of the above.
- Focal property, *i.e.,* $SP - S'P = 2a$.
- Transverse and Conjugate axis, Latus rectum, coordinates of vertices, foci and centre; and equations of the directrices and the axes.
- General second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, represents a parabola if $h^2 = ab$, ellipse if $h^2 < ab$, and hyperbola if $h^2 > ab$.

  Condition that $y = mx + c$ is a tangent to the conics.

4. Inverse Trigonometric Function

- Principal values.
- $\sin^{-1} x$, $\cos^{-1} x$, $\tan^{-1} x$ etc., and their graphs.
- $\sin^{-1}x = \cos^{-1} \sqrt{1-x^2} = \tan^{-1} \dfrac{x}{\sqrt{1-x^2}}$.
- $\sin^{-1} x = \text{cosec}^{-1} \dfrac{1}{x}$, $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$ and similar relations for $\cot^{-1} x$, $\tan^{-1} x$, etc.
- Addition formulae.

  $\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right)$

  $\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{1-y^2}\ \sqrt{1-x^2}\right)$

  Similarly $\tan^{-1} x \pm \tan^{-1} y = \tan^{-1} \dfrac{x \pm y}{1 \mp xy}$, $xy < 1$.

  Similarly establish formulae for $2 \sin^{-1}x$, $2 \cos^{-1}x$, $2 \tan^{-1}x$ and $3 \tan^{-1}x$ using the above formulae.
- Application of these formulae.

5. Calculus

(*i*) Differential Calculus

- Revision of topics done in Class XI : mainly the differentiation of product of two functions, quotient rule etc.
- Derivatives of trigonometric functions.
- Derivatives of exponential functions.
- Derivatives of logarithmic functions.
- Derivatives of inverse trigonometric functions : differentiation by means of substitution.
- Derivatives of implicit functions and chain rule for composite functions.
- Derivatives of Parametric functions.
- Differentiating of a function with respect to another function, *e.g.*, differentiation of $\sin x^3$ with respect to x^3.
- Logarithmic Differentiation : Finding dy/dx when $y = x^{x^{x^{\cdots}}}$
- Successive differentiation up to 2nd order.
- L'Hospital's theorem.
- $\dfrac{0}{0}$ form, $\dfrac{\infty}{\infty}$ form, 0° form, ∞^∞ form etc.
- Rolle's Mean Value Theorem : its geometrical interpretation.
- Lagrange's Mean Value Theorem : its geometrical interpretation.
- Maxima and minima.

(*ii*) Integral Calculus

- Revision of formulae from Class XI.
- Integration of $1/x$, e^x, $\tan x$, $\cot x$, $\sec x$, $\text{cosec}\ x$.
- Integration by simple substitution.
- Integrals of the type $f'(x)\,[f(x)]^n$, $\dfrac{f'(x)}{f(x)}$.
- Integration by parts.
- Integration by means of substitution.
- Integration using partial fractions.

- Expressions of the form $\frac{f(x)}{g(x)}$ when degree of $f(x)$ < degree of $g(x)$

  e.g. $\frac{x+2}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$

  $\frac{x+2}{(x-2)(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$

  $\frac{x+1}{(x^2+3)(x-1)} = \frac{Ax+B}{x^2+3} + \frac{C}{x-1}$

  When degree of $f(x) \geq$ degree of $g(x)$,

  e.g. $\frac{x^2+1}{x^2+3x+2} = 1 - \left(\frac{3x+1}{x^2+3x+2}\right)$.

- Integrals of the type :

  $\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{px+q}{ax^2+bx+c}\,dx, \int \frac{px+q}{\sqrt{ax^2+bx+c}}\,dx$

  and expressions reducible to this form.

- Integrals of the form :

  $\int \frac{dx}{a\cos x + b\sin x}, \int \frac{dx}{a+b\cos x}, \int \frac{dx}{a+b\sin x}, \int \frac{1 \pm x^2}{1+x^4}\,dx,$

  $\int \frac{dx}{1+x^4}, \int \sqrt{\tan x}\,dx, \int \sqrt{\cot x}\,dx.$

- Properties of the definite integrals.

  Problems based on the following properties of definite integrals are to be covered.

  $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

  $\int_a^b f(x)\,dx = -\int_a^b f(x)\,dx$

  $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_a^b f(x)dx$, where $a < c < b$.

  $\int_a^b f(x)\,dx = \int_a^c f(a+b-x)dx$

  $\int_0^b f(x)\,dx = \int_a^c f(a-x)dx$

  $$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$$

  $$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, & \text{if } f \text{ is an even function} \\ 0, & \text{if } f \text{ is an odd function} \end{cases}$$

- Application of definite integrals : area bounded by curves, lines and coordinate axes is required to be covered.

6. Correlation and Regression

- Definition and meaning of correlation and regression coefficient.
- Coefficient of correlation by Karl Pearson.

If $x - \bar{x}$, $y - \bar{y}$ *are small non-fractional numbers, we use*

$$r = \frac{\Sigma (x - \bar{x})\,(y - \bar{y})}{\sqrt{\Sigma (x - \bar{x})^2}\ .\sqrt{\Sigma (y - \bar{y})^2}}$$

If x and y are small numbers, we use

$$r = \frac{\Sigma xy - \frac{1}{N}\Sigma x\,\Sigma y}{\sqrt{\Sigma x^2 - \frac{1}{N}(\Sigma x)^2}\ .\sqrt{\Sigma y^2 - \frac{1}{N}(\Sigma y)^2}}$$

Otherwise, we use assumed means A and B, where $u = x - A$, $v = y - B$

$$r = \frac{\Sigma uv - \frac{1}{N}(\Sigma u)\,(\Sigma v)}{\sqrt{\Sigma u^2 - \frac{1}{N}(\Sigma u)^2}\ .\sqrt{\Sigma v^2 - \frac{1}{N}(\Sigma v)^2}}$$

- Rank correlation by Spearman's (Correction included).
- Lines of regression of x on y and y on x.

NOTE: *Scatter diagrams* and the following topics on regression are required.

(*i*) *The method of least squares.*

(*ii*) *Lines of best fit.*

(*iii*) Regression coefficient of x on y and y on x.

(*iv*) $b_{xy} \times b_{yx} = r^2$, $0 \le b_{xy} \times b_{yx} \le 1$

(*v*) Identification of regression equations.

7. Probability

- Random experiments and their outcomes.
- Events : sure events, impossible events, mutually exclusive events, independent events and dependent events.
- Definition of probability of an event.
- Laws of probability : addition and multiplication laws, conditional probability. (excluding Baye's theorem)

8. Complex Numbers

- Argument and conjugate of complex numbers.
- Sum, difference, product and quotient of two complex numbers additive and multiplicative inverse of a complex number.
- Simple locus questions on complex numbers; proving and using :

$$z \cdot \bar{z} = |z|^2 \text{ and } \overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2 \text{ and } \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$$

- Triangle inequality.
- Square root of a complex number.
- Demoivre's theorem and its simple applications.
- Cube roots of unity : 1, ω, ω^2; application problems.

9. Differential Equations

- Differential equations, order and degree.
- Solution of differential equations.
- Variables separable.
- Homogeneous equations and equations reducible to homogeneous form.
- Linear form $\frac{dy}{dx} + Py = Q$ where P and Q are functions of x only. Similarly for $\frac{dx}{dy}$.

NOTE: Equations reducible to variable separable type are included. The second order differential equations are excluded.

SECTION – B

10. Vectors

- Scalar (dot) product of vectors.
- Cross product – its properties, area of a triangle, collinear vectors.
- Scalar triple product – volume of a parallelopiped, co-planarity.

Proof of Formulae (Using Vectors)

- Sine rule
- Cosine rule
- Projection formula
- Area of a $\Delta = \frac{1}{2} ab \sin c$.

NOTE: Simple geometric applications of the above are required to be covered.

11. Coordinate geometry in 3-dimensions

(*i*) Lines

- Cartesian and vector equations of a line through one and two points.
- Coplanar and skew lines.
- Conditions for intersection of two lines.
- Shortest distance between two lines.

NOTE: Symmetric and non-symmetric forms of lines are required to be covered.

(*ii*) Planes

- Cartesian and vector equation of a plane.
- Direction ratios of the normal to the plane.
- One point form.
- Normal form.
- Intercept form.
- Distance of a point from a plane.
- Angle between two planes, a line and a plane.
- Equation of a plane through the intersection of two planes, *i.e.*, $P_1 + kP_2 = 0$.

Simple questions based on the above.

12. Probability

Baye's theorem; theoretical probability distribution, probability distribution function; binomial distribution : its mean and variance.

NOTE: Theoretical probability distribution is to be limited to binomial distribution only.

SECTION – C

13. Discount

True discount : banker's discount; discounted value; present value; cash discount, bill of exchange.

NOTE: Banker's gain is required to be covered.

14. Annuities

Meaning; formulae for present value and amount; deferred annuity, applied problems on loans, sinking funds, scholarships.

NOTE: Annuity due is required to be covered.

15. Linear Programming

Introduction, definition of related terminology such as constraints, objective function, optimization, isoprofit, isocost lines; advantages of linear programming; limitations of linear programming; application areas of linear programming; different types of linear programming (L.P.), problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solution, optimum feasible solution.

16. Application of Derivatives in Commerce and Economics in the Following

Cost function, average cost, marginal cost, revenue function and break even point.

17. Index Numbers and Moving Averages

- Price index or price relative.
- Simple aggregate method.
- Weighted aggregate method.
- Simple average of price relatives.
- Weighted average of price relatives (cost of living index, consumer price index).

NOTE: Under moving averages the following are required to be covered.

- Meaning and purpose of the moving averages.
- Calculation of moving averages with the given periodicity and plotting them on a graph.
- If the period is even, then the centered moving average is to be found out and plotted.

NOTATIONS

SC	stands for School Certificate (Cambridge).
GCE	stands for General Certificate of Education (Cambridge).
HSC	stands for High School Certificate (Cambridge).
ISC	stands for Indian School Certificate.
CU	stands for Cambridge University.
OU	stands for Oxford University.
LU	stands for London University.
NU	stands for Northern University.
IS	stands for Intermediate Science, London University.
OC	stands for Oxford and Cambridge.
NDA	stands for National Defence Academy of India.
IAS	stands for Indian Administrative Service.
IIT	stands for Indian Institute of Technology.
MNR	stands for Motilal Nehru Regional College.
ISCBM	stands for Indian School Certificate Business Mathematics.
NMOC	stands for National Mathematics Olympiad Council.
AP	stands for Andhra Pradesh.
HB	stands for Haryana Board.
PB	stands for Punjab Board.
AIEEE	stands for All India Engineering Entrance Examination.
EAMCET	stands for Engineering Agriculture and Medicine Common Entrance Test.

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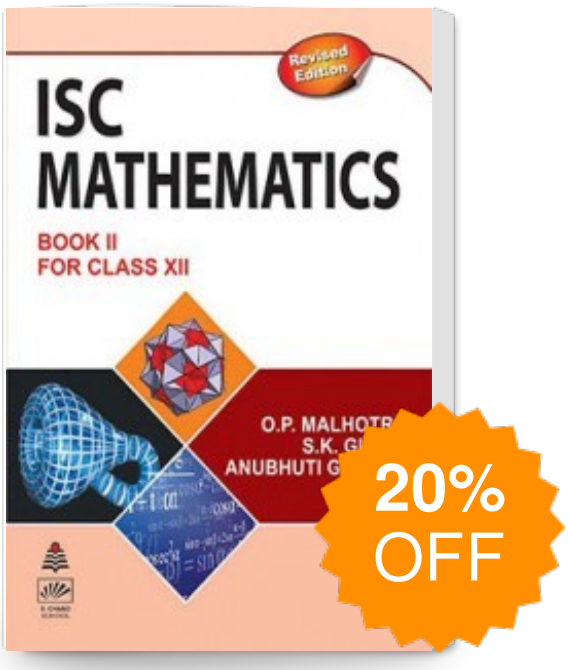

Publisher : SChand Publications ISBN : 9788121905565

Author : O P Malhotra And S K Gupta

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