MODERN'S abc OF

OBJECTIVE PHYSICS For MH–CET

Maharashtra State Engineering & Pharmacy, BAMS, BHMS, Paramedical Entrance Examinations

By

O.P. KAKKAR
Head of Physics Deptt.,
A.P.J. College of Fine Arts, Jalandhar.

SAMEER ARORA
Assistant Professor in Physics,
A.P.J. College of Fine Arts, Jalandhar.

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Preface

We feel pleasure in presenting the book 'Modern's abc of Objective Physics' for students appearing in MH-CET examinations . The book has been thoroughly written in accordance with the latest syllabus and changing trends of examinations.

REVISION NOTES

This provides thorough discussion on the chapter emphasising all the important formulae, facts and terms. This can serve as quick revision of the chapter before the examination. This part has been enriched with new materials, Key points, Learning Plus, Facts to Memorise, In Focus, etc.

QUESTIONS

These include a variety of questions in the form of multiple choice questions. These questions are graded according to degree of difficulty. These are : Level I (Basic Conceptual Questions), Level II (Comprehensive Questions), Recent Competitive questions (RCQ) (AIEEE, JEE-Main, AIPMT, MH-CET) from 2011 to 2016 are given separately at the end of each chapter.

ANSWERS

The book provides answers to all relevant questions. This is an important feature of this new edition. Some of the difficult questions have been solved whereas hints to many other questions are given.

Except for the Mock-test, complete solutions to ALL other questions have been given in the part 'HINTS AND SOLUTIONS' appended to each chapter.To aquaint the readers with style and type of questions asked, RECENTLY set competitive questions (RCQ) with complete solutions have been added at the end of each chapter. They will further make the students familiar with the standard and style of the test in details

MOCK TESTS

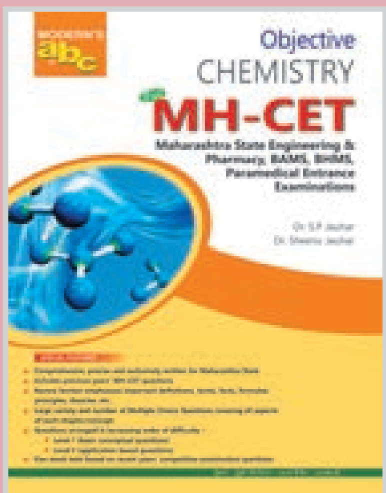
This is yet another new feature of this edition. This will help students to check their performance after they have gone through the chapters. Three mock test papers covering the complete syllabus are also given at the end of the book. All these will provide sufficient materials to students for practice during their preparation.

In fact no pains have been left to render the book flawless, errorless and authentic. Inspite of the best care and effort, if any error, omission or misprint comes to the notice of the reader, it may kindly be brought to the notice of the authors or publisher. It will be thankfully acknowledged and ammended in the subsequent edition. Any useful suggestion from the learned colleagues or student readers which may be relevent to the enhancement of utility and improvement of the contents of the book shall be acknowledged and incorporated in the subsequent editions.

In the presence of ONE SINGLE TEST for the eligibility and admission to MANY of the colleges situated in the Maharashtra, our book which is selective and contains VERY IMPORTANT and TYPICAL questions in the form of EXCELLENT, SUPERIOR, USEFUL and carefully SELECTIVE material becomes UNIQUE and a “self teacher” in itself and we hope it will cater to the need of ONE and ALL aspirants appearing for the test and would form a popular choice with each one of the readers. The price of book has been kept comparatively LOW to suit all stratas of students inspite of the fact that it has colourful, artistic presentation in shape and form.

The authors take this opportunity to thank **Mr. Balwant Sharma (NSH), Mr. Manik Juneja National Head (Content Operation)** and his entire team of Modern Publishers, in general and **Mr. Ravinder Pathania (Manager Publication)** in particular, for rendering all type of cooperation and guidance during the preparation and printing of the book in its present form within a shortest possible time.

—Authors



MODERN'S abc OF

OBJECTIVE CHEMISTRY

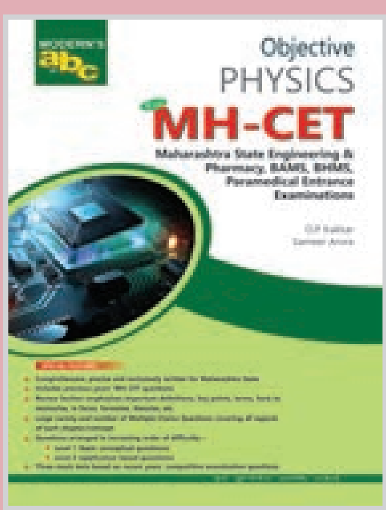
For MH–CET

ABOUT THE BOOK

We feel pleasure in presenting the book **'Modern's abc of Objective Chemistry'** for students appearing in MH-CET examinations . The book has been thoroughly written in accordance with the latest syllabus and changing trends of examinations.

SPECIAL FEATURES

- Comprehensive, precise and exclusively written for Maharashtra
- Includes previous years' MH-CET questions
- Review Section emphasizes important definitions, terms, facts, formulae, principles, theories, etc.
- Large variety and number of Multiple Choice Questions covering all aspects of each chapter/concept
- Questions arranged in increasing order of difficulty –
  - Level 1 (basic conceptual questions)
  - Level 2 (application-based questions)
- Five mock tests based on recent years' competitive examination questions



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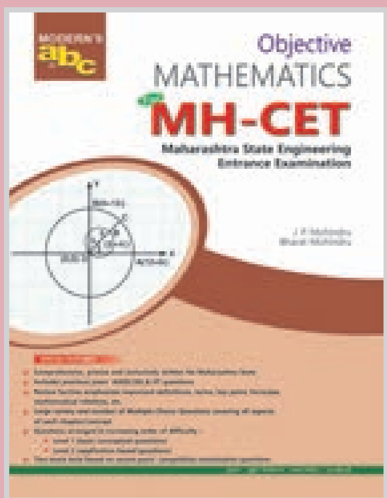

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OBJECTIVE MATHEMATICS

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We feel pleasure in presenting the book **'Modern's abc of Objective Mathematics'** for students appearing in MH-CET examinations. The book has been thoroughly written in accordance with the latest syllabus and changing trends of examinations.

SPECIAL FEATURES

- Comprehensive, precise and exclusively written for Maharashtra
- Includes previous years' AIEEE/JEE/IIT questions
- Review Section emphasizes important definitions, terms, facts, principles, theories, etc.
- Large variety and number of Multiple Choice Questions covering all aspects of each chapter/concept
- Questions arranged in increasing order of difficulty –
  - Level 1 (basic conceptual questions)
  - Level 2 (application-based questions)
- Two mock tests based on recent years' competitive examination questions

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MODERN'S abc OF PHYSICS

The book presents the subject matter in full conformity with the syllabi prescribed by C.B.S.E., New Delhi and Education Boards of other Indian states. To keep pace with changing trends in education at national level, the whole text has been arranged strictly according to N.C.E.R.T. pattern. The main stress has been laid on SI. The symbols and signs used for various physical quantities are also in keeping with the recommendations at national and international levels.

The book provides a result-oriented training to young students. The whole text of the book is embedded with short notes in the form of **The jargon** (introducing apparently a new physical term with a proper definition), Key point (highlighting an important point in the text) and **Watch out** (bringing out the difference between the physical and apparent meaning of a physical term). Further, the text has been studded with simple **Self-Study Questions,** so as to provide an insight and a proper grip over the topic, as one learns it. The article work in each chapter of a unit is coupled with well graded and carefully selected **Solved Numerical Problems** for easy comprehension of the beginners. So that the students can prepare for the Annual Examinations in an independent manner, a large number of **Very Short Answer Questions** and **Short Answer Questions** have been incorporated in the book with proper Answers/Hints. **Unsolved Numerical Problems** for self-practice have been categorised into various types, so as to enable the students to choose the appropriate formula with ease. Further, detailed Hints/Solutions have been provided to Unsolved Numerical Problems. **Techie-Stuff** offers a special feature of the book. It contains real **Conceptual Numerical Problems** and **Conceptual Short Answer Questions.** It is aimed to provide intensive understanding and deep insight of the subject to the students, so that they get the feel of the type of questions asked in competitive examinations, such as I.I.T.

The Competitive Examination File in each unit forms another special feature of the book and consists of three parts. **Revision at a Glance** of the contents of a unit is for easy and handy reference of various physical laws, principles, terms and formulae in that unit and for its quick revision. **Numerical Problems from Competitive Examinations,** such as **I.I.T., Roorkee** and **I.S.M., Dhanbad** have been provided with solutions by adopting a novel technique in the form of **Thought Process.** Armed with this technique, the students will be able to attack the otherwise brainteasing and seemingly incomprehensible numerical problems with great ease. **Multiple Choice Questions** set in various competitive examinations, such as **C.B.S.E., A.I.I.M.S., A.F.M.C., M.N.R., C.P.M.T., I.I.T.,** etc have been thoroughly covered in the book. For the sake of easy preparation, these questions have been categorised into **Text-Based** and **Thought-Based** Multiple Choice Questions. The author is of the firm opinion that the learning is a continual and gradual process. With the Competitive Examination Files on different units at their disposal, the students would be able to master them steadily all through the academic year, while preparing for admission to professional courses.

MODERN'S abc OF + CHEMISTRY

ABOUT THE BOOK

The book in your hands is strictly based upon the new syllabus prescribed by C.B.S.E., New Delhi and Educational Boards of Indian states. The book has been written according to N.C.E.R.T. pattern and keeping in view the changing trends of different examinations. The book has been number ONE among the teachers and the students all over India for its clear presentation, effective approach of solving numerical problems and attractive figures.

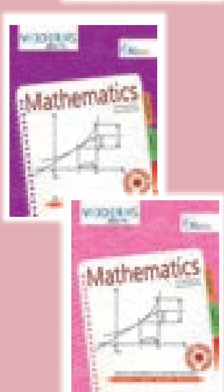
FEATURES OF THE BOOK

✤ Simple language and easily reproducible diagrams. ✤ Large variety of SOLVED NUMERICAL PROBLEMS. ✤ Additional numerical problems under the heading PRACTICE PROBLEMS for self assessment and practice. ✤ REVISION EXERCISES in the form : Very Short Answer Type, Short Answer Type and Long Answer Type Questions with HINTS and SOLUTIONS to some questions. ✤ CONCEPTUAL QUESTIONS solved at the end of each chapter. ✤ COMPETITION FILE covering additional information, graded numerical problems and objective questions to prepare for COMPETITIVE EXAMINATIONS for entrance to Medical and Engineering colleges. ✤ COMPLETE coverage of previous year questions from all types of Boards' examinations and competitive examinations such as I.I.T., Roorkee University, C.B.S.E. (PMT) and other State Boards.

In a NUTSHELL the book provides EXCELLENT GUIDANCE to students for Board's examinations as well as for competitive examinations for entrance to professional colleges.

A THOROUGH & SINCERE STUDY OF THE UNIQUE & UNMATCHED BOOK WILL BOOST THE STUDENTS TO ACHIEVE THEIR TARGET

MODERN'S abc OF + MATHEMATICS



ABOUT THE BOOK

The book designed for Higher Secondary class fulfils the student's need for a basic study of the concepts, methods and logic of modern discrete Mathematics. Its subject matter is simple, up-to-date and in accordance with the changing trends of different examinations. Solved examples and unsolved problems have been selected very carefully and graded properly. Keeping in view the modern trend, the exercises have been divided into three groups viz. " Very Short Answer Type Questions", "Short Answer Type Questions" and "Long Answer Type Questions." Almost each unit is followed by "Competition Corner" in order to meet the requirements of those students who are to appear in various competitive examinations for admission in I.I.T., Roorkee and other Engineering colleges of the country.

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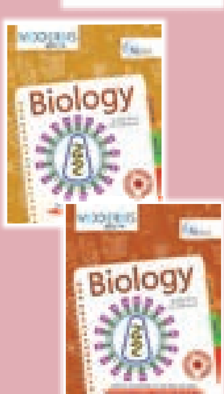


ABOUT THE BOOK

Modern's abc + of Biology has been written specially for students of XII under 10+2 system of education of CBSE and other Boards following NCERT pattern of Examination. Ever since the publication of the first edition, the book has been receiving the overwhelming response from teachers and taught alike. Keeping in view the recent edition of the book has been re-written as per latest syllabus. The book has been supplemented with practice problems and some interesting facts for competitive examinations at the end of each chapter. All the objective type questions and very short answer questions have been answered. Special attempt has been made to make the book useful for students preparing for competitive examinations for entrance to various medical Colleges. Superfluous details present the text material in most practical and original way.

With all these exclusive features, the book is bound to be the first choice of students all over India for Board and various competitive examinations.

CONTENTS

1 UNIFORM CIRCULAR MOTION

Revision Notes

IMPORTANT FORMULAE, FACTS AND TERMS

1. Circular motion

When a particle moves in a circular path with constant speed such that the magnitude of its velocity remains constant, while the direction of velocity changes continuously, the motion is said to be **uniform circular motion.**

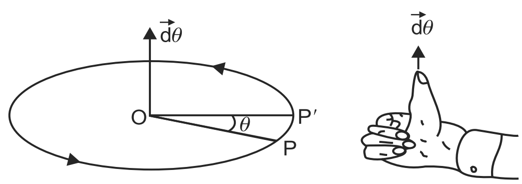
Key Point

The angular displacement θ, angular velocity ω and angular acceleration α are called *pseudo vectors* or **axial vectors** because they are directed along the axis of the circular path. Their sense of direction is given by the right hand thumb rule as stated below.

2. Angular displacement

The angle traced out by the particle moving in a circular path in a certain interval of time is called the ***angular displacement.*** It is expressed in *radians.* It is a dimensionless quantity. *Small angular displacements are taken to be vectors.*

Angular displacement as vector : Let us consider a particle P moving in a circular path of radius '*r*'. Let it trace out a small angle $d\vec{\theta}$ in a time *dt*. It can be shown that when $d\vec{\theta}$ *is infinitesimally small, it is a* ***Vector.***



Direction : The direction of *angular displacement vector* and the sense of rotation are related to each other and is given by Right Hand Rule which states that *if the fingers of the right hand are curled round the axis of rotation in the direction of rotation, then the thumb represents the direction of angular displacement vector.* The length of the arrow gives the magnitude $\left|d\vec{\theta}\right|$.

3. Angular Velocity

Angular velocity is defined as the rate of change of angular displacement of a body.

If a body describes a small angle Δθ in small interval of time Δ*t*, then the angular velocity ω is given by the relation $\omega = \frac{\Delta\theta}{\Delta t}$ and the instantaneous value of angular velocity will be given by

$$\omega = \underset{\Delta t \to 0}{\text{Limit}} = \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}.$$

Unit. Angular velocity is measured in radians per second (rad s^{-1}). Dimension of angular velocity is $[T^{-1}]$.

4. Relation between angular velocity, time period and frequency

We know that in one complete revolution a revolving body describes an angle of 2π radian in time T, the time period.

Thus $$\boxed{\omega = \frac{2\pi}{T}}$$

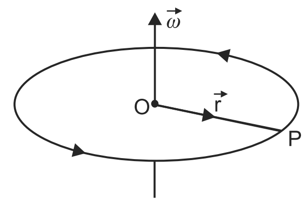
Further, $\frac{1}{T} = \nu$, where ν is the frequency.

Hence, $\omega = 2\pi \times \frac{1}{T} = 2\pi\nu$

5. Relation between angular velocity and linear velocity

Now $\omega = \frac{\theta}{t} = \frac{l}{rt}$ $(\because \theta = l/r)$

where l = length of the arc of radius 'r', traced out in time t by the moving particle. But $\frac{l}{t} = v$ = linear velocity, $\therefore$ $v = r\omega$.



6. Angular velocity as vector

We know that $\omega = \frac{d\theta}{dt}$. If $d\theta$ is small it is vector *i.e.* $\frac{d\theta}{dt}$ is **vector.** Hence $\vec{\omega}$ is a vector and $\vec{\omega} = \frac{d\vec{\theta}}{dt}$. The direction of $\vec{\omega}$ is the *same* as that of $d\vec{\theta}$.

7. Vector relation between linear velocity and angular velocity

Let $d\vec{\theta}$ be the angular displacement. The corresponding linear displacement $d\vec{r}$ is represented along PP'. *By right hand rule for the vector product*

$$d\vec{r} = d\vec{\theta} \times \vec{r}$$

or $$\frac{d\vec{r}}{dt} = \frac{d\vec{\theta}}{dt} \times \vec{r}$$

or $$\vec{v} = \vec{\omega} \times \vec{r}$$

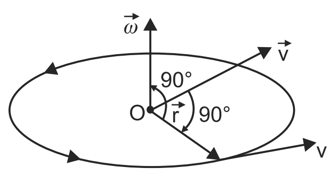


Fig. (*a*)

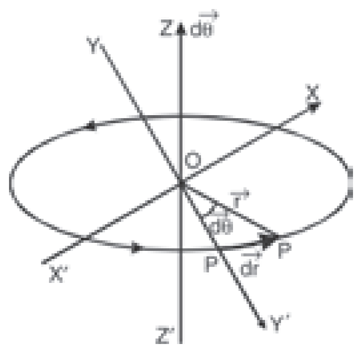


Fig. (*b*)

The direction of $\vec{v}$ is along the tangential direction as is represented in Fig. (*b*).

8. Angular acceleration

Consider a particle having variable angular velocity. Let ω_1, ω_2 be its angular velocities at times t_1 and t_2 respectively.

Let $\omega_2 - \omega_1 = \Delta\omega$ and $t_2 - t_1 = \Delta t$

Average angular acceleration is defined as the ratio of the change in angular velocity to the time taken by the particle to undergo this change.

$$\therefore \quad \vec{\alpha} = \frac{\Delta\omega}{\Delta t} \quad ...(1)$$

Instantaneous angular acceleration (α) : *It is defined as the limit of the ratio* $\frac{\Delta\omega}{\Delta t}$ *as* Δt *tends to zero.*

i.e. $$\alpha = \underset{\Delta t \to 0}{\text{Lt.}} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} \quad ...(2)$$

The units of α are radian/s^2 (rad/s^2) or degree/s^2.

For small change in velocity $d\vec{\omega}$ is vector and hence $\vec{\alpha}$ is vector.

Key Point

(*i*) For a particle in *non-uniform* motion in a circular path of constant radius the acceleration has two components.

(*a*) $\vec{a_t}$ = tangential component $= \vec{\alpha} \times \vec{r}$

(*b*) $\vec{a_c}$ = radial component $= \vec{\omega} \times \vec{v}$.

The resultant acceleration is given by $\vec{a} = \vec{a_t} + \vec{a_c}$ $= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$.

(*ii*) In case of the *uniform* circular motion $\vec{a_t} = 0$ but $\vec{a_c}$ is not ZERO.

9. Vector relation between linear acceleration and angular acceleration

We know that $\vec{v} = \vec{\omega} \times \vec{r}$

Differentiating both sides w.r.t. t, we get

$$\frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt} \quad \text{... (3)}$$

∴ From (2) and (3),

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$

Hence, the resultant acceleration of particle P consist of two components

(*i*) $\vec{\alpha} \times \vec{r}$ and (*ii*) $\vec{\omega} \times \vec{v}$

The first is called the **tangential component** given by $\vec{a_t} = \vec{\alpha} \times \vec{r}$ and second is called the **radial component** $\vec{a_c}$ or Centripetal Component given by $\vec{a_c} = \vec{\omega} \times \vec{v}$.

$$\boxed{a_t = r\alpha} \quad \text{and} \quad \boxed{a_c = v\omega = \frac{v \times v}{r} = \frac{v^2}{r} = r\omega^2}$$

Since $\vec{\alpha}; \vec{\omega}; \vec{v}$ and $\vec{r}$ are vectors the directions of $\vec{a_c}$ and $\vec{a_t}$ are represented as shown. As $\vec{\alpha}$ and $\vec{r}$ are at right angles and angle between $\vec{\omega}$ and $\vec{v}$ is also 90°, the acceleration of particle in a non-uniform circular motion at any instant is the resultant of two components $\vec{a_t}$ and $\vec{a_c}$ and

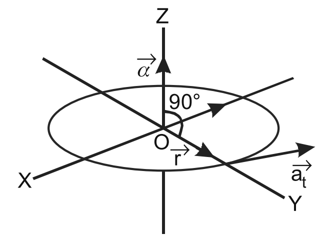
$$|a| = \sqrt{a_t^2 + a_c^2}$$

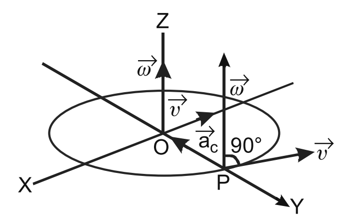
In case the particle moves with uniform speed, $\vec{\alpha} = 0$ and as such $\vec{a_t} = 0$. But $\vec{a_c}$ is not zero and is given by $\vec{a_c} = \vec{\omega} \times \vec{v}$. $\vec{a_c} = 0$ only when $\omega = 0$ *i.e.*, when the particle moves in a straight line.

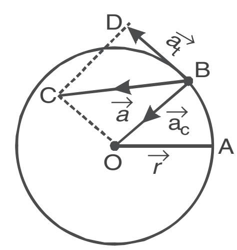
Thus, a_t **exists only for non-uniform circular motion** whereas a_c is **always present** whenever a particle moves in a circular path.

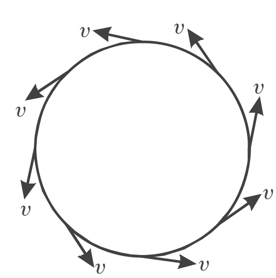
10. Centripetal force. When a body moves along a circular path with a constant speed its direction of motion changes every moment, as is shown in Fig. According to Newton's first law of motion, *nobody can change its straight line path itself, unless it is forced to do so.* Therefore, an external force must be applied on the body to move it along a circular path. Since the magnitude of velocity is not to change, the force applied at every point should have no component along the direction of velocity *i.e.* **along the tangent at the point.** To achieve it, the force should act at right angles to the tangent at every point (∵ Component of force F at 90° to it = F cos 90° = 0). As radius of a circle through any point is always perpendicular to the tangent to the circle at that point, *the force should act along the radius of the circle.* This centre seeking force is called **centripetal force.**

In uniform circular motion the directions of velocity, acceleration and tension in the string go on changing continuously.









Magnitude of centripetal force : Thus, centripetal acceleration of the revolving body $a = \frac{v^2}{r}$

$\therefore$ Centripetal force = Mass × Centripetal acceleration

or $$F = \boxed{\frac{mv^2}{r}}$$

If ω be its angular velocity, then $v = r\omega$ and hence

$$F = \frac{m}{r} . r^2\omega^2 = \boxed{mr\omega^2}$$

Also $\omega = \boxed{2\pi\nu,}$ where ν = frequency of revolution, hence

$$F = mr.\ 4\pi^2\nu^2 = 4\pi^2 mr\nu^2$$

$\therefore$ Centripetal force $F = \frac{mv^2}{r} = mr\omega^2 = \boxed{4\pi^2\ mr\nu^2} = \frac{4\pi^2\ m.r}{T^2}$

where T is the period of revolution.

Key Point

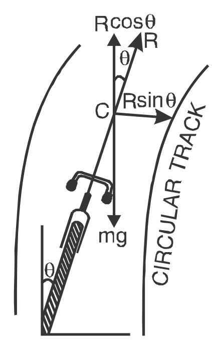
(*a*) A pseudo force which is equal and opposite to contripetal force is called *centrifugal force*. It acts on the agency which exerts the centripetal force. They cannot balance as they act on two different bodies.

(*ii*) Centripetal force cannot change the K.E. of the body as work done by this force is ZERO.

(*iii*) A torque is not essential for uniform rotation.

11. Applications of centripetal force.

(*a*) Bending of cyclist : When a cyclist goes round a circular track, a centripetal force is required to keep him in circular path. *If the force of friction is negligible,* he has to lean inward in order to provide this force by the component of the reaction. As is shown in the Fig. For equilibrium, the component R cos θ must balance the weight (mg) of the system and the component R sin θ should provide the centripetal force required.



$\therefore$ $$R\cos\theta = mg \quad ...(i)$$

and $$R\sin\theta = \frac{mv^2}{r} \quad ...(ii)$$

Dividing (*i*) by (*ii*), $\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2/r}{mg}$ or $\tan\theta = \boxed{\frac{v^2}{rg}}$ ... (*iii*)

or $$\theta = \boxed{\tan^{-1}\left(\frac{v^2}{rg}\right)}$$

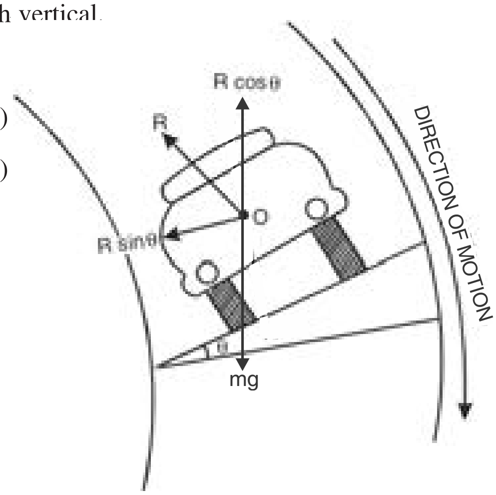
From (*iii*), the angle of bending (θ) will be small, if

(*i*) velocity (v) of turning is small, (*ii*) radius (r) of the circular path is large.

(*b*) Banking of tracks : To avoid overturning of cars and other vehicles the road is banked in such a way that the outer edge of the curved road is slightly raised above the level of inner edge as shown in Fig.

Let θ be the angle of banking of curved road. Clearly the total normal reaction acting effectively at the centre of gravity G of the car is also inclined at an angle θ with vertical.

Thus, in the equilibrium state



$$R\sin\theta = \frac{mv^2}{r} \quad ...(i)$$

and $$R\cos\theta = mg \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\tan\theta = \boxed{\frac{v^2}{rg}} \quad \text{or } \theta = \tan^{-1}\left[\frac{v^2}{rg}\right]$$

Maximum Safe Spped

Now $$v^2 = g\,r\tan\theta$$

or $$v = \sqrt{g\,r\tan\theta}$$

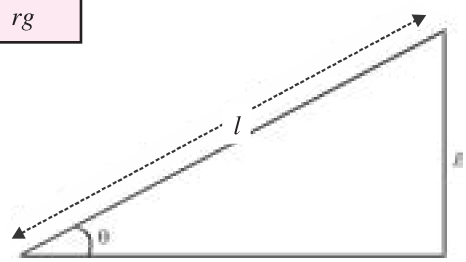
'v' gives us the **maximum safe speed** of the vehicle along the banked road.

If l is the width of the road and h is the height of the outer edge over the inner edge,

$$\sin\theta = \frac{h}{l} \approx \tan\theta \approx \theta.$$

$$\therefore \qquad \boxed{\tan\theta = \frac{h}{l} = \frac{v^2}{rg}}$$

$$\text{or} \qquad \boxed{h = \frac{lv^2}{rg}}$$



(*c*) **Aeroplane rounding a curve :** An aeroplane in curved flight, must also bend (bank) inwardly for the same reason as for a cyclist. In this case reaction is produced by the air. The angle of banking is :

$$\boxed{\tan\theta = \frac{v^2}{rg}.}$$

(*d*) **Maximum speed of a vehicle along rough road :** If a vehicle with its C.G. 'h' above the ground is in equilibrium while moving with a velocity v_{max} and turns round the curve of radius 'r' and if '$2a$' is the distance between its two wheels on the unbanked road, then for equilibrium,

$$\frac{mv^2_{max.}}{r \times AD} = \frac{mg}{CD}$$

or *Maximum* safe speed is given by :

$$v_{max} = \sqrt{\frac{g \times r \times AD}{CD}} = \sqrt{\frac{g \times r \times a}{h}}.$$

(*e*) *Let us now take into account the frictional force between the tyres and the road on a banked surface.*

In this case folowing three forces are being considered.

(*i*) Weight mg of the vehicle vertically downward through the C.G. of the body.

(*ii*) The reaction R of the road normal to its surface making an angle θ with vertical force.

(*iii*) Force of friction F, parallel to the plane surface.

Resolve R and F into two rectangular components *i.e.* R cos θ and F cos θ vertical and R sin θ and F cos θ horizontal.

Since the vehicle does not have any vertical motion, we have

$$mg + F\sin\theta = R\cos\theta$$

$$\text{or} \qquad mg = R\cos\theta - F\sin\theta$$

$$\text{But} \qquad F = \mu R$$

$$\therefore \qquad mg = R\cos\theta - \mu R\sin\theta$$

The force R sin θ and F cos θ acting horizontally provide the necessary centripetal force, thus

$$\frac{mv^2}{r} = R\sin\theta + F\cos\theta$$

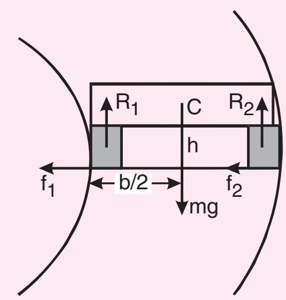
Key Point

When a vehicle is moving along unbanked road with forces of friction f_1 and f_2 and normal reactions R_1 and R_2, then taking moments about centre. We get

$$(f_1 + f_2)\,h = (R_1 - R_2)\,b/2$$

$$\left(\frac{mv^2}{r}\right)h = (R_1 - R_2)\,b/2$$

$$\text{or } R_1 - R_2 = \frac{2\,mv^2h}{rb}.$$

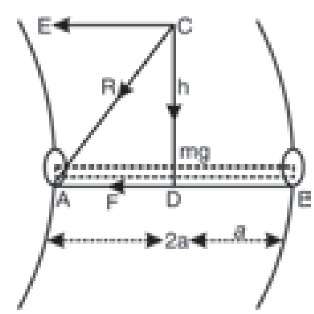


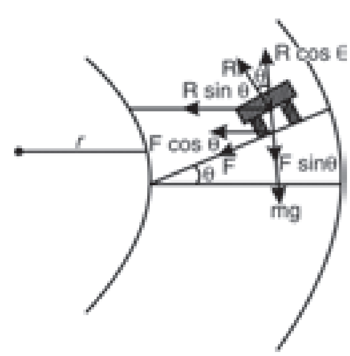
Also $R_1 + R_2 = mg$ solving

$$R_1 = \frac{1}{2}\left[mg - \frac{2\,mv^2h}{rb}\right]$$

$$R_2 = \frac{1}{2}\left[mg + \frac{2\,mv^2h}{rb}\right].$$

As v increases R_1 goes on decreasing and a stage is reached when $R_1 = 0$ and inner wheel leaves the ground.





or $\frac{mv^2}{r} = \text{R} \sin\theta + \mu\text{R}\cos\theta$

Dividing two equations, we get

$$\frac{\frac{mv^2}{r}}{mg} = \frac{\text{R}\sin\theta + \mu\text{R}\cos\theta}{\text{R}\cos\theta - \mu\text{R}\sin\theta} \quad \text{or} \quad \frac{v^2}{rg} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$$

or $$\frac{v^2}{rg} = \frac{\cos\theta\,[\tan\theta + \mu]}{\cos\theta\,[1 - \mu\tan\theta]}$$

or $$v = \sqrt{\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right) gr}$$

If the track is without friction μ = 0 and the safest speed is

$$v = \sqrt{gr\tan\theta}$$

If the road is **unbanked** but has a frictional force, then putting θ = 0, we get the **maximum safest speed** *i.e.*

$$v_{\text{max}} = \sqrt{\mu gr}$$

Key Point

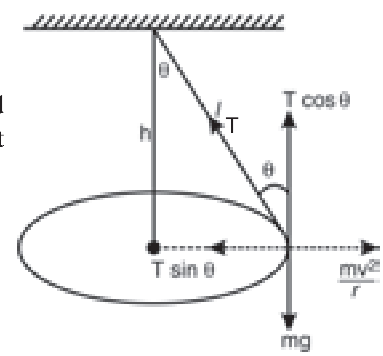
If the road is not banked at the curve but there is a force of friction just sufficient to avoid skidding, then let the force of friction be F, R be the normal reaction and μ the coefficient of friction, then F/R = μ or F= μ R. This force provides the centripetal force *i.e.*

$$\text{F} = \frac{mv^2}{r}$$

or $\mu\,\text{R} = \mu\,mg = \frac{mv^2}{r}$

or $\mu = \frac{v^2}{rg}$ gives least value of coeff. of friction.

(*f*) Conical pendulum or motion in horizontal circle : When a mass '*m*' is suspended from one end of a string and other end is fixed it forms a conical pendulum because it describes a horizontal circle and vertical cone with its apex at its fixed point. Then



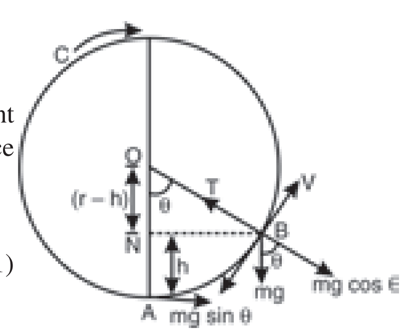
$$mg = \text{T}\cos\theta \text{ and } \frac{mv^2}{r} = \text{T}\sin\theta$$

and time period of pendulum $t = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{l\cos\theta}{g}}$.

(*g*) Motion in a vertical circle : Let us consider a particle (say stone) of mass '*m*' tied to a string of length '*r*' in motion from the lowest point A in anti-clockwise direction with an initial velocity '*u*' rising at height '*h*', in time '*t*'.

Let at any instant '*t*' the particle reaches point B describing angle θ from its original position. Considering equilibrium of particle at B, the forces acting on the stone are :

1. The weight *mg* acting vertically downward, and
2. The tension T in the string acting along BO.



The weight *mg* may be resolved into two rectangular components. The component *mg* cos θ acts along OB, *i.e.*, opposite to the direction of tension T. Thus, their difference T–*mg* cos θ provides the necessary centripetal force for circular motion.

$$\therefore \quad \text{T} - mg\cos\theta = \frac{mv^2}{r} \qquad \text{... (1)}$$

where v = velocity of the particle at point B.

The other component of weight *i.e.*, *mg* sin θ acts tangentially and in a direction opposite to that of velocity *v*. This component is responsible for change in velocity of the particle from *u* at lowest point to *v* here. If the effective vertical height through which particle has risen, while moving from A to B, be *h* (*i.e*, AN = *h*), then we have

$$v^2 = u^2 - 2\,gh \qquad \text{...(2)}$$

Substituting value of v^2 in (1), we have

$$T - mg\cos\theta = \frac{m}{r}\left(u^2 - 2gh\right)$$

or $$T = mg\cos\theta + \frac{m}{r}\left(u^2 - 2gh\right)$$

But from ΔOBN, $\cos\theta = \frac{ON}{OB} = \frac{OA - AN}{OB} = \frac{r-h}{r}$

$\therefore$ $$T = mg\left(\frac{r-h}{r}\right) + \frac{m}{r}\left(u^2 - 2gh\right)$$

$$= \frac{m}{r}\left[gr - gh + u^2 - 2gh\right]$$

or $$T = \frac{m}{r}\left[u^2 + gr - 3gh\right] \qquad ...(3)$$

This is the general equation of motion in a vertical circle giving us value of tension present in the string at any point during its motion.

Special cases :

1. At the lowermost (or starting) point A, $h = 0$ and hence tension at the lowest point :

$$T_L = \frac{m}{r}\left[u^2 + gr\right] \qquad ...(4)$$

2. At the uppermost point C, $h = 2r$ and hence tension at the point is given by

$$T_H = \frac{m}{r}\left[u^2 + gr - 6gr\right]$$

or $$T_H = \frac{m}{r}\left[u^2 - 5gr\right] \qquad ...(5)$$

Subtracting (5) from (4), we have

$$T_L - T_H = \frac{m}{r}\left[u^2 + gr\right] - \frac{m}{r}\left[u^2 - 5gr\right] = 6\,mg. \qquad ...(6)$$

3. If the initial velocity u is just sufficient to make the stone cross the highest point C without any slackening of string, then $T_H = 0$ and hence

$$0 = \frac{m}{r}\left[u^2 - 5gr\right] \text{ or } u^2 = 5gr \qquad [\textit{From equation (5)}]$$

or $$\boxed{u = \sqrt{5gr}}$$

Hence, the least velocity with which a particle (say stone) must be projected from the lowest position of a vertical circle so as to reach the highest point without leaving the track is $\sqrt{5gr}$ *. This is the minimum velocity with which the particle can* **LOOP the LOOP.**

4. Moreover, if V is the velocity which the particle possesses at highest point C, then

$$V^2 = u^2 - 2g.\,2r \quad [\because \text{ At highest point } h = 2r]$$
$$= u^2 - 4gr$$
$$= 5gr - 4gr = gr$$

or $\boxed{V = \sqrt{gr}}$ is commonly called as the **'Critical Velocity'.**

(*h*) Bucket revolving in vertical circle : If a bucket containing water is revolved fast enough in a vertical plane, water does not spill out of a bucket even when the bucket is just in inverted position at the highest point. This is possible if

$$\frac{mv^2}{r} = mg$$

or $$\boxed{v = \sqrt{rg}}$$

Key Point

Difference in the tensions at the lowest point and the highest point.

Now $T_L = \frac{mr}{r}[u^2 + gr]$ $(\because h = 0)$

$T_H = \frac{m}{r}[u^2 + gr - 6gr]$ $(\because h = 2r)$

Now difference of tensions

$T_L - T_H = \frac{m}{r}[u^2 + gr - u^2 + 5gr] = 6\,mg$

i.e., Six times the weight of the body.

Key Point

When the velocity of the particle at the lowest point is just sufficient to loop the loop, then

(*a*) velocity at lowest point

$u = \sqrt{5gr}$

(*b*) Velocity at the highest point

$v = \sqrt{gr}$

(*c*) Velocity at the middle point where $h = r$ is

$u_m^2 = 5gr - 2g.r = 3gr$

or $\boxed{u_m = \sqrt{3gr}}$.

Also the corresponding tensions are given by

$T = \frac{m}{r}(u^2 - 3gh + gr)$

(*a*) Tension at lowest point

$T_L = \frac{m}{r}(5gr - 0 + gr) = 6\,mg.$

(*b*) Tension at highest point

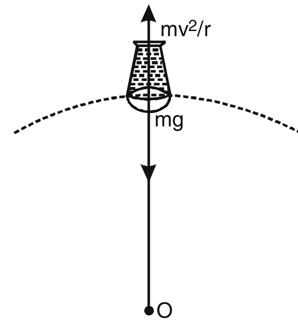
$T_H = \frac{m}{r}(5gr - 6gr + gr)$

= ZERO.

(*c*) Tension at the middle point

$T_M = \frac{m}{r}(5gr - 3gr + gr) = 3\,mg.$

(*i*) **Height of inclined plane for looping the loop of circle of radius '*r*' :** If v is the velocity at the bottom of the incline of height '*h*' then according to the principle of conservation of energy, P.E. at the top of the incline = K.E. at the bottom of the incline

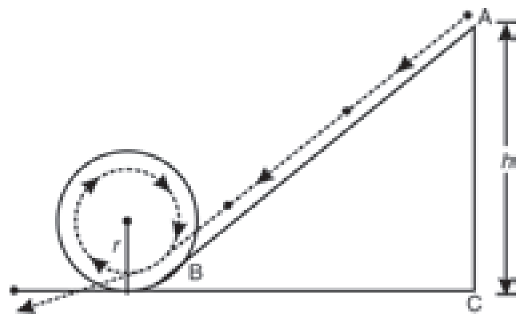


i.e., $mgh = \frac{1}{2}mv^2$

or $h = \frac{v^2}{2g}$ ...(*i*)

Further, the motor-cyclist will be able to go around the loop if the velocity at the lowest point of the loop is at least $\sqrt{5\,gr}$, putting in (*i*)

$\therefore$ $h = \frac{5\,gr}{2\,g} = \frac{5\,r}{2}$ *i.e.,* height of the plane should be two and a half times the radius. If 'D' is the diameter of the circle then D = 2r or $r = \frac{D}{2}$ and $h = \frac{5}{2} \times \frac{D}{2} = \frac{5}{4}D$.



Important Concepts and Tips

(*i*) The *speed, kinetic energy* and *angular momentum* of the particle moving in the circular path in uniform circular motion remain **constant.**

(*ii*) The work done by centripetal force is **zero.**

(*iii*) On an unbanked road, the minimum radius of curvature of the road for safety is $R = \frac{v^2}{\mu g}$.

(*iv*) To prevent skidding on unbanked road while going round a curve, the maximum velocity $v_{max} = \sqrt{\mu rg}$;

(*v*) The maximum speed with which a body can go round the vertical curve (say speed breaker) without loosing contact with ground is $v_{max} = \sqrt{gr}$.

(*vi*) Minimum velocity of the body at the bottom to go looping the loop is $\sqrt{5\,gr}$, where '*r*' is the radius of the loop.

(*vii*) Total energy of the particle while revolving in vertical circle is $\frac{5}{2}mgr$, where r = radius of the circle.

(*viii*) At a point in level with the centre of the vertical circle, the tension in string is 3 mg and velocity is $\sqrt{3\,gr}$.

(*ix*) The K.E. of body moving in horizontal circle is the same at all points while it is different at different points in case of motion in vertical circle.

(*x*) If the linear velocity of the body in a circular path of radius '*r*' is equal to the velocity of free fall through a height $\frac{r}{2}$, the centripetal acceleration is equal to '*g*'.

(*xi*) For looping the loop of circle of radius '*r*', the height '*h*' of an inclined plane from which body should be made to slide down should be two and a half times the radius.

Multiple Choice Questions

1. Torque produced by the centripetal force $\vec{F}_c$ in circle of radius $\vec{r}$ is given by :

(*a*) $\vec{r} \times \vec{F}_c$ (*b*) $\vec{F}_c \times \vec{r}$

(*c*) Null vector (*d*) None of these.

2. A body of mass m is moving in a horizontal circle of radius '*r*'. If the centripetal force is F, the kinetic energy of the body is :

(*a*) $\frac{F\,.\,r}{2}$ (*b*) $F\,.\,r$

(*c*) $\frac{F\,.\,r^2}{2}$ (*d*) $\sqrt{F\,.\,r}$

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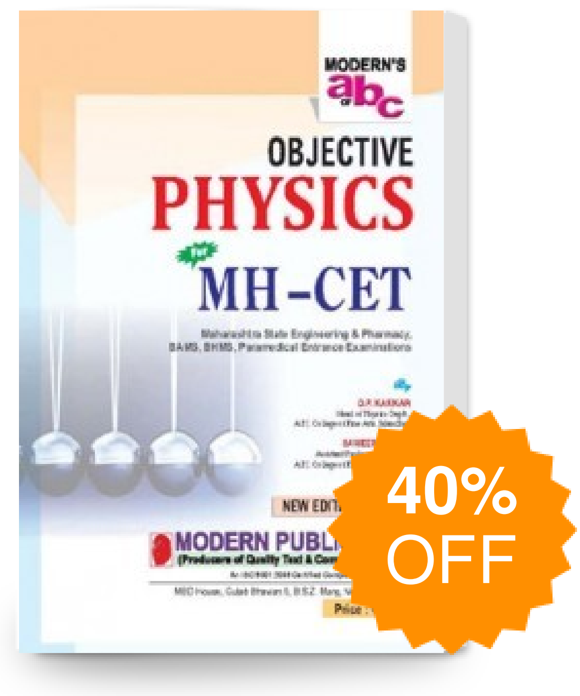

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