

**SPPU**

**First Year Engineering (F.E.)**

# ENGINEERING MATHEMATICS-I

**I<sup>st</sup> Semester** (Common to all branches)

- Strictly as per the SPPU syllabus.
- Introduction and key points to remember for every chapter.
- Applications of the topics to various branches of engineering explained
- Concepts explained with the help of flow chart.
- Large number of MCQ's with solution and for practice.
- MCQ's based test paper after every unit to gain confidence.
- May 2017 university paper with solution.

**Free  
MCQ's  
Booklet**



**Dr. Daljeet Kaur**

**Prof. Yuvraj M. Karad**



**Gigatech Publishing House**

*Igniting Minds*

# Engineering Mathematics - I

**Semester I**

**First Year Engineering Common to All Branches**

**As per the new revised syllabus of SSPU. w.e.f. academic year 2012-2013**

**Dr. Daljeet Kaur**

M.Sc (Maths) Ph.D. (Maths)

**Professor and First Year Head,  
Sinhgad Technical Education Society's,  
Sinhgad Academy of Engineering,  
Kondhwa, Pune - 411048**

**Prof. Yuvraj M. Karad**

B.E. (Mech.) M.E. (Design)

**Formaly Assistant Professor in  
Mahavir College of Engineering,  
Nashik**



**Gigatech Publishing House**

**Igniting Minds**

## **Engineering Mathematics - I**

First Year Engineering Common to All Branches

As per new revised syllabus of SPPUw.e.f. academic year 2012-2013

Dr. Daljeet Kaur, Prof. Yuvraj M. Karad

**First Edition:2017**

**Published By –**

**Gigatech Publishing House**

631/32, Budhawar Peth,

Office No. 105, First Floor, Shan Bramha Complex, Pune – 411 002.

Phone No. 952 952 0952

**Copyright ©Gigatech Publishing House**

All rights reserved. No part of this publication can be stored in any retrieval system or reproduced in and form or by any means without prior permission of the Publisher.

### **LIMITS OF LIABILITY AND DISCLAIMER OF WARRANTY**

The Authors and Publisher of this book have tried their best to ensure that the program, procedure and function described in the book are correct. However the author and publisher make no warranty with regard to the program and documentation contained in the book.

**ISBN NO. : 978-81-934-1403-3**

**Price : ₹460/-**

---

**Available at All Leading Book Stalls**

**Distributor :**

**PRADEEP BOOK DISTRIBUTORS**

102/3, 1<sup>ST</sup> Floor, Shan Bramha Complex,  
Pune. Telephone No. 020 – 24458333,

942 208 7031 / 844 699 5959

**E-mail : pbd0909@gmail.com**

**Note:** For any queries, doubts, suggestions & complaints regarding the subject, please feel free to contact on the below e-mail or phone number. We will be more than thankful for your feedback.

**Ph: 952 952 0952**

**E-mail : info.gigatech1@gmail.com**

## Preface

This book 'Engineering Mathematics-I' is intended to be a textbook for students of first year engineering of Savitribai Phule Pune University. In most Sciences, one generation years down what another has built and what one has established another undoes. In mathematics alone, each generation adds a new story to the old structure. Keeping this in mind, this book is written to have a better introduction of the Applied Mathematics. This book is presented with simple but exact explanation of subject matter, application of each topic to real-life, engineering problems, large number of illustrative examples followed by well graded exercises. We have tried to be rigorous and precise in presenting the mathematical concepts in very simple manner. We hope that the students will not only learn some powerful mathematical tools, but also will develop their ability to understand the concept and apply it properly to solve engineering problems. We feel that faculty members will also enjoy reading this book which is enriched with applications of each topic.

### Acknowledgement

We express our sincere thanks to Hon. Prof. M. N. Navale, Founder-President STES, Hon. Dr. (Mrs.) Sunanda M. Navale, Founder- Secretary STES, Hon. Mr Rohit Navale, Vice president (HR), STES and Hon. Mrs. Rachana Navale – Ashtekar, Vice President (Admin), STES as they are constant source of inspiration to us right from the first day of our teaching career. We would also like to thank Principal SCOE, SKNCOE, NBNSOE, SAE, SITS, RMDSOE, SIT, SKNSITS for their co-operation and support. We would also thank our department colleagues for their suggestions and timely help.

Any suggestions for the improvement of this book will be sincerely acknowledged and incorporated in the next edition.

We are thankful to Mr. Harshal J Potadar and Rohit K Dongare from **Gigatech Publishing House (GPH)** for their continuous support and patience in preparing this book.

# SYLLABUS

---

## Unit 1: (09 Hrs.)

**Matrices:** Rank, Normal form, System of Linear Equations, Linear Dependence and Independence, Linear and Orthogonal Transformations. Eigen values, Eigen Vectors, Cayley – Hamilton Theorem. Application to problems in Engineering (Translation and Rotation of Matrix).

(Ref. Chapter 1, 2 and 3)

## Unit 2:

**Complex Numbers & Applications:** Argand's Diagram, De'Moivre's theorem and its application to find roots of algebraic equations. Hyperbolic Functions, Inverse Hyperbolic Functions, Logarithm of Complex Numbers, Separation into Real and Imaginary parts, Application to problems in Engineering

(Ref. Chapter 4 and 5)

## Unit 3: (09 Hrs.)

**Infinite Series:** Infinite Sequences, Infinite Series, Alternating Series, Tests for Convergence, Absolute and Conditional Convergence, Range of Convergence.

**Differential Calculus:** Successive Differentiation, Leibnitz Theorem

(Ref. Chapter 6 and 7)

## Unit 4: (09 Hrs.)

**Expansion of Functions:** Taylor's Series and Maclaurin's Series.

**Differential Calculus:** Indeterminate Forms, L' Hospital's Rule, Evaluation of Limits.

(Ref. Chapter 8 and 9)

## Unit 5: (09 Hrs.)

**Partial Differentiation and Applications:** Partial Derivatives, Euler's Theorem on Homogeneous Functions, Implicit functions, Total Derivatives, Change of Independent Variables

(Ref. Chapter 10)

## Unit 6: (09 Hrs.)

**Jacobian:** Jacobians and their applications. Errors and Approximations.

**Maxima and Minima:** Maxima and Minima of Functions of two variables, Lagrange's method of undetermined multipliers.

(Ref. Chapter 11)

# CONTENTS

## UNIT - I

### 1. Matrices 1.1 – 1.32

- 1.1 Introduction
- 1.2 Applications of Matrices
- 1.3 Matrix
  - 1.3.1 Types of Matrices
- 1.4 Operations of Matrices
  - 1.4.1 Addition and Subtraction of Matrices
  - 1.4.2 Matrix Multiplication
  - 1.4.3 Equality of Two Matrices
  - 1.4.4 Elementary Transformations of Matrices
- 1.5 Rank of a Matrix
- 1.6 Echelon Form or Canonical Form of a Matrix
  - 1.6.1 Examples on Echelon Form or Canonical Form of a Matrix
- 1.7 Normal Form of Matrix
- 1.8 Find non-singular matrices P and Q such that PAQ in normal form
- 1.9 Computation of Inverse by Adjoint Method
- 1.10 Key Points

### 2. System of Linear Algebraic Equations 2.1 – 2.70

- 2.1 Introduction
- 2.2 Geometrical Interpretation of Consistency of Non-Homogeneous Equations
- 2.3 Type I: Problems on Non - Homogeneous System of Equations using Matrix method or Method of Reduction
- 2.4 Type 2: Non - Homogeneous Equations with Coefficient matrix A OR R.H.S. matrix B containing some  $\lambda$  or  $\mu$  or  $k$ 
  - 2.4.1 Solved Examples
- 2.5 Non - Homogeneous System of Equations for 3 Equations in 3 Unknowns
  - 2.5.1 Solved Examples
- 2.6 System of Homogeneous Linear Equations
  - 2.6.1 Solved Examples of System of Homogeneous Linear Equations
- 2.7 Homogeneous System of Equations with Three Equations in Three Unknowns 2-30
  - 2.7.1 Solved Examples
- 2.8 Linear Dependent and Independent Vectors
  - 2.8.1 Solved Examples
- 2.9 Linear Transformation
  - 2.9.1 Solved Problems on Linear Transformation
- 2.10 Orthogonal Transformation and Orthogonal Matrix
  - 2.10.1 Solved Examples on Orthogonal Transformation and Orthogonal Matrix
- 2.11 Key Points

### **3. Eigen Values and Eigen Vectors** **3.1 – 3.53**

- 3.1 Introduction
- 3.2 Applications of Eigen Values and Eigen Vectors
- 3.3 Mathematical Representation of Eigen Values and Eigen Vectors
- 3.4 Properties of Eigen Values and Eigen Vectors
- 3.5 Method for Finding Eigen Values and Eigen Vectors
  - 3.5.1 Type I: A is Non-symmetric, Eigen Values are Non-repeated
  - 3.5.2 Type 2: A is Non-symmetric, Eigen values are Repeated
  - 3.5.3 Type 3: A is Symmetric. Eigen values are Non-repeated
  - 3.5.4 Type 4: A is Symmetric, Eigen values are Repeated
  - 3.5.5 Type 5: Examples Involving only Two Eigen Vector  
OR Only One Eigen Vector for 'n' Eigen Values
- 3.6 Cayley Hamilton Theorem
- 3.7 Key Points

## **UNIT - II**

### **4. Complex Numbers** **4.1 – 4.62**

- 4.1 Introduction
- 4.2 Applications of Complex Numbers
- 4.3 Algebra of Complex Numbers
- 4.4 Geometrical Representation of A Complex Number : The Argand Diagram
- 4.5 Polar Form of A Complex Number : (Modulus /Argument)
- 4.6 Solved Examples on Basic Concepts
- 4.7 Problems on Argand Diagram and Locus
- 4.8 Powers of Complex Numbers : De moivers Theorem
  - 4.8.1 Examples on De moivers Theorem
- 4.9 Applications of De Moivres Theorem to solve Algebraic Equations
  - 4.9.1 Problems on Roots of Equations
- 4.10 Key Points

### **5. Hyperbolic Functions** **5.1 – 5.47**

- 5.1 Introduction
- 5.2 Application of Hyperbolic Functions
- 5.3 Relation Between Circular and Hyperbolic Functions
- 5.4 Derivatives of Hyperbolic Functions
- 5.5 Examples on Basic Concepts, Definitions, Identities and Formulae
- 5.6 Examples on Type 2: Separation of Real and Imaginary Parts
- 5.7 Logarithm of Complex Numbers
- 5.8 Problems on Basic Concepts, Definition of  
General Value and Principal Value of Logarithm
- 5.9 Separation of Real and Imaginary Parts
  - 5.9.1 Type I: Problems of the Type  $(x + iy)^{a+ib}$

5.9.2 Type 2: Problems on the type  $\log_{a+ib} (x + iy)$  Method  
Use Change of Base Rule

5.9.3 Type 3: Problems of the Type  $\frac{(a + ib)^{x + iy}}{(p + iq)^{c + id}} = r + is$

5.10 Key Points

### UNIT - III

#### 6. Infinite Series

6.1 – 6.56

6.1 Introduction

6.2 Applications of Infinite Series

6.3 Infinite series

6.3.1 Sequence of Partial sums

6.3.2 Tests of Convergence and Divergence

If  $\lim_{n \rightarrow \infty} u_n \neq 0$ , then  $\sum_{n=1}^{\infty} u_n$  is Divergent

6.3.3 Geometric Series

6.3.4 The P-Series

6.3.5 Comparison Test

6.3.6 Solved Examples

6.4 D'Alembert's Ratio Test

6.4.1 Solved Examples

6.4.2 Ratio Test

6.4.3 Solved Examples

6.5 Raabe's Test

6.5.1 Solved Examples

6.6 Cauchy's  $n^{\text{th}}$  Root Test

6.6.1 Solved Examples

6.7 Logarithmic Test

6.7.1 Solved Examples

6.8 Alternating Series

6.8.1 Leibnitz's Test

6.8.2 Solved Examples

6.9 Key Points

#### 7. Successive Differentiation

7.1 – 7.47

7.1 Introduction

7.2  $n^{\text{th}}$  Derivative of Some Elementary Function

7.3 Applications of Successive Differentiation

7.4 Type I: Basic Type

7.4.1 Solved Examples

7.5 Types II: Based on Standard  $n^{\text{th}}$  Derivatives

7.5.1 Solved Examples

- 7.6 Type 3: Based on Partial Fractions
  - 7.6.1 Solved Examples
- 7.7 Type 4: Based on De Moivre's Theorem
  - 7.7.1 Solved Examples
- 7.8 Type 5: Miscellaneous Examples
  - 7.8.1 Solved Examples
- 7.9 Leibnitz's Theorem [for Finding  $n^{\text{th}}$  Derivative of Product of Two Functions]
  - 7.9.1 Solved Examples on Type I
  - 7.9.2 Type II
  - 7.9.3 Solved Example on Type II
- 7.10 Key Points

## UNIT - IV

### 8. Taylor's and Maclaurin's Theorems 8.1 – 8.43

- 8.1 Introduction
- 8.2 Applications of Maclaurin's Series
- 8.3 Maclaurin's Series
  - 8.3.1 Examples on Maclaurin's Theorem
- 8.4 Standard Expansions using Maclaurin's Theorem
- 8.5 Methods of Expansions of Functions
- 8.6 Type I: Use of Standard Expansions
- 8.7 Type II: Use of Differentiation and Integration
  - 8.7.1 Solved Examples
- 8.8 Type DI: Based on Substitution
- 8.9 Type IV: Use of Leibnitz Theorem
  - 8.9.1 Solved Examples
- 8.10 Introduction of Taylor Series
- 8.11 Applications of Taylor Series
- 8.12 Examples on Taylor's Series
- 8.13 Key Points

### 9. Indeterminate Forms 9.1 – 9.50

- 9.1 Introduction
- 9.2 Applications of L'Hospital's Rule
- 9.3 L'Hospital's Rule for  $\frac{0}{0}$  and  $\frac{\infty}{\infty}$
- 9.4 Other Indeterminate Forms
- 9.5 Type 1: Indeterminate Form  $\frac{0}{0}$  Form
  - 9.5.1 Examples
  - 9.5.2 Type I (B): To Find the Values of Unknowns

- 9.6 Type 2: Indeterminate Form  $\frac{\infty}{\infty}$ 
  - 9.6.1 Examples
- 9.7 Type 3:  $\infty - \infty$  Form
  - 9.7.1 Examples
- 9.8 Type 4: Indeterminate  $0 \cdot \infty$  Form
  - 9.8.1 Examples
- 9.9 Type 5: Indeterminate Forms  $0^0, \infty^0, 1^\infty$ 
  - 9.9.1 Examples
- 9.10 Type 6: Problems on Series Expansions
  - 9.10.1 Examples
- 9.11 Key Points

## UNIT - V

### 10. Partial Differentiation

**10.1 – 10.96**

- 10.1 Introduction
- 10.2 Partial Derivatives of First Order
- 10.3 Applications of Partial Differentiation
- 10.4 Type I: Problems Based on Direct Partial Differentiation, Basic Rules
- 10.5 Variables to be Treated as Constants
  - 10.5.1 Solved Examples
- 10.6 Homogenous Functions
  - 10.6.1 Homogenous Functions for Two Variables
  - 10.6.2 Homogenous Function of Three Variables
  - 10.6.3 Euler's Theorem on Homogenous Function
  - 10.6.4 Solved Examples
- 10.7 Composite Functions
- 10.8 Total Derivative
  - 10.8.1 Theorem of Total Differential Coefficient
  - 10.8.2 Implicit Differentiation
  - 10.8.3 Solved Examples
- 10.9 Change of Independent Variables
  - 10.9.1 Solved Examples
- 10.10 Key Points

## UNIT - VI

### 11. Jacobians

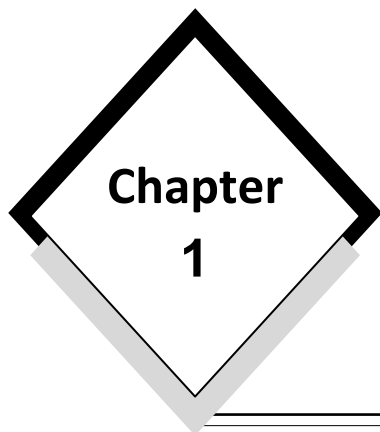
**11.1 – 11.82**

- 11.1 Introduction
- 11.2 Definition
- 11.3 Applications of Jacobian

- 11.4 Evaluation of Jacobian
  - 11.4.1 Type I: Explicit Form
  - 11.4.2 Solved Examples
  - 11.4.3 Type 2: Verify  $JJ' = 1$  Where  $J = \frac{\partial (u, v)}{\partial (x, y)}$  and  $J' = \frac{\partial (x, y)}{\partial (u, v)}$
  - 11.4.4 Solved Examples
  - 11.4.5 Type 3: Jacobian of Composite Functions [Chain Rule]
  - 11.4.6 Solved Examples
  - 11.4.7 Type 4: Jacobian of Implicit Function
  - 11.4.8 Solved Examples
  - 11.4.9 Type 5: Partial Derivative of Implicit Functions
  - 11.4.10 Solved Examples
  - 11.4.11 Type 6: Functional Dependence
  - 11.4.12 Solved Examples
- 11.5 Errors and Approximations
  - 11.5.1 Relative Error
  - 11.5.2 Absolute Error
  - 11.5.3 Solved Examples
- 11.6 Maxima and Minima
  - 11.6.1 Solved Examples
- 11.7 Lagrange's Multipliers
  - 11.7.1 Working Rule for Lagrange's Multipliers
  - 11.7.2 Solved Examples
- 11.8 Key Points

**SOLVED QUESTIONS PAPER MAY-2017****Q.1-Q.23****QUESTIONS PAPERS DEC-2012 TO MAY-2016****1-13**

◆◆◆



## Unit I

# Matrices

### Syllabus

**Matrices** : Rank, Normal form,

#### 1.1 Introduction

Matrices are a very important tool in expressing and discussing problem which arise in daily life. For example, if two families A and B have expenses such as utilities, food, health and entertainment then how can this data collected be represented? There are many ways available but one of them has an advantage of combining the data so that it is easy to manipulate. Indeed, We write the data as,

$$\text{Month} = \begin{bmatrix} \text{Family} & \text{Food} & \text{Utilities} & \text{Health} & \text{Entertainment} \\ A & a & b & c & d \\ B & e & f & g & h \end{bmatrix}$$

For simplicity and convenience, we write.

$$A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix}$$

← Row

↑  
Column

This is what we call a matrix. The size of the matrix is defined by the number of rows and number of columns. In this case, the above matrix has 2 rows and 4 columns. We can have a matrix with  $m$  rows and  $n$  columns. In This case, we say that matrix is a  $(m \times n)$  matrix, Pronounced as  $m$  by  $n$  matrix. The first entry meaning  $(m)$  is the number of rows while the second entry  $(n)$  is the number of columns. Our above matrix is a  $2 \times 4$  matrix.

A simplest example of a matrix is a wall calendar. It is organized as a set of days, where each row is a week. Another example of matrix in daily life is a sensor in a camera. It is a rectangular organized set of components that convert light energy to electric signals.

Matrices have a wider application in Engineering. Many problems can be transformed into simultaneous equations and their solution can easily be found with the help of matrices. Matrix is not only a convenient notation that summaries in a natural and convenient form whole group of operations but it is actually possible to solve set of differential or algebraic equations written in matrix form in a most convenient manner.

### 1.2 Applications of Matrices :

---

There are many applications of matrices in everyday life.

- 1) Matrices are used in cryptography. Cryptography is concerned with keeping communication private. Today the governments use sophisticated methods of coding and decoding messages. One type of code, which is extremely difficult to break, makes use of a large matrix to encode a message. Then the inverse of the matrix is used by the receiver, to decode the message. The first matrix is called the encoding matrix and its inverse is called the decoding matrix. Also the internet could not function without encryption and neither could banks because they now use these same means to transmit private and sensitive data.
- 2) Matrices provide a solid, general tool to represent and combine all common transformation. The most power full feature that matrices give is concatenation where by several transformation (rotation, translation, Scaling etc).

Can be combined into a simple way matrix. You can do translation rotation Scaling etc. operations without using matrices. But as soon as you want a general representation for any transformation it becomes necessary to use matrices. In computer graphics concatenation (combining) is a very common operation so it is beneficial to have a single representation and a simple way to combine all sorts of transformations.

Matrix multiplication is particularly useful in computer graphics since a digital image is basically a matrix. The rows and columns of pixels, and the numerical values corresponding to the pixels colour value. Matrix multiplication is also useful in decoding video signals and it can help process digital sound

- 3) The graph paper you use is also a matrix. The latitudes and longitudes are also a matrix. The pixels of a TV are also a matrix.
- 4) In computer based applications, matrices play an important role in the projection of three dimensional image into a two dimensional screen, creating the realistic seeming motions. Computer have embedded matrix arithmetic in graphic, processing algorithm, especially to render reflection and refraction.

- 5) In robotics and automation, matrices are the base elements for the robot movements. The movements of the robots are programmed with the rows and columns of matrices. The inputs for controlling the robots are given based on the calculation from matrices.
- 6) Matrices are the best representation methods for plotting common survey things like the traits of people's population, habits etc.
- 7) Matrices are used in calculating the gross domestic products in economics which helps in calculating the goods production efficiency.
- 8) In physics related applications, matrices are applied in the study of electrical circuits, quantum mechanics and optics. Matrices play an important role in the calculation of battery power outputs, resistor, conversion of electrical energy into another useful energy, in solving problems using Kirchhoff's laws of voltages and current.

### 1.3 Matrix

#### Definition :

The arrangement of set of elements in the form of rows and columns is called a matrix. The elements of the matrix can be real (or) complex numbers. In general a  $m \times n$  matrix is given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} \begin{matrix} \leftarrow \text{Row} \\ \\ \text{Column} \end{matrix} \quad m \times n$$

#### Order of the Matrix :

#### Definition :

The number of rows and columns represents the order of the matrix. It is denoted by  $m \times n$ , where  $m$  is the number of rows and  $n$  is the number of columns.

**Note :** Matrix is a system of representation and it does not have any numerical value.

e.g. : The array of numbers below is an example of a matrix.

$$\begin{bmatrix} 21 & 62 & 33 & 93 \\ 44 & 95 & 66 & 13 \\ 77 & 38 & 79 & 33 \end{bmatrix}$$

The order of the above matrix is  $3 \times 4$ , meaning that it has 3 rows and 4 columns.

**Matrix Notation :** A matrix is denoted by capital letters such as A, B, C, ... etc.

#### 1.3.1 Type of Matrices

1. **Row Matrix :** A matrix is said to be a row matrix, if it contains only one row.

e.g.  $A = [1 \ 5 \ 4 \ 3]$  is a row matrix.

2. **Column Matrix** : A matrix is said to be a column matrix, if it contains only one column.

e.g.  $A = \begin{bmatrix} -4 \\ 5 \\ 2 \end{bmatrix}$  is a column matrix.

3. **Rectangular matrix** : A matrix is said to be rectangular, if the number of rows and number of columns are not equal.

e.g.  $A = \begin{bmatrix} 3 & -4 & 1 \\ -6 & 2 & -9 \end{bmatrix}$  is a rectangular matrix.

4. **Square Matrix** : A matrix is said to be square, if the number of rows and number of columns are equal.

e.g.  $A = \begin{bmatrix} -2 & 6 \\ -4 & 5 \end{bmatrix}$  is a square matrix.

5. **Diagonal matrix** : A square matrix is said to be a diagonal matrix, if all the elements except the principal diagonal elements are zeros.

e.g.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

6. **Scalar Matrix** : A square matrix is said to be a scalar matrix if all the diagonal elements are equal.

e.g.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

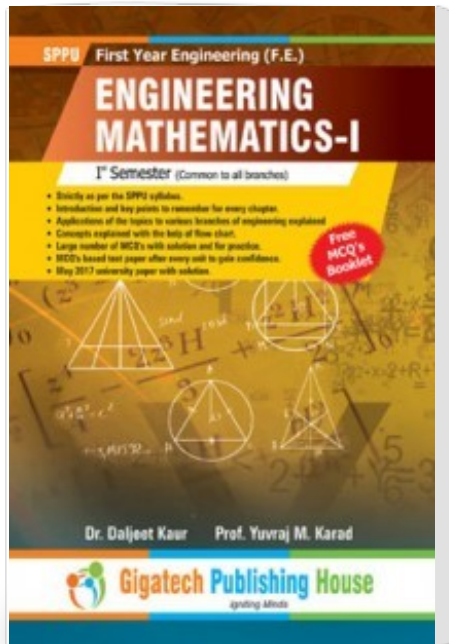
7. **Upper Triangular Matrix** : It is a square matrix in which all the elements below the principal diagonal are zeros.

e.g.

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 7 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

8. **Lower Triangular Matrix** : It is a square matrix in which all the elements above the principal diagonal are zeros.

# Engineering Mathematics-I Ist Semester (Common to all branches)



Publisher : Gigatech Publishing  
House

ISBN : 9788193414033

Author : Dr. Daljeet Kaur,  
Prof. Yuvraj M. Karad

Type the URL : <http://www.kopykitab.com/product/11954>



## Get this eBook