

Third Edition

STRUCTURAL ANALYSIS

Volume-II



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STRUCTURAL ANALYSIS

VOLUME II

By

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STRUCTURAL ANALYSIS

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FOREWORD

The two volume publication on Structural Analysis by Dr. R.Vaidyanathan and his disciple Dr. P. Perumal is totally geared to meet the syllabus of Anna University.

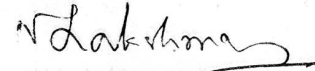
The first volume deals comprehensively with analytical methods while volume two deals with moving loads and analysis of different structural configurations.

The speciality of the book is that it addresses the students as if a teacher would address them in a classroom.

Thanks to several decades of teaching Structural Engineering subjects in almost all the Government colleges in the state, the authors have had access to the students' minds and most of their doubts and difficulties and have used this book to answer them and clarify many a mystery relating to the behaviour of various structural shapes.

They have rendered the mathematical parts to painfree simplicity and concentrated on all practical applications of every structural shape like arches, suspension cable, rigid frames and curved beams.

I am sure students of Civil Engineering all over the world find these volumes immensely helpful in aiding their comprehension of structural behaviour and analysis.


(N. Lakshmanan)



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PREFACE TO THE THIRD EDITION

We feel happy to note the continuous increase in the reception of the second edition of Structural Analysis – Volume II. The thoroughly revised book, now in its third edition includes improved/simplified calculations, addition of new problems on influence lines for trusses, influence lines for indeterminate structures, Arches, Cables and suspension bridges and space trusses. We have made the contents appropriate for a wide range of readers so that any student or professional would find the desired contents.

M/s Laxmi Publications have been very tolerant and patient with the Authors. Our thanks are due to M/s Laxmi Publications whose professional comments and suggestions have made value addition to this book. The authors owe the publishers a lot for their flexibility and accommodation.

The continued reinforcing feed backs and encouraging words from Professors and students all over, in various forms, acts as catalysts in our effort.

—Authors

PREFACE TO THE FIRST EDITION

The authors have great pleasure in presenting to the student community the second volume of their book on Structural Analysis. This book meets the syllabus for Structural Analysis II of Anna University, Tamil Nadu. However, this also will serve the students and teachers of all other Universities, AMIE students and practicing Engineers. The objective of this book is to present the material in a simple way. The authors have taken care to answer the doubts that they themselves had when they were students of Civil Engineering.

This volume contains eight chapters, namely Rolling loads, Influence lines, Influence Lines for Indeterminate Structures, Arches, Cables and Suspension bridges, Beams Curved in Plan, Space Trusses and Plastic Analysis of Structures. All the chapters are presented in a simple and lucid style with a lot of numerical examples and illustrative sketches for the benefit of the teachers and students.

A large number of problems are also available at the end of each chapter to enable the student to test his/her understanding of the subject. The authors have freely consulted many good books on the subject and the help is gratefully acknowledged.

The authors are aware that the book is not perfect. They would appreciate if errors discovered, if any, are brought to their attention. Suggestions from the readers for improvement of this book will be thankfully appreciated. The next edition is sure to take into account all such suggestions and would be more free from errors and omissions.

The authors are very confident that the staff and student community who are in need of a single, simple book covering the entire syllabus with simple presentation will get the fullest benefits.

—Authors

ACKNOWLEDGEMENTS

We are not sure we saw farther than others, but we had a lot of giants on whose slippery shoulders we could stand to peek into the intricacies of Structural Analysis. We thank them all.

First and foremost, we acknowledge the following authors whose books enlightened us in several aspects.

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M/s Laxmi Publications have been really very quick, prompt, understanding and considerate when we were regularly passing deadlines and overhauling our own texts. We are thankful to them for their accommodative partnership and hope to deserve the same in future.

We both are greatly beholden to Seetha and Saraswathi, our wives and our family for putting up with our night owl habits and irregular mealtimes. They have suffered a lot and we are trying to prove they have not suffered in vain.

A lot of people gave us words of encouragement which sustained us in our grind. We say, “thank you folks; the words may not have cost you much, but they were invaluable to us”.

Several sets of students have served as sounding boards for us, the brilliant, the slow, the inattentive and the indifferent. But all of them have helped us decide what kind of presentation would be effective.

We therefore dedicate this book to the student community.

—Authors

CHAPTER 1

ROLLING LOADS

1.1 INTRODUCTION

In Civil Engineering structures, loads can change their positions. In such a case, certain load positions can be critical to certain parts or components of the structure. Shifting of load positions is common enough in buildings. But they are more pronounced in bridges, over which vehicles keep rolling. Certain wheel positions can be again crucial. We should be able to identify such positions and their influence on structural members. In this chapter we will consider beams and try to evolve criteria for load positions causing maximum bending moments or maximum shears.

1.2 SIMPLY SUPPORTED BEAMS

1.2.1 Load Categories

We can consider 5 categories of loads on beams :

1. Single concentrated loads.
2. udl longer than the beam span
3. udl shorter than the beam span
4. Two wheel axles separated by a fixed distance
5. Multiple wheel axles (train of loads).

The objectives of this study on rolling loads are:

1. To find the load position and values of maximum shear force and bending moment at a given section due to a given system of rolling loads.
2. To find the location and values of the absolute maximum shear force and bending moment that may occur on the span due to the given system of rolling loads and
3. To find the equivalent udl due to a given system of rolling loads to make the designer's work simple.

1.2.2 Simply Supported Beam with a Single Concentrated Load W Moving from the Left End to the Right

As a load W moves from the left support A to the right we will try to get the maximum positive shear force, maximum negative shear force and the maximum bending moment at any typical section X , at a distance x from A . We will call the distance of the load from A as a . (see Fig. 1.1).

(a) Maximum positive shear at X (Load in XB)

For any load position (for any a),

$$R_A = \frac{W \cdot (l - a)}{l} \quad R_B = \frac{Wa}{l}$$

[when $a < x$, (when W is on AX), SF at X is equal to R_B and is negative ;
when $a > x$ (when W is on XB), SF at X is equal to R_A and is positive.]

SF at X,
$$F_X = R_A = \frac{W(l-a)}{l}, \text{ when } a \geq x. \quad \dots(1.1)$$

The maximum value of positive F_x occurs when a is least, *i.e.*, $a = x$

and
$$F_{x \max} = \frac{W(l-x)}{l} \quad \dots(1.2)$$

This is plotted in Fig. 1.1 (c).

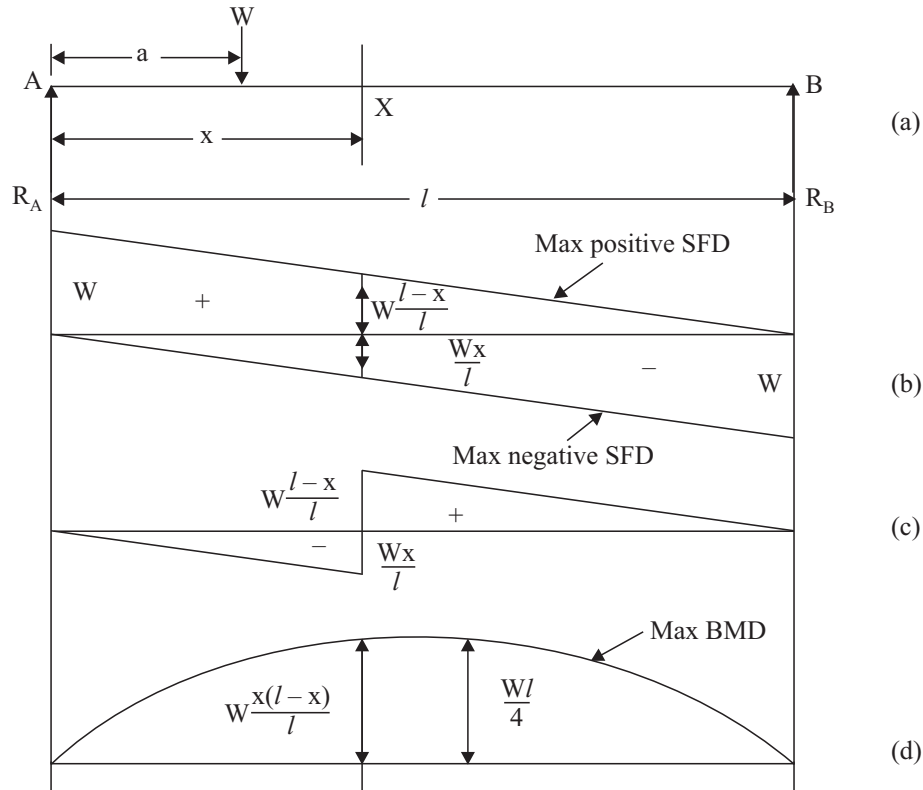


Fig. 1.1

(b) Maximum negative shear at X

Negative shear occurs at X when the load is on AX ($a \leq x$) and

$$F_x = -R_B = -\frac{Wa}{l} \quad \dots(1.3)$$

The maximum value of R_B occurs at X, when $a = x$. This is seen in Fig. 1.1 (c). Maximum negative shear, $F_{x \max}$ is then $-\frac{Wx}{l}$

From the above 2 cases, we can conclude that the maximum positive shear occurs at a section X when the load is on the section and just to the right of it. Its value is $\frac{W(l-x)}{l}$. The maximum negative shear at X occurs when the load is at X (and just to the left of X). Its value is $\frac{Wx}{l}$.

(c) Absolute maximum shear ($F_{\max \max}$)

To get this, we can vary x to get $F_{\max \max}$.

$$F_{\max \max} + = \frac{W(l-0)}{l} = W, \quad \text{when } x = 0 \quad \dots(1.4)$$

$$F_{\max \max} - = \frac{-W \cdot l}{l} = -W \quad \text{when } x = l \quad \dots(1.5)$$

$F_{\max} +$ and $F_{\max} -$ are plotted in Fig. 1.1 (b)

(d) Maximum bending moment diagram

(i) Load on AX

$$M_x = R_B \cdot (l-x) = \frac{Wa}{l} (l-x) \quad \dots(1.6)$$

Here, the max value of a is x

$$M_{x \max} = \frac{Wx}{l} (l-x) \quad \dots(1.7)$$

(ii) Load on XB

$$M_x = R_A x = \frac{W(l-a)}{l} \cdot x \quad \dots(1.8)$$

The maximum value of this occurs at $a = x$ and its value is

$$M_{x \max} = \frac{W(l-x)}{l} \cdot x \quad \dots(1.9)$$

Obviously, (1.7) and (1.9) happen to be identical. It represents the bending moment at X when the load is at X ($a = x$)

When we plot (1.7), we get the diagram of maximum bending moment at X, when x varies from 0 to l . This is a parabola, [Fig. 1.1 (d)] and its maximum value occurs at $x = \frac{l}{2}$ and $M_{\max \max} = \frac{Wl}{4}$, this being the absolute maximum bending moment in a S.S. beam due to a moving load W . For $M_{\max \max}$ to occur, the load position is, $a = \frac{l}{2}$ and the section is at $x = \frac{l}{2}$.

1.2.3 S.S. Beams with a Moving udl Longer than the Span

This happens when long trains traverse small span bridges. The train wheels if they are close enough can be treated as transmitting a udl to the bridge girder.

Again we will consider the load front to be at a variable distance ' a ' from the L.H. support A. Let us consider a typical section X, at a distance x from A.

(a) Maximum positive shear at X

When the load head H is left of X, section X will have only negative shear, equal to R_B . For considering positive shear, the tail of the load T, has to be at a variable distance b from A. [Fig. 1.2 (a)]

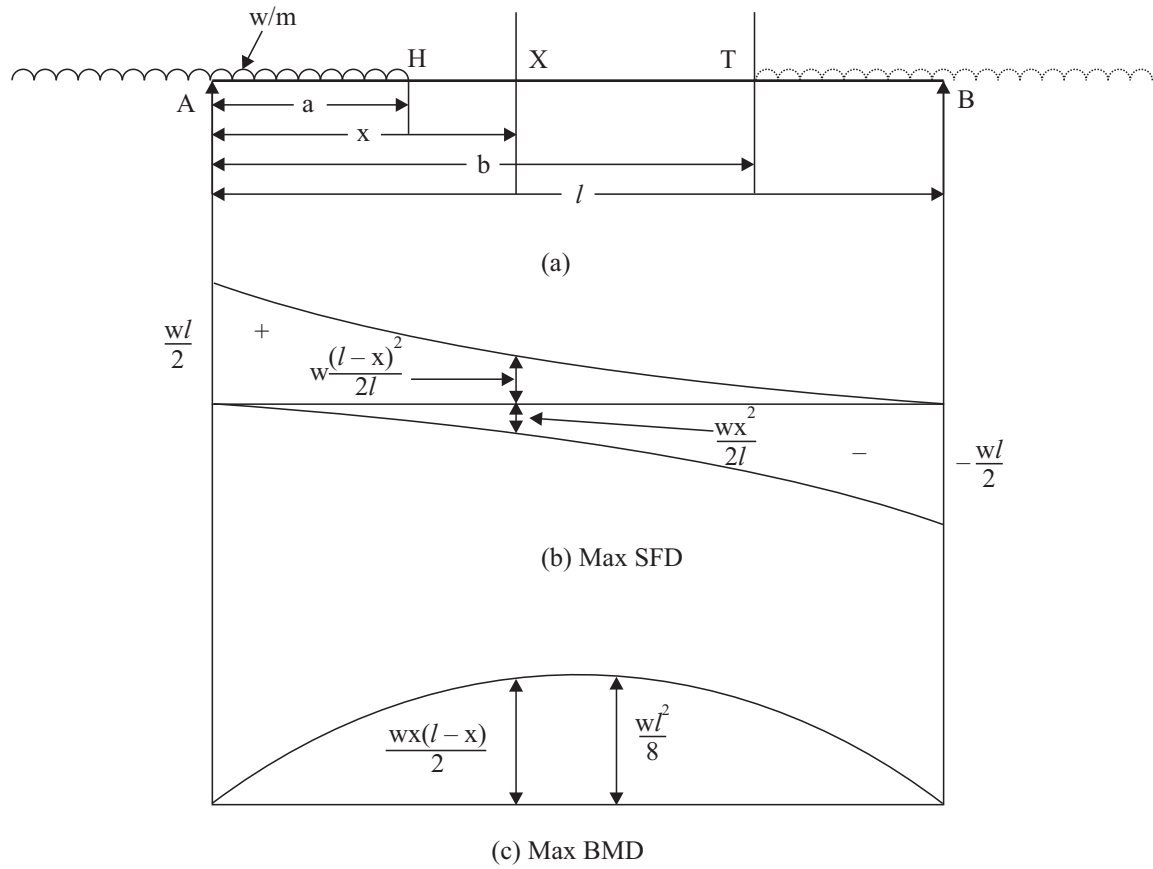


Fig. 1.2

(i) $b \leq x$

when T is on AX, $R_A = w \frac{(l-b)(l-b)}{2l}$.

Then positive shear at X is

$$F_x = R_A - w(x-b) = w(l-b) \frac{(l-b)}{2l} - w(x-b)$$

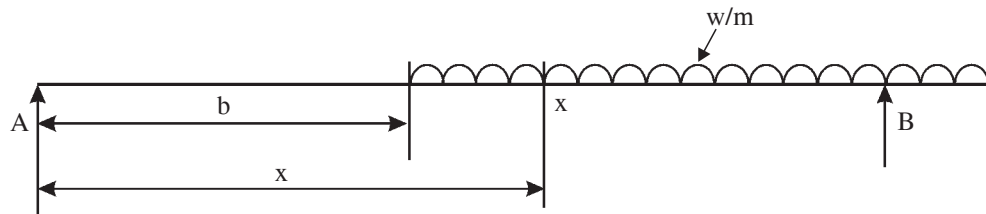


Fig. 1.3

when b is less than x , $(x-b)$ is positive. Hence F_x is a maximum when $b = x$

$$\text{Hence, } F_{x \max} = \frac{w(l-b)^2}{2l} = \frac{w(l-x)^2}{2l} \quad \dots(1.10)$$

We can plot $F_{x \max}$ as x varies from 0 to l . This is a second degree curve and is shown in Fig. 1.2 (b). $F_{x \max}$ is maximum when b is zero ; *i.e.*, when the load covers the entire span and occurs at A. Its value is $\frac{wl}{2}$.

(ii) $b > x$ need not be considered

(b) **Maximum negative shear**

By the reasoning made in section (a) above, the maximum negative shear at X occurs when the load head H is at X. Its value is

$$F_{x \max} = -R_B = -\frac{wa^2}{2l} = -\frac{wx^2}{2l} \quad \dots(1.11)$$

As ' a ' varies from 0 to l , F_x varies parabolically from 0 to $-\frac{wl}{2}$. This is also plotted in Fig. 1.2 (b).

(c) **Maximum bending moment**

(i) Load head H on AX. ($a \leq x$)

$$M_x = R_B \cdot (l-x) = \frac{w \cdot a^2 \cdot (l-x)}{2l}$$

This obviously is maximum when $a = x$ and

$$M_{x \max} = \frac{w x^2 (l-x)}{2l} \quad \dots(1.12)$$

(ii) Load head H on XB ($a \geq x$)

$$\begin{aligned} M_x &= R_B (l-x) - \frac{w(a-x)^2}{2} \\ M_x &= \frac{w \cdot a^2 (l-x)}{2l} - \frac{w(a-x)^2}{2} \end{aligned} \quad \dots(1.13)$$

To maximize M_x , let us equate $\frac{\partial M_x}{\partial a}$ to zero.

$$+ \frac{w a (l-x)}{l} - w(a-x) = 0$$

$$wa - \frac{w \cdot a \cdot x}{l} - wa + wx = 0$$

$$a = l, \text{ when } x \neq 0$$

Hence the load shall cover the entire span.

$$M_{x \max} = \frac{wl}{2} (l-x) - \frac{w(l-x)^2}{2} = \frac{wx(l-x)}{2} \quad \dots(1.14)$$

This will have a maximum value when $x = \frac{l}{2}$,

$$\left(\text{Since } \frac{\partial}{\partial x} M_{x \max} = -\frac{wl}{2} + wx = 0 \quad \text{and} \quad x = \frac{l}{2} \right)$$

And $M_{\max \max} = \frac{wl^2}{8}$, and occurs at mid span when the load covers the entire span. It so happens that the max BM diagram is also the same as the BMD when the entire span is loaded uniformly.

1.2.4 S.S. Beams with a Moving udl Shorter than the Span

This situation occurs in long span railway bridges. This would give rise to more number of load positions than in the long udl case. Let us assume a udl of intensity w and length c . The load head H is at a distance ' a ' from the left end A of the span. Let us consider the shears and bending moments at a typical section X, distance x from A. (Fig. 1.4)

(a) Positive shear force F_x

(i) $a < x$. (load head to the left of X)

Shear force at X = $-R_B$. This will not give any positive shear at X.

(ii) Let us shift the load such that the tail of the load is at a distance b from A (T to the right of X), as in Fig. 1.4 (b).

$$F_{x \max} = R_A = \frac{wc}{l} \left(l - b - \frac{c}{2} \right) \quad \dots(1.15)$$

F_x will have its maximum value when b is a minimum, i.e., $b = x$. Then

$$F_x = \frac{wc}{l} \left(l - x - \frac{c}{2} \right) \quad \dots(1.16)$$

Now let us explore if shifting the load train to the left of x would increase or decrease F_x . When b becomes $x - \Delta b$, equation (1.15) becomes

$$F_{x \max} = \frac{wc}{l} \left(l - x + \Delta b - \frac{c}{2} \right) - w\Delta b, \quad [\text{see Fig. 1.4 (c)}]$$

$$F_x = \frac{wc}{l} \left(l - x - \frac{c}{2} \right) + \boxed{\frac{wc}{l} \Delta b - w\Delta b}$$

In the above equation, the quantity in the box is negative since $\frac{c}{l}$ is less than 1. So we conclude that if we move the load train to the left of X, the shear force does not increase. Hence equation (1.16) represents $F_{x \max}$.

(b) Negative shear force F_x

(i) $a < x$

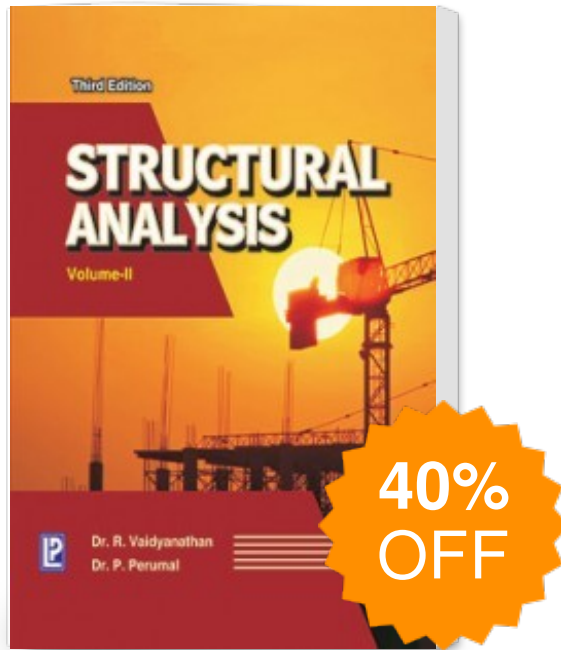
$$F_x = -R_B = -wc \cdot \frac{\left(a - \frac{c}{2} \right)}{l}$$

This will have the maximum value when $a = x$.

$$F_{x \max} = -R_B = -\frac{wc}{l} \left(x - \frac{c}{2} \right) \quad \dots(1.17)$$

Moving the load head H beyond X will not increase this value, by the same reasoning as for the positive shear force.

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