Mechanics and Electrodynamics

For B.Sc. Classes as per
UGC Model Syllabus, Course-1 and Course-2

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AND
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PREFACE TO THE SEVENTH EDITION
[MECHANICS AND ELECTRODYNAMICS]

This is the revised and enlarged edition of the book **"Properties of Matter"** by Brij Lal and N. Subrahmanyam. As per UGC Model Syllabus the name of the book is changed to **"Mechanics and Electrodynamics".**

The present edition of the book is revised as per the U.G.C. Model Syllabus. Topics from Mechanics, Properties of Matter, Mathematical Physics, Methods in Physics, Electrodynamics which constitute Paper I and Paper II for the First year degree course have been covered.

Questions and problems including solved problems have been updated.

We hope this book will also benefit students appearing for IAS, AMIE and other Competitive Examinations.

We are grateful to the students and teachers who have appreciated the book. Suggestions for further improvement of the book will be highly appreciated.

Our grateful thanks to all the staff of S. Chand and Co. Ltd., Shri Ravindra Kumar Gupta, Managing Director, Shri Navin Joshi GM (S&M), Shri Bhagirath Kaushik, Regional Manager for getting the book printed in time and to Mr. D.R. Parab, Branch Manager for the encouragement and co-ordination work.

JIVAN SESHAN

Preface to the First Edition
[PROPERTIES OF MATTER]

This book on "*Properties of Matter*" is intended to suit the needs of B.Sc. (Pass, Honours and Subsidiary) and Engineering students. The book covers the topics included in the syllabi of various Indian Universities.

The subject matter is divided into ten chapters. Each chapter is self contained and is treated in a comprehensive way, using the S.I. system of units. Non-relativistic motion, dynamics of rigid bodies, conservation laws, harmonic oscillators and superposition principle, gravity, gravitation, gravitational field and potential, solar system, Universe, Galilean Transformation and Special Theory of Relativity are some of the topics which have been given special attention. Appendix at the end of the book contains some important topics *viz.* Minkowski space, centre of mass system, moving charges in magnetic and electric fields, space rocket, cathode ray oscillograph and cyclotron.

While preparing the book, it was assumed that the student is familiar with the basic principles of Physics. However, some of the elementary discussions are also included (at some places) to initiate an advanced treatment of the subject. Solved numerical problems, wherever necessary, are given in the text. Exercises in the end of each chapter contain questions which are largely drawn from the university question papers of recent years.

We hope that this book will be found useful by students and teachers. We will appreciate any suggestions for the improvement of the book.

In the end, we sincerely thank Shri Shyam Lal Gupta, Director Eurasia Publishing House, New Delhi for publishing the book.

May 15, 1980

BRIJ LAL
N. SUBRAHMANYAM

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VECTORS AND SCALARS 1

1.1 Displacement

Suppose a body has to go from the position A to the position B. It can travel from A to B by different paths (Fig. 1.1). It can go from A to B along the shortest straight line path 1, or the curved paths 2, 3, 4 etc. Whatever be the path taken by the body, the displacement of the body is always represented by the shortest distance between the two positions A and B. Thus, the displacement from A to B is equal to the distance covered along the path 1. Hence displacement is defined as the shortest distance between the initial and the final positions of the body.

THIS CHAPTER

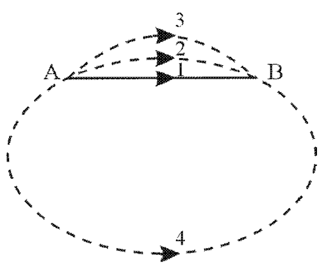

Fig. 1.1

A quantity that has only magnitude and no direction is called a *scalar* quantity. For example, the length of a bridge, area of a surface, volume and mass of a body, density, speed, potential etc., are scalar quantities.

A quantity that has both magnitude and direction is called a *vector* quantity. For example, displacement, velocity, acceleration, force, momentum etc., are vector quantities. A vector quantity is usually represented by a line with an arrow head. The magnitude of the vector is shown by the length of the line and the direction of the arrow represents the direction.

The magnitude of a vector quantity is called the **modulus of the vector**.

In Fig. 1.1, the displacement from A to B along path 1 is represented by a vector $\vec{AB}$. The displacement B to A is represented by a vector $\vec{BA}$. Here

$$\vec{AB} = -\vec{BA}$$

The modulus of the vector AB is represented as $\vec{A}\ AB\,|$

To find the resultant of scalar quantities, they are simply added algebraically. To find the resultant of two or more vector quantities, simple algebraic addition is not applicable. The addition of vectors is done by applying the parallelogram law or the polygon law of vectors.

1.2 Addition and Subtraction of Vectors

(1) Addition

Let a particle P be influenced simultaneously by two vectors $\vec{A}$ and $\vec{B}$ To find the resultant of these two vectors $\vec{A}$ and $\vec{B}$, draw the vector $\vec{A}$ and then draw the vector $\vec{B}$ in order (Fig. 1.2). The resultant is given in magnitude and direction by the vector $\vec{C}$.

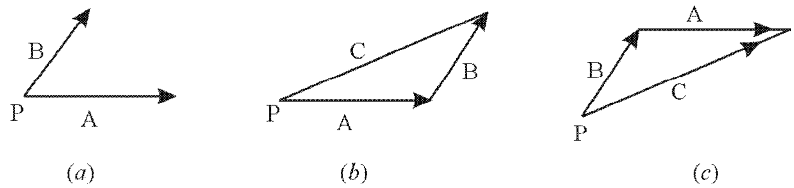

Fig. 1.2

$$\therefore \qquad \vec{A} + \vec{B} = \vec{C} \qquad ...1.1$$

The resultant of B and A is also equal to C.

$$\vec{A} + \vec{B} = \vec{B} + \vec{A} \qquad ...1.2$$

Hence, in the addition of two or more vectors, the order of addition is not important. The same result is obtained irrespective of the order of addition. Equation (*ii*) refers to the "**Commutative Law of Addition of Vectors**."

Consider three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ acting simultaneously on a particle.

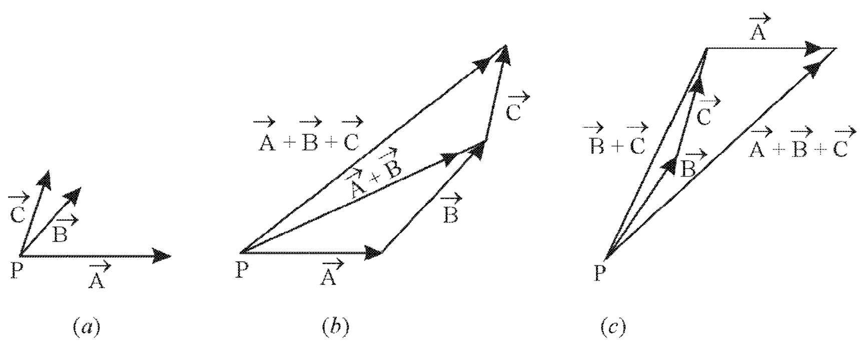

Fig. 1.3

The resultant of these three vectors can be obtained by adding the resultant of A and B to C or by adding the resultant of $\vec{B}$ and $\vec{C}$ to A [Fig. (1.3.)]. In either case, The same result is obtained.

$$\therefore \qquad (\vec{A} + \vec{B}) + \vec{C} = (\vec{B} + \vec{C}) + \vec{A}$$

$$= \vec{A} + (\vec{B} + \vec{C}) \qquad ...1.3$$

Equation (*iii*) represents **Associative Law** for the addition of vectors.

(2) Subtraction

The difference between two vectors A and B is obtained by taking the vector A first and adding to it an equal and opposite vector B. The resultant of $A - B$ is shown in Fig. 1.4

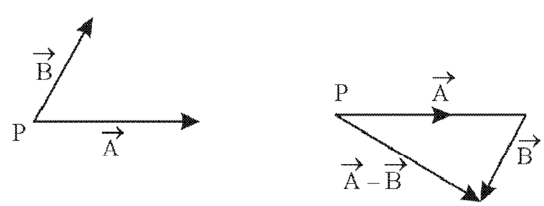

Fig. 1.4

Note. Addition or subtraction of vectors is possible only when the vectors represent the same physical quantity.

1.3 Rectangular Components of a Vector

Let $\vec{R}$ be a vector acting on the particle at O. The two rectangular components of $\vec{R}$ along two perpendicular directions (say X-axis and Y-axis) are given by $\vec{A}$ and $\vec{B}$. as in Fig. 1.5.

$$\vec{R} = \vec{A} + \vec{B}$$

Suppose $\vec{A}$ makes an angle θ with $\vec{R}$. The magnitudes of the component vectors are given by

$$|\vec{A}| = |\vec{R}| \cos\theta \qquad ...1.4$$

$$|\vec{B}| = |\vec{R}| \sin\theta \qquad ...1.5$$

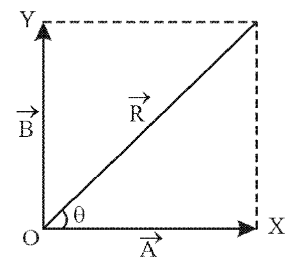

Fig. 1.5

1.4 Multiplication of a Vector by a Scalar

When a vector $\vec{A}$ is multiplied by a scalar n, the resultant vector $\vec{R}$ is equal to nA. The direction of $\vec{R}$ is the same as that of $\vec{A}$.

$$\vec{R} = n\vec{A}$$

1.5 Scalar Product (Dot Product)

If A and B are two vectors making an angle θ with each other, then their scalar product,

$$\vec{R} = \vec{A} . \vec{B} = |\vec{A}||\vec{B}| \cos\theta \qquad ...1.6$$

Here, $\vec{A} . \vec{B}$ is also called the ***dot product*** and it is a scalar quantity.

If θ is zero, $\cos\theta = 1$,

$$\vec{A} . \vec{B} = |\vec{A}||\vec{B}| \qquad ...1.7$$

If θ is 90°, $\cos 90 = 0$

$$\vec{A} . \vec{B} = 0 \qquad ...1.8$$

Also

$$\vec{A} . \vec{B} = \vec{B} . \vec{A} \qquad ...1.9$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| (|\vec{B}| \cos\theta) = AB\cos\theta$$

$$\vec{A} \cdot \vec{B} = (|A| \cos\theta) |B| = AB\cos\theta$$

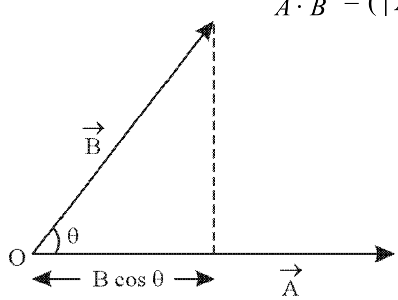

Fig. 1.6 (*a*)

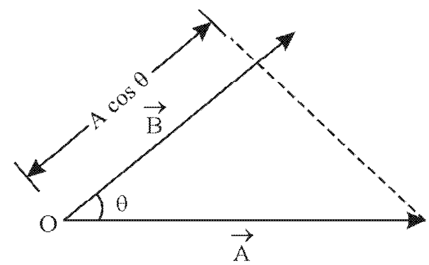

Fig. 1.6 (*b*)

Example. *Suppose a force F acts on the particle P and moves it through a displacement S as in Fig. 1.7. The work done,*

$$W = \vec{F} . \vec{S} \qquad ...1.10$$

$$W = |\vec{F}||\vec{S}| \cos\theta \qquad ...1.11$$

or

$$W = FS\cos\theta \qquad ...1.12$$

Here W is a scalar quantity.

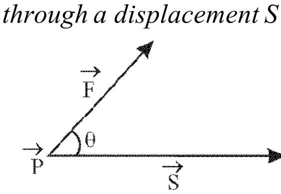

Fig. 1.7

1.6 Law of Cosines

Using the dot product

$$\because \quad \vec{R} = \vec{A} \cdot \vec{B}$$

$$R^2 = \vec{R}\cdot\vec{R} = (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B})$$

$$= \vec{A}\cdot\vec{A} + \vec{B}\cdot\vec{B} + 2\,\vec{A}\cdot\vec{B}$$

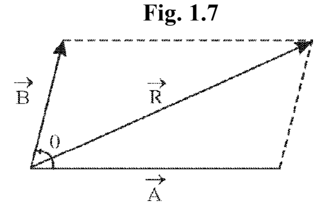

Fig. 1.8

$$= A^2 + B^2 + 2\,AB\cos(\vec{A}, \vec{B})$$
$$= A^2 + B^2 + 2\,AB\cos\theta \qquad \text{...1.14}$$

1.7 Vector Product (Cross Product)

If $\vec{A}$ and $\vec{B}$ are two vectors making an angle θ with each other then their vector product is given by

$$\vec{R} = \vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}|\sin\theta \qquad \text{...1.13}$$

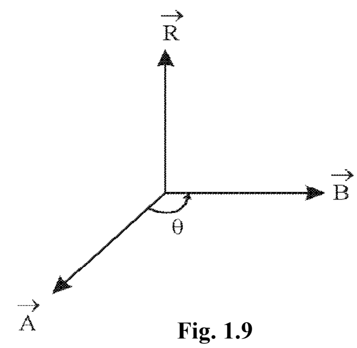

Fig. 1.9

Here $\vec{A} \times \vec{B}$ is also called the ***cross product*** and it is a vector quantity. The direction of $\vec{R}$ is perpendicular to the plane containing the vectors $\vec{A}$ and $\vec{B}$.

It is to be noted that if the order of vectors is reversed the sign of the vector product changes.

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$
$$\vec{A} \cdot \vec{B} = (|A|\cos\theta)\,|B| = AB\cos\theta$$

Vector product obeys the distributive law :

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

1.8 Area of a Parallelogram

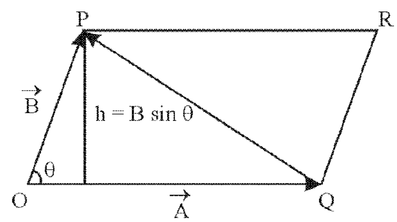

Fig. 1.10

Area of parallelogram = Base × Height

$$= |\vec{A}| \times |B| \sin\theta = AB\sin\theta$$
$$= \vec{A} \times \vec{B} \qquad \text{...1.15}$$

1.9 Volume of a Parallelopiped Scalar Triple Product

The quantity $(\vec{A} \times \vec{B}) \cdot \vec{C}$ is a scalar. If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are the sides of a parallelopiped then the scalar product represents the volume of the parallelopiped.

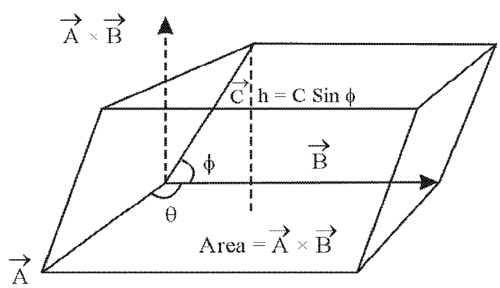

Fig. 1.11

$$\text{Volume} = (\text{base area}) \times \text{height}$$
$$= |\vec{A} \times \vec{B}|\,h$$
$$= |\vec{A} \times \vec{B}|\,C\sin\phi$$

or $$\text{Volume} = (\vec{A} \times \vec{B}) \cdot \vec{C} \qquad \text{...1.16}$$

Note. Scalar product is commutative *i.e.*

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B}) \qquad \text{...1.17}$$

1.10 Vector Triple Product

Three vectors can be multiplied so that the resulting product is a vector

$$\vec{A}\,(\vec{B} \cdot \vec{C}) = \vec{B}\,(\vec{C} \cdot \vec{A}) = \vec{C}\,(\vec{A} \cdot \vec{B})$$

$\vec{B} \cdot \vec{C}$ is a scalar. Hence $\vec{A}\,(\vec{B} \cdot \vec{C})$ is a vector multiplied by a scalar quantity $(\vec{B} \cdot \vec{C})$. The direction of the new vector is in the direction of $\vec{A}$

Also, $\vec{A} \times (\vec{B} \times \vec{C})$

$(\vec{B} \times \vec{C})$ is a vector $\perp r$ to the plane of $\vec{B}$ and $\vec{C}$.

$\therefore$ $\vec{A} \times (\vec{B} \times \vec{C})$ is a vector $\perp r$ to both $\vec{A}$ and $\vec{B} \times \vec{C}$.

$\therefore$ $\vec{A} \times (\vec{B} \times \vec{C})$ is a vector in the plane of $\vec{B}$ and $\vec{C}$.

It can be shown that $\vec{A} \times \vec{B} \times \vec{C} = \vec{B}\,(\vec{C} \cdot \vec{A}) - \vec{C}\,(\vec{A} \cdot \vec{B})$ ...1.18

1.11 Unit Vector

A unit vector is a vector of unit magnitude and has a direction. For the usual three-dimensional Castesian Co-ordinate system the unit vectors are denoted by $\hat{i}$, $\hat{j}$, $\hat{k}$ where $\hat{i}$ is along the x-axis, $\hat{j}$ along the y-axis and $\hat{k}$ along the z-axis as in Fig. 1.12.

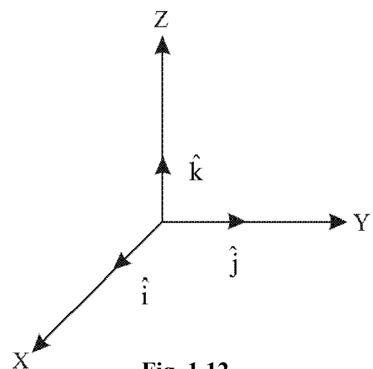

Fig. 1.12

In component form $\hat{i} = [1, 0, 0]$

$\hat{j} = [0, 1, 0]$

and $\hat{k} = [0, 0, 1]$

Some other notations used for unit vectors are $\hat{x}, \hat{y}, \hat{z}$ or $\hat{x}_i$, $\hat{e}_i$, or $\hat{u}_i$;

where $i = 1, 2, 3$

Since the three axis are mutually $\perp r$, the following relationship holds among the unit vectors :–

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1 \quad ...1.19$$

and

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0 \quad ...1.20$$

Also, from the definition of the vector product,

$$\hat{i} \times \hat{j} = \hat{k} \qquad \hat{j} \times \hat{k} = \hat{i} \qquad \text{and } \hat{k} \times \hat{i} = \hat{j}$$

Positive when clockwise *i-j-k-i* (alphabetical) and negative when anti-clockwise *i-k-j-i*.... (not alphabetical)

$$\hat{j} \times \hat{i} = -\hat{k}; \quad \hat{i} \times \hat{k} = -\hat{j}; \quad \hat{k} \times \hat{j} = -\hat{i}$$

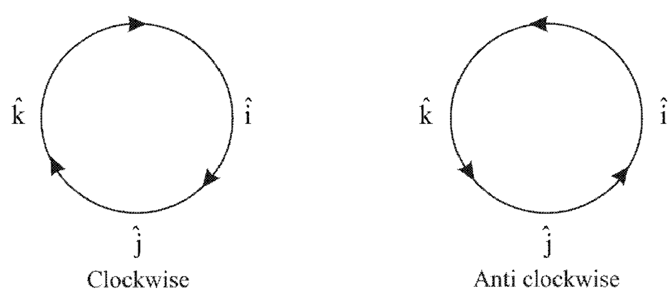

Fig. 1.13

Note that $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

1.12 Components of a Vector

In terms of unit vectors, $\vec{A}$ can be written as the vector sum of unit vectors, each multiplied by the corresponding component.

$\therefore$ $\vec{A} = Ax\hat{i} + Ay\hat{j}$ in 2 Dimension.

and $\vec{A} = Ax\hat{i} + Ay\hat{j} + Az\hat{k}$ in 3 dimension

$Ax = \vec{A} \cdot \vec{i}$

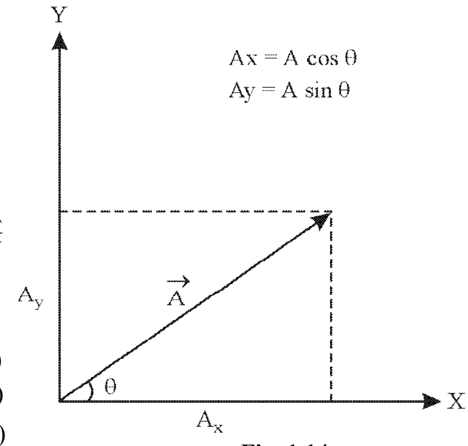

Fig. 1.14

Note. Using unit vector notation, we have

$$\vec{A} = Ax\hat{i} + Ay\hat{j} + Az\hat{k}$$

$$\vec{B} = Bx\hat{i} + By\hat{j} + Bz\hat{k}$$

$$\therefore \quad \vec{A} + \vec{B} = (Ax + Bx)\,\hat{i} + (Ay + By)\,\hat{j} + (Az + Bz)\,\hat{k}$$

Similarly for $\vec{A} - \vec{B}$

For scalar product of two vectors, we have

$$\vec{A} \cdot \vec{B} = (Ax\hat{i} + Ay\hat{j} + Az\hat{k}) \cdot (Bx\hat{i} + By\hat{j} + Bz\hat{k})$$

$$= AxBx\,(\hat{i}\cdot\hat{i}) + AyBy\,(\hat{j}\cdot\hat{j}) + AzBz\,(\hat{k}\cdot\hat{k}) + AxBy\,(\hat{i}\times\hat{j})$$
$$+ AxBy\,(\hat{i}\times\hat{k}) + AyBx\,(\hat{j}\times\hat{i}) + AyBz\,(\hat{j}\cdot\hat{k})$$
$$+ AzBx\,(\hat{k}\cdot\hat{i}) + AzBy\,(\hat{k}\cdot\hat{j})$$

$\because \quad \hat{i}\cdot\hat{i} = 1$ and $\hat{i}\cdot j = \ldots\ldots = 0$

$$\therefore \quad \vec{A}\cdot\vec{B} = Ax\,bx + Ay\,By + Az\,Bz \qquad \ldots 1.21$$

The vector product can be written as

$$\vec{A}\times\vec{B} = (Ax\hat{i} + Ay\hat{j} + Az\hat{k}) \times (Bx\hat{i} + By\hat{j} + Bz\hat{k})$$
$$= AxBx\,(\hat{i}\times\hat{i}) + AyBy\,(\hat{j}\times\hat{j}) + AzBz(\hat{k}\times\hat{k})$$
$$+ AxBy\,(\hat{i}\times\hat{j}) + AxBz\,(\hat{i}\times\hat{k}) + AyBx\,(\hat{j}\times\hat{i})$$
$$+ AyBz\,(\hat{j}\times\hat{k}) + AzBx\,(\hat{k}\times\hat{i}) + AzBy\,(\hat{k}\times\hat{j})$$

$\because \quad \hat{i}\times\hat{i} = \hat{j}\times\hat{j} = 0$ and $i\times j = k \quad j\times i = -\hat{k}$ etc.

$$\therefore \quad \vec{A}\times\vec{B} = AxBy\hat{k} + AxBz\,(-\hat{j}) + AxBx\,(-\hat{k})$$
$$+ Ay\,Bz\,(\hat{i}) + Az\,Bx\,(\hat{j}) + Az\,By\,(-\hat{i})$$
$$= \hat{i}\,(Ay\,Bz - Az\,By) + \hat{j}\,(Az\,Bx - Ax\,Bz) + \hat{k}\,(Ax\,By - Ay\,Bx)$$

In determinant form $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ Ax & Ay & Az \\ Bx & By & Bz \end{vmatrix}$

In component form

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = Ax\,(By\,Cz - Bz\,Cy) + Ay\,(Bz\,Cy - Bx\,Cz) + Az\,(Bx\,Cy - By\,Cx) \qquad \ldots 1.22$$

or $\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} Ax & Ay & Az \\ Bx & By & Bz \\ Cx & Cy & Cz \end{vmatrix}$

1.13 Vector Triple Product

$$\vec{A}\times(\vec{B}\times\vec{C}) = B\,(\bar{A}\cdot\bar{C}) - \bar{C}\,(\bar{A}\cdot\vec{B})$$

Now $\vec{A}\times(\vec{B}\times\vec{C})$ is a vector which is Co-planar with $\vec{B}$ and $\vec{C}$

Let $\quad \vec{A}\times(\vec{B}\times\vec{C}) = \lambda\vec{B} + \mu\vec{C} \qquad \ldots 1.23$

where λ and μ are scalars

Now $\vec{A}\times(\vec{B}\times\vec{C})$ is $\perp r$ to $\vec{A}$

Implies $\quad \therefore \quad \vec{A}\cdot[\vec{A}\times(\vec{B}\times\vec{C})] = 0$

Hence $\lambda\vec{A}\cdot\vec{B} + \mu\,\vec{A}\cdot\vec{C} = 0$ (Multiplying R.H.S of Eq. 1.23)

Let $\dfrac{\lambda}{\vec{A}\cdot\vec{C}} = \dfrac{-\mu}{\vec{A}\cdot\vec{B}} = x$ (say)

$\therefore$ $\lambda = x\cdot(\vec{A}\cdot\vec{C})$

$\mu = -x\cdot(\vec{A}\cdot\vec{B})$

$\therefore$ Eq. 1.23 reduces to

$$\vec{A}\times(\vec{B}\times\vec{C}) = x\cdot(\vec{A}\cdot\vec{C})\,\vec{B} - x\,(\vec{A}\cdot\vec{B})\,\overline{C}$$

To find x, let $\vec{A}=\vec{B}=i$ and $\vec{C}=j$

$\therefore$ $\vec{A}\times(\vec{B}\times\vec{C}) = x\,(\hat{i}\cdot\hat{j})\,\hat{i} - x\,(\hat{i}\cdot\hat{i})\,\hat{j}$

Subt $\hat{i}\times(\hat{i}\times\hat{j}) = 0 - x\,j$

$\hat{i}\times\hat{k} = -\hat{j} = -x\,\hat{j}$ $\quad\therefore x = 1$

$(\because \hat{i}\times\hat{j} = \hat{k} \quad \hat{i}\cdot\hat{j} = 0 \quad \hat{i}\cdot\hat{i} = 1)$

Hence rule, $B_{ac} - C_{ab}$ rule

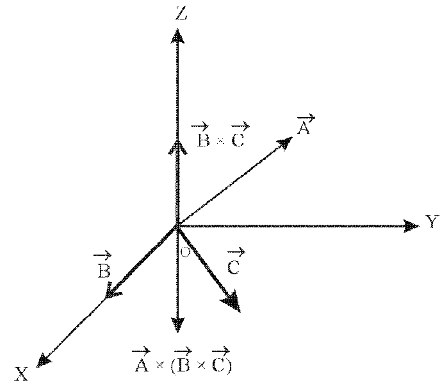

Fig. 1.15

1.14 Reciprocal Vectors

Consider a system of non-Coplanar vectors $\vec{a}, \vec{b}, \vec{c}$ and another system also of non-coplan *ar* vectors $\vec{a}', \vec{b}', \vec{c}'$.

They are reciprocal vectors, if

$$\vec{a}' = \frac{\vec{b}\times\vec{c}}{\vec{a}\cdot(\vec{b}\times\vec{c})},\ \vec{b}' = \frac{\vec{c}\times\vec{a}}{\vec{a}\cdot(\vec{b}\times\vec{c})} \text{ and } \vec{c}' = \frac{\vec{a}\times\vec{b}}{\vec{a}\cdot(\vec{b}\times\vec{c})} \qquad \text{...1.24}$$

This is so because $\vec{a}\cdot\vec{a}' = \vec{b}\cdot\vec{b}' = \vec{c}\cdot\vec{c}' = 1$

i.e. $\vec{a}\cdot\vec{a}' = \dfrac{\vec{a}\cdot(\vec{b}\times\vec{c})}{\vec{a}\cdot(\vec{b}\times\vec{c})}$ similarly for $\vec{b}'$ and $\vec{c}'$.

From Eq. 1.24 it is clear that $\vec{a}'$ is $\perp r$ to the plane of $\vec{b}$ and $\vec{c}$.

Hence $\vec{a}'\cdot\vec{b} = \vec{a}'\cdot\vec{c} = 0$

or $\vec{b}'\cdot\vec{a} = \vec{b}'\cdot\vec{c} = 0$ also

$\vec{c}'\cdot\vec{a} = \vec{c}'\cdot\vec{b} = 0$

Since the two systems are reciprocal of each other the inverse relation follows :

$$\vec{a} = \frac{\vec{b}'\times\vec{c}'}{\vec{a}'\cdot(\vec{b}'\times\vec{c}')},\ \vec{b} = \frac{\vec{c}'\times\vec{a}'}{\vec{a}'\cdot(\vec{b}'\times\vec{c}')};\ \vec{c} = \frac{\vec{a}'\times\vec{b}'}{\vec{a}'\cdot(\vec{b}'\times\vec{c}')}$$

Example. If $\vec{a}_1, \vec{a}_2, \vec{a}_3$ and $\vec{b}_1, \vec{b}_2, \vec{b}_3$ are reciprocal system of vectors, prove that

$$\vec{a}_1\times\vec{b}_1 + \vec{a}_2\times\vec{b}_2 + \vec{a}_3\times\vec{b}_3 = 0.$$

Solution. By def. of reciprocal vectors we have

$$\vec{b}_1 = \frac{\vec{a}_2\times a_3}{\vec{a}_1\cdot(\vec{a}_2\times\vec{a}_3)};\ \vec{b}_2 = \frac{\vec{a}_3\times\vec{a}_1}{\vec{a}_1\cdot(\vec{a}_2\times\vec{a}_3)};\ \vec{b}_3 = \frac{\vec{a}_1\times\vec{a}_2}{\vec{a}_1\cdot(\vec{a}_2\times\vec{a}_3)}$$

$$\therefore \quad \vec{a}_1\times\vec{b}_1 = \frac{\vec{a}_1\times(\vec{a}_2\times\vec{a}_3)}{\vec{a}_1\cdot(\vec{a}_2\times\vec{a}_3)}$$

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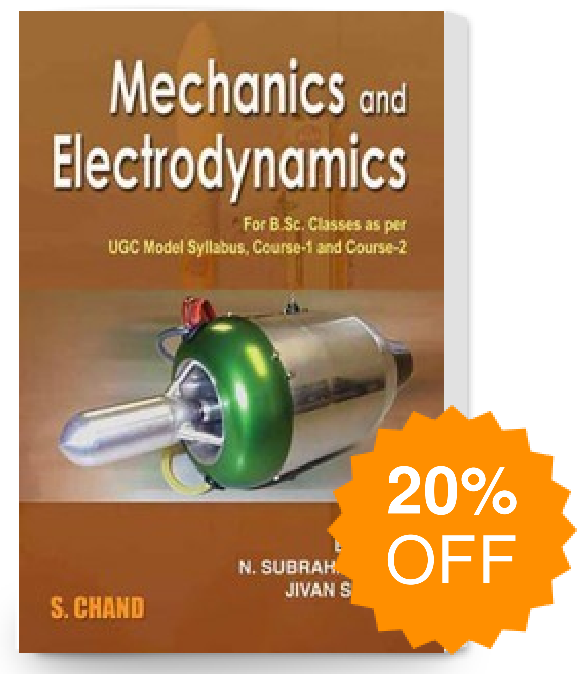

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