BASIC ENGINEERING MATHEMATICS

Volume-1

AS PER NEW SYLLABUS OF
RAJIV GANDHI PROUDYOGIKI VISHWAVIDYALAYA
(RGPV)

Subject Code: MA110

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S. CHAND

BASIC
ENGINEERING MATHEMATICS

VOLUME – I

[For B.E. First Year Semester I (all branches). Strictly according to the syllabi of Rajiv Gandhi Proudyogiki Vishwavidyalaya, Bhopal (M.P.) and all Engineering Colleges affiliated to Ravi Shankar University, Raipur (Chhatisgarh)]

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PREFACE TO THE EIGHTH EDITION

The book is primarily written to the unifid syllabus 2015 ownward of Mathematics of first semester of all Engineering colleges affiliated to Rajiv Gandhi Prouyo giki vishwavidyalaya, Bhopal.

This book also course of B. Tech / B.E. / B.Tech First year course of Indian Engineering Colllage.

Keeping in view of the new examination scheme, more than 500 **Objective Questions** are included at the end of each exercise in this revised edition.

The subject matter is presented in a very systematic and logical manner. Every endeavour has been made to make the contents as simple and lucid as possible. Emphasis has been laid on making the concepts clear. A lot of pains and concentration on the part of the author have gone in solving the examples in the best possible way. In providing the solution of the problems, care has been taken not to miss even minor step so that the students can follow the subject even without the guidance of the teacher.

The book contains fairly large number of solved examples from question papers of examinations recently conducted by different universities and engineering colleges so that the students may not find any difficulty while answering these problems in their final examination. Latest question papers of First Semester of Rajeev Gandhi Technical University, Bhopal for the years June 2013, June 2014, Dec. 2014, June 2015, have been fully solved in this book.

The author possesses very long and rich experience of teaching Mathematics to the engineering students of degree classes, and has first hand experience of the problems and difficulties that they generally face.

I take this opportunity to express my deep sense of gratitude to the management and the editorial team of M/s. S. Chand & Co. Pvt. Ltd. for publishing this book in a short time.

I will feel amply rewarded, if suggestions and constructive feedback is provided by the students and teachers, which will enable me to make the book error-free and more user-friendly.

For any clarification, I may be contacted on the following address:

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RAJIV GANDHI PROUDYOGIKI VISHWAVIDYALAYA BHOPAL

Mathematics-I

Course Objective

The objective of this foundational course is to review mathematical concepts already learnt in higher secondary. This course will also introduce fundamentals of mathematical functions, derivatives and aspects of calculus to students.

Course content

UNIT - I: Recapitulation of Mathematics

Basics of differentiation, Rolle's and Lagranges theorem, Tangents and Normals, Indefinite integral (Substitution, Integration using Trigonometric Identity & Integration by Parts & Definite integral).

UNIT - II: Ordinary Derivatives & Applications

Expansion of functions by Maclaurin's & Taylor's Theorem (One Variable), Maxima and Minima of functions of two variables. Curvature (Radius, Center & Circle of Curvature for Cartesian Coordinates), Curve Tracing.

UNIT - III: Partial Derivatives & Applications

Definition, Euler's Theorem for Homogeneous Functions, Differentiation of implicit functions, Total Differential Coefficient, Transformations of independent Variables, Jacobians, Approximation of Errors.

UNIT - IV: Integral Calculus

Definite integrals as a Limit of Sum, Application in Summation of series, Beta and Gamma functions (Definitions, Relation between Beta and Gamma functions, Duplication formula, Applications of Beta & Gama functions).

UNIT - V: Applications of Integral Calculus

Multiple integral (Double & Triple Integrals), Change of Variables, Change the Order of Integration, Applications of Multiple Integral in Area, Volume, Surface & Volume of Solid of Revolution about X-Axis & Y-Axis.

COURSE OUTCOMES

The curriculum of the Department is designed to satisfy the diverse needs of students. Coursework is designed to provide students the apportunity to learn key concepts of mathematical functions, partial derivatives as well as fundamentals and applications of integral calculus.

Evaluation

Evaluation will be continuous an integral part of the class as well through external assessment.

CONTENTS

UNIT – III : PARTIAL DERIVATIVES AND APPLICATIONS

UNIT – IV : INTEGRAL CALCULUS

UNIT – V : APPLICATION OF INTEGRAL CALCULUS

USEFUL FORMULAE

Chapter 1 Basics of differentiation

Rate of change of y with respect to $x = \frac{dy}{dx}$.

$$\boxed{\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}} \quad \lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to \infty} (1+x)^{\frac{1}{x}} = e \quad \lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$$

Continuity of a Function

The function $f(x)$ is said to be continuous at $x = a$ if

Left-hand $\lim_{x \to a^-} f(x)$ = Right-hand $\lim_{x \to a^+} f(x) = f(a)$

Differntiability of a Function

The function $f(x)$ is said to be differentiable at $x = a$ if

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$$

Chapter 2 Roll's and Lagranges theorem

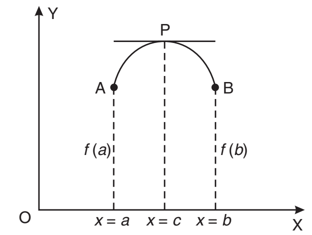
Rolle's theorem

Let f be real valued function defined on the closed interval $[a, b]$ such that:

(*i*) It is continuous on closed interval $[a, b]$

(*ii*) It is differentiable on open interval (a, b)

(*iii*) $f(a) = f(b)$



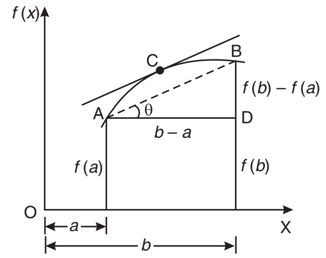
Lagrange's Mean-value theorem

Statement. If a function $f(x)$ is

(*i*) Continuous in the closed interval $[a, b]$,

(*ii*) Derivable in the open interval (a, b), then, there exists at least one point $c \in (a + b)$ such that

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$



Chapter 3 Tangents and normals

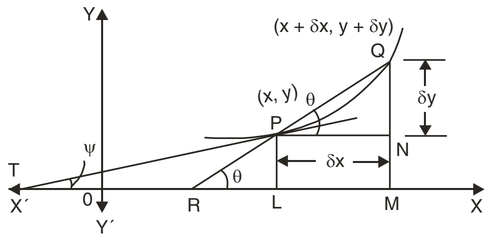
$$\boxed{\frac{dy}{dx} = \text{slope of the tangent at } P.}$$

(*b*) The equation of the tangent at (x_1, y_1) is

$$\boxed{y - y_1 = \frac{dy}{dx}(x - x_1),}$$

Equation of the normal at (x_1, y_1) is

$$\boxed{y - y_1 = -\frac{dx}{dy}(x - x_1).}$$



CHAPTER 4 INDEFINITE INTEGRAL

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
2. $\int dx = x + C$
3. $\int \frac{1}{x} dx = \log|x| + C$
4. $\int e^x dx = e^x + C$
5. $\int a^x dx = \frac{a^x}{\log_e a} + C$
6. $\int \sin x \, dx = -\cos x + C$
7. $\int \cos x \, dx = \sin x + C$
8. $\int \sec^2 x dx = \tan x + C$
9. $\int \text{cosec}^2 x \, dx = -\cot x + C$
10. $\int \sec x \tan x \, dx = \sec x + C$
11. $\int \text{cosec}\, x \cot x \, dx = -\text{cosec}\, x + C$
12. $\int \cot x \, dx = \log|\sin x| + C$
13. $\int \tan x \, dx = -\log|\cos x| + C$
14. $\int \sec x \, dx = \log|\sec x + \tan x| + C$
15. $\int \text{cosec}\, x dx = \log|\text{cosec}\, x - \cot x| + C$
16. $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
17. $\int -\frac{1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + C$
18. $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$
19. $\int -\frac{1}{a^2 + x^2} dx = \frac{1}{a} \cot^{-1}\left(\frac{x}{a}\right) + C$
20. $\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C$
21. $\int -\frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \text{cosec}^{-1}\left(\frac{x}{a}\right) + C$

CHAPTER 5 INTEGRATION BY SUBSTITUTION

1. $\int f(ax+b)\, dx = \frac{1}{a} \phi(ax+b) + C$
2. $\boxed{\int \frac{f'(x)dx}{f(x)} dx = \log f(x) + c}$
3. $\boxed{\int \{f(x)\}^n f'(x)\, dx = \frac{lf(x)n+1}{n+1} + c}$

INtegration Using Trigonometric Identity

Trigonometric identities

1. $\sin^2 \theta + \cos^2 \theta = 1$
2. $1 + \tan^2 \theta = \sec^2 \theta$
3. $1 + \cot^2 \theta = \text{cosec}^2 \theta$
4. $\sin 2\theta = 2 \sin \theta \cos \theta$
5. $\cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$
6. $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$
7. $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$

CHAPTER 6 INTEGRATION BY PARTS

$$\boxed{\int f(x) \cdot \phi(x)\, dx = f(x) \int \phi(x)\, dx - \int [f'(x) \cdot \int \phi(x)\, dx]\, dx}$$

In words

Integral of the product of two functions

= first function × integral of the second – Integral of [diff. coeff. of first function × Integral of the second]

CHAPTER 7 DEFINITE INTEGRAL

1. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

2. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

3. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx,\ c(\in(a,b))$

4. $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

5. $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

6. $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

7. $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx,$ if $f(2a-x) = -f(x)$

$= 0,$ if $f(2a-x) = -f(x)$

8. $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx,$ if f is an even function i.e., $f(-x) = f(x)$.

$\int_{-a}^a f(x)\,dx = 0,$ if f is an odd function i.e., $f(-x) = -f(x)$.

CHAPTER 8 ORDINARY DERIVATIVES

1. $\frac{1}{dx}(x^n) = nx^{n-1}$

2. $\frac{d}{dx}(\log x) = \frac{1}{x}$

3. $\frac{d}{dx}(\sin x) = \cos x$

4. $\frac{d}{dx} = (\cos x) = -\sin x$

5. $\frac{d}{dx}(\tan x) = \sec^2 x$

6. $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$

7. $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$

8. $\frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x \cdot \cot x$

9. $\frac{d}{dx}(a^x) = a^x \log_e a$

10. $\frac{d}{dx}(e^x) = e^x$

11. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

12. $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

13. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

14. $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

15. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

16. $\frac{d}{dx}(\mathrm{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

17. $\frac{d}{dx}(uv) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

18. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$

19. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

20. $\frac{d}{dx}(\sinh x) = \cosh x$

21. $\frac{d}{dx}(\cosh x) = \sinh x$

22. $\frac{d}{dx}(\tanh x) = \mathrm{sec\,h}^2\, x$

23. $\frac{d}{dx}(\coth x) = -\mathrm{cosech}^2 x$

24. $\frac{d}{dx}(\mathrm{sech}\, x) = -\mathrm{sech}\, x \cdot \tanh x$

25. $\frac{d}{dx}(\text{cosech } x) = -\text{cosech } x \cdot \coth x$ **26.** $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$

27. $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$ **28.** $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

29. $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}, \; x > 1 \text{ or } x < -1$ **30.** $\frac{d}{dx}(\text{sech}^{-1} x) = \frac{1}{x\sqrt{x^2+1}}, \; 0 < x < 1$

31. $\frac{d}{dx}(\text{cosech}^{-1} x) = \frac{1}{x\sqrt{x^2+1}}$

$\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$, $\cosh^2 x - \sinh^2 x = 1$, $\text{sech}^2 x = 1 - \tanh^2 x$

$\text{cosech}^2 x = -1 + \coth^2 x$, $\sinh^{-1} x = \log[x + \sqrt{x^2+1}]$, $\cosh^{-1} x = \log[x + \sqrt{x^2-1}]$

CHAPTER 9 EXPANSION OF FUNCTIONS

Maclaurin's Series: $f(x) = f(0) + \frac{x}{11} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \ldots + \frac{x^n}{n!} f^n(0) + \ldots$

Taylor's Theorem: If $f(a + h)$ can be expanded in a series of ascending powers of h, then the expansion is

$$f(a + h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \frac{h^3}{3!} f'''(a) + \ldots$$

If $a + h = x$ or $h = x - a$,

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \ldots$$

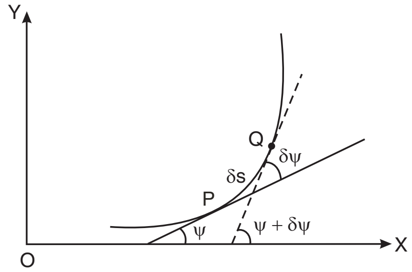
CHAPTER 10 CURVATURE

$$\text{Curvature at } P = \lim_{\delta x \to 0} \frac{\delta\psi}{\delta s} = \frac{d\psi}{ds}$$

Reciprocal of curvature is called the radius of curvature.

Thus, Radius of curvature $(\rho) = \frac{ds}{d\psi}$.

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$



Centre of curvature, $\bar{x} = x - \frac{y'}{y''}(1 + y'^2)$ $\bar{y} = y + \frac{1}{y''}(1 + y'^2)$

Evoluate. The locus of the centre of curvature of a curve is called its evolute and a curve is said to be an involute of evolute.

CHAPTER 11 CURVE TRACING

Method of Tracing a Curve

Symmetry: (*i*)A curve is symmetric about x-axis if the equation remains the same by replacing y by $-y$. Here y should have even power only.

2. **Curve through origin.** The curve passes through the origin, if the equation does not contain constant term.
3. **The points of intersection with the axes.** By putting $y = 0$ in the equation of the curve we get the co-ordinates of the point of intersection with the x-axis.
4. **Regions in which the curve does not lie.**
   If the value of y is imaginary for certain value of x then the curve does not exist for such values.
5. **Asymptotes are the tangents to the curve at infinity.**
   (*a*) **Asymptote parallel to the *x*-axis** is obtained by equating to zero, the coefficient of the highest power of x.
   (*b*) **Asymptote parallel to the *y*-axis** is obtained by equating to zero, the coefficient of highest power of y.

Remember **:** POSTAR. Where,

$P \equiv$ Point of Intersection, $O \equiv$ Origin, $S \equiv$ Symmetry, $T \equiv$ Tangent, $A \equiv$ Asymptote, $R =$ Region.

CHAPTER 12 PARTIAL DERIVATIVES

Let $z = f(x, y)$ be function of two independent variables x and y. If we keep y constant and x varies then z becomes a function of x only. The derivative of z with respect to x, keeping y as constant is called partial derivative of 'z', w.r.t. 'x' and is denoted by symbols.

$$\frac{\partial z}{\partial x}, \frac{\partial f}{\partial x}, \quad f_x(x, y) \text{ etc.} \qquad \text{Then } \frac{\partial z}{\partial x} = \lim_{\delta x \to 0} \frac{f(x + \delta x, y) - f(x, y)}{\delta x}$$

The process of finding the partial differential coefficient of z w.r.t. 'x' is that of ordinary differentiation, but with the only difference that we treat y as constant.

Similarly, the partial derivative of 'z' w.r.t. 'y' keeping x as constant is denoted by

$$\frac{\partial z}{\partial y}, \frac{\partial f}{\partial x}, \quad f(x, y) \qquad \frac{\partial z}{\partial y} = \lim_{\delta y \to 0} \frac{f(x, y + \delta y) - f(x, y)}{\delta x}$$

Homogeneous Function

A function $f(x, y)$ is said to be homogeneous function in which the power of each term is the same.

Euler's Theorem on Homogeneous Function

Statement. If z is a homogeneous function of x, y of order n, then $x.\frac{\partial z}{\partial x} + y.\frac{\partial z}{\partial y} = n z$

CHAPTER 13 TOTAL DIFFERENTIATION

Let $z = f(x, y)$ ...(1)

If δx, δy be the increments in x and y respectively, let δz be the corresponding increment in z.

Then $z + \delta z = f(x + \delta x, y + \delta y)$ ...(2)

Subtracting (1) from (2), we have

$$\delta z = f(x + \delta x, y + \delta y) - f(x, y) \qquad ...(3)$$

Adding and subtracting $f(x, y + \delta y)$ on R.H.S. of (3), we have

$$\delta z = f(x + \delta x, y + \delta y) - f(x, y + \delta y) + f(x, y + \delta y) - f(x, y)$$

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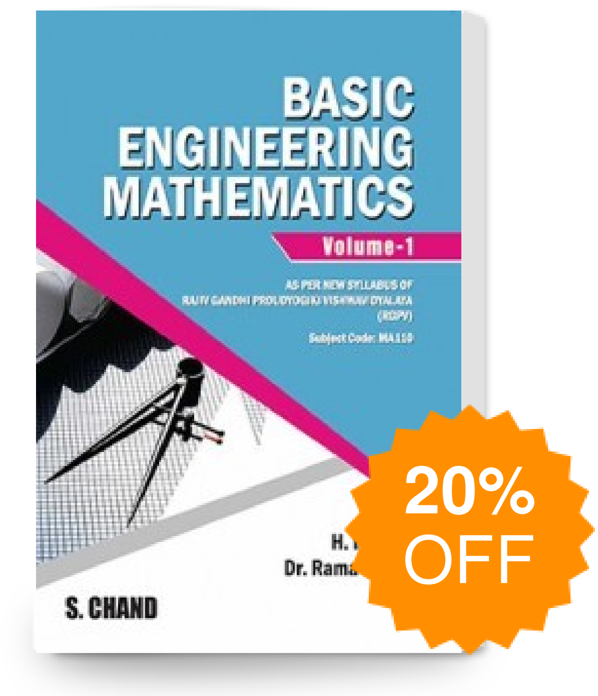

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