2nd Revised Edition

Fundamentals of STRUCTURAL ANALYSIS

WITH COMPUTER ANALYSIS AND APPLICATIONS





SUJIT KUMAR ROY
SUBRATA CHAKRABARTY

S. CHAND

FUNDAMENTALS OF
STRUCTURAL ANALYSIS

With Computer Analysis & Applications

[For B.E. / B.Tech. in Civil Engineering and also Useful for M.E./M.Tech. Students]

Dr. SUJIT KUMAR ROY
B.E., M.E., Ph.D. (IIT, Kharagpur), MIE (India)
Assistant Professor of Civil Engineering
and
Dr. SUBRATA CHAKRABARTY
B.E., M.Tech, Ph.D. (IIT, Kharagpur)
Bengal Engineering College
Howrah - 711 103

S. CHAND & COMPANY LTD.
(AN ISO 9001 : 2000 COMPANY)
RAM NAGAR, NEW DELHI-110 055

S. CHAND & COMPANY LTD.
(An ISO 9001 : 2000 Company)
Head Office: 7361, RAM NAGAR, NEW DELHI - 110 055
Phone: 23672080-81-82, 9899107446, 9911310888
Fax: 91-11-23677446
Shop at: **schandgroup.com;** e-mail: **info@schandgroup.com**

Branches :

AHMEDABAD	: 1st Floor, Heritage, Near Gujarat Vidhyapeeth, Ashram Road, **Ahmedabad** - 380 014, Ph: 27541965, 27542369, ahmedabad@schandgroup.com
BENGALURU	: No. 6, Ahuja Chambers, 1st Cross, Kumara Krupa Road, **Bengaluru** - 560 001, Ph: 22268048, 22354008, bangalore@schandgroup.com
BHOPAL	: Bajaj Tower, Plot No. 243, Lala Lajpat Rai Colony, Raisen Road, **Bhopal** - 462 011, Ph: 4274723. bhopal@schandgroup.com
CHANDIGARH	: S.C.O. 2419-20, First Floor, Sector - 22-C (Near Aroma Hotel), **Chandigarh** -160 022, Ph: 2725443, 2725446, chandigarh@schandgroup.com
CHENNAI	: 152, Anna Salai, **Chennai** - 600 002, Ph: 28460026, 28460027, chennai@schandgroup.com
COIMBATORE	: No. 5, 30 Feet Road, Krishnasamy Nagar, Ramanathapuram, **Coimbatore** -641045, Ph: 0422-2323620 coimbatore@schandgroup.com **(Marketing Office)**
CUTTACK	: 1st Floor, Bhartia Tower, Badambadi, **Cuttack** - 753 009, Ph: 2332580; 2332581, cuttack@schandgroup.com
DEHRADUN	: 1st Floor, 20, New Road, Near Dwarka Store, **Dehradun** - 248 001, Ph: 2711101, 2710861, dehradun@schandgroup.com
GUWAHATI	: Pan Bazar, **Guwahati** - 781 001, Ph: 2738811, 2735640 guwahati@schandgroup.com
HYDERABAD	: Padma Plaza, H.No. 3-4-630, Opp. Ratna College, Narayanaguda, **Hyderabad** - 500 029, Ph: 24651135, 24744815, hyderabad@schandgroup.com
JAIPUR	: A-14, Janta Store Shopping Complex, University Marg, Bapu Nagar, **Jaipur** - 302 015, Ph: 2719126, jaipur@schandgroup.com
JALANDHAR	: Mai Hiran Gate, **Jalandhar** - 144 008, Ph: 2401630, 5000630, jalandhar@schandgroup.com
JAMMU	: 67/B, B-Block, Gandhi Nagar, **Jammu** - 180 004, (M) 09878651464 **(Marketing Office)**
KOCHI	: Kachapilly Square, Mullassery Canal Road, Ernakulam, **Kochi** - 682 011, Ph: 2378207, cochin@schandgroup.com
KOLKATA	: 285/J, Bipin Bihari Ganguli Street, **Kolkata** - 700 012, Ph: 22367459, 22373914, kolkata@schandgroup.com
LUCKNOW	: Mahabeer Market, 25 Gwynne Road, Aminabad, **Lucknow** - 226 018, Ph: 2626801, 2284815, lucknow@schandgroup.com
MUMBAI	: Blackie House, 103/5, Walchand Hirachand Marg, Opp. G.P.O., **Mumbai** - 400 001, Ph: 22690881, 22610885, mumbai@schandgroup.com
NAGPUR	: Karnal Bag, Model Mill Chowk, Umrer Road, **Nagpur** - 440 032, Ph: 2723901, 2777666 nagpur@schandgroup.com
PATNA	: 104, Citicentre Ashok, Govind Mitra Road, **Patna** - 800 004, Ph: 2300489, 2302100, patna@schandgroup.com
PUNE	: 291/1, Ganesh Gayatri Complex, 1st Floor, Somwarpeth, Near Jain Mandir, **Pune** - 411 011, Ph: 64017298, pune@schandgroup.com **(Marketing Office)**
RAIPUR	: Kailash Residency, Plot No. 4B, Bottle House Road, Shankar Nagar, **Raipur -** 492 007, Ph: 09981200834, raipur@schandgroup.com **(Marketing Office)**
RANCHI	: Flat No. 104, Sri Draupadi Smriti Apartments, East of Jaipal Singh Stadium, Neel Ratan Street, Upper Bazar, **Ranchi** - 834 001, Ph: 2208761, ranchi@schandgroup.com **(Marketing Office)**
SILIGURI	: 122, Raja Ram Mohan Roy Road, East Vivekanandapally, P.O., **Siliguri**-734001, Dist., Jalpaiguri, (W.B.) Ph. 0353-2520750 **(Marketing Office)**
VISAKHAPATNAM	: Plot No. 7, 1st Floor, Allipuram Extension, Opp. Radhakrishna Towers, Seethammadhara North Extn., **Visakhapatnam** - 530 013, (M) 09347580841, visakhapatnam@schandgroup.com **(Marketing Office)**

First Edition 2003
Revised Edition 2009
Reprint 2010 (Twice)
Reprint with Corrections 2011

ISBN : 81-219-2195-3 **Code :** 10 245

PRINTED IN INDIA

By Rajendra Ravindra Printers Pvt. Ltd., 7361, Ram Nagar, New Delhi -110 055 and published by S. Chand & Company Ltd., 7361, Ram Nagar, New Delhi -110 055.

PREFACE TO THE SECOND EDITION

In this edition the book has been thoroughly revised and corrected. A chapter on Plastic Analysis has been added. Hope the students will find the book more useful.

Kolkata **Dr. S.K. ROY**
Dr. S. CHAKRABARTY

PREFACE TO THE FIRST EDITION

The book "Fundamentals of Structural Analysis" is the outcome of the authors' long experience of teaching structural engineering to undergraduate and post-graduate students in Civil Engineering. The book takes an integral look at structural engineering starting with fundamentals and ending with computer analysis. Computer applications have been introduced early though the full fledged analysis has been postponed to later stages when physical conception on structural behaviour is clear.

The book is suitable for 5th, 6th and 7th semesters of undergraduate course. The sixteenth chapter on Matrix Analysis and Computers can be taught in the 7th semester which will prepare the students for extensive study on Computer Analysis in the 8th semester and will be useful transition from matrix method to more general finite element study at P.G. level. Similarly the seventeenth chapter on "Dynamic Analysis" is introductory in character.

In writing this book we have consulted large number of Indian and foreign books on structural analysis. A select bibliography is given at the end of the book. We are grateful to all authors whose works form the background of this book. The senior author is grateful to Dr. N.C. Sinha, former Professor and Head of C.E. Deptt. of Jalpaiguri Govt. Engineering College and B.E. College (D.U.) for initiating him into the field of textbook writing.

A large number of examples have been worked out in the book so that students can master the subject by practising the examples and problems. It is quite possible to have some computational and typographical errors. Readers will earn our gratitude by pointing them to us.

We are grateful to Mr. Ravindra Kumar Gupta, Managing Director of M/s S. Chand and Co. Ltd. for publishing the book with great care and also in a very short time.

Our grateful thanks are due to Mr. R.S. Saxena, Advisor, Delhi office and to Mr. R.M. Nath, Manager, Calcutta Branch for coordinating the publication of the book.

Kolkata **Dr. S.K. ROY**
Dr. S. CHAKRABARTY

CONTENTS

1. **BASIS OF STRUCTURAL ANALYSIS** ... **1–11**

Introduction–Relation between Analysis and Design–Structural Idealisation–Classification of structures–Joint and Support Conditions–Material Characteristic–Super Position–Loads.

2. **EXTERNAL EQUILIBRIUM OF STRUCTURE** ... **12–42**

Introduction–Free Body Diagram–Calculation of Reactions–Matrix Formulation of Reaction Problems–Matrix Solution and Structural Behaviour–General Criteria for Determining Stability, Determinacy and Indeterminacy of a Structural Member–Calculation of Indeterminacy by Formulae.

3. **TRUSS ANALYSIS** ... **43–123**

Introduction–Basic Assumptions of Truss Analysis–Analysis of Plane Truss–Further Comments on Truss–Matrix Method of Solution of Trusses–Space Truss.

4. **AXIAL FORCE, SHEAR AND MOMENT DIAGRAMS–BEAM** ... **124–162**

Introduction–Internal Equilibrium of a Flexural Member–Relationship between Load, Shear and Moment–Drawing Axial Force, S.F. and B.M. Diagrams–Superposition of Moment Diagrams–Additional Methods for Drawing the S.F. and B.M. Diagrams–Loading Diagram from S.F. and/or Bending Moment Diagram.

5. **THRUST, SHEAR AND MOMENT DIAGRAMS–FRAMES** ... **163–187**

Introduction–Analysis of Rigid Frames–Sign Convention–Transformation of Concentrated and Distributed Loads.

6. **CABLE, SUSPENSION BRIDGE AND THREE HINGED ARCH** ... **188–245**

Introduction–Analytical Solution of Cable Structure–Graphical Solution of Cable Problems–Analytical Solution of Cables under Concentrated Loads–The Length of a Cable–Elastic Stretch of a Cable–Suspension Bridge–Three Hinged Arch–Analysis of Three Hinged Arch.

7. **FORCE DISPLACEMENT RELATIONS : GEOMETRIC METHODS** ... **246–312**

Introduction–Geometrical Method–Differential Equation of the Deflection Curve–Singularity Function and Macaulay's Notation–Moment Area Method–Areas and Centre of Gravities of Standard Figures–Further Examples on Moment Area Methods–Methods of Concentrated Angle Changes–Correction for Pinned Structures–Conjugate Beam Method–Change of Support Conditions–Sign Convention–Method of Superposition.

8. **FORCE DISPLACEMENT RELATIONS–ENERGY METHOD** ... **313–391**

Introduction–Conservation of Energy, Work and Strain–Work and Complementary Work–Expressions for Strain Energy and Complementary Energy–Expressions for Strain Energy for Linear Elastic Member–Computations of Deflection Through Real Work–Virtual Work and Complementary Virtual Work–Principle of Virtual Work and Complementary Virtual Work (Rigid Bodies)– Principle of Virtual Work and Complementary Work (Deformable

Bodies)–Derivation of Principle of Complementary Virtual Work for a General Elastic Body–Expressions for Internal Complementary Virtual Work–Application of Complementary Virtual Work–Important Energy Theorems–Linear and Nonlinear Problems–Betti's Theorem and Maxwell's Law of Reciprocal Deflection.

9. INFLUENCE LINES FOR STATICALLY DETERMINATE STRUCTURES ... 392–457

Introduction–Influence Line for Determinate Structure–Influence Lines by Virtual Work–Muller Breslau's Principle–Uses of Influence Lines–Moving Concentrated Loads–Girders with Floor Beams–Statical Analysis of a Girder with Floor Beams–Influence Lines for Girders with Floor Beams–Maximum Shear in a Girder with Floor Beams–Influence Line for Truss–Influence Line for the Three Hinged Arch–Influence Line for three Hinged Suspension Bridge.

10. CLASSICAL COMPATIBILITY (FORCE) METHOD OF ANALYSIS ... 458–547

Introduction–Consistent Deformation Method–Notes for Drawing the B.M. Diagram for Portals–Solution of Statically Indeterminate Trusses–Indeterminate Structures Subjected to Support Settlements, Temperature Variation, Fabrication Error etc–Application of Energy Methods in Statically Indeterminate Problems–Generalised Solution of Fixed Portal Frames (Elastic Centre Method)–Theorem of Three Moments–Derivation of the Equation–Temperature Change and Fabrication Error.

11. CLASSICAL EQUILIBRIUM (DISPLACEMENT) METHODS OF ANALYSIS ... 548–707

Introduction–Slope Deflection Method–Derivation of Slope Deflection Equations–Slope Deflection Equations for Frames–Iterative Solution of Slope Deflection Equations–Moment Distribution Method–Preliminary Ideas–Application of Preliminary Ideas–Computation of Displacement Variables in M.D. Process–Systematic Steps of Moment Distribution Method–Distribution Factor at a Pinned End and at a Fixed End–Moment Distribution for Frames with Sloping Members–Kani's Method–Derivation of Rotation Contribution Method End Conditions–Symmetrical Frames without Lateral Sway–Frames with Lateral Sway, Vertical Loading only–Frames with Lateral Sway–Derivation of Special Cases.

12. TWO HINGED ARCH ... 708–728

Introduction–Standard Cases of Two Hinged Semicircular Arch of Constant Flexural Rigidity EI.–Standard Cases of Two Hinged Parabolic Arch–Effect of Temperature Change and Yielding of Supports on Horizontal Thrust–Derivation of Horizontal Thrust without Neglecting Axial Force.

13. COLUMN ANALOGY ... 729–761

Introduction–General Equation of Stresses in a Column–Derivation of Column Analogy Method–Indeterminate and Corresponding Determinate Structures–Analogous Column–The Main Characteristic of Analogous Column–Properties of Analogous Column Sections–Determination of Stiffness and Carry Over Factors by Column Analogy Method.

14. INFLUENCE LINES FOR STATICALLY INDETERMINATE STRUCTURES ... 762–806

Introduction–Influence Line Diagrams for Plane Frames–Qualitative Influence Line–Influence Lines for Indeterminate Truss–Influence Line for Two Hinged Arch.

15. APPROXIMATE METHODS OF ANALYSIS OF INDETERMINATE STRUCTURES ... 807–833

Introduction–Approximate Analysis of Statically Indeterminate Trusses–Approximate Analysis of Beams and Frames– Portals and Mills Bents Subjected to Lateral Forces–Building Frames Subjected to Vertical Forces only – Building Frames subjected to Lateral Forces.

16. MATRIX METHOD OF STRUCTURAL ANALYSIS ... 834–943

Introduction–Classification of Framed/Sketetal Structure–Degree of Freedom Boundary Conditions–Force Displacement Relationships–Fundamental Concept of Matrix Method of Structural Analysis–Matrix Method of Analysis of Plane Trusses–Matrix Method of Analysis for Beam Structures–Matrix Method of Analysis of Frame Structure–Matrix Method of Analysis of Grillages–Space Frame.

17. DYNAMIC ANALYSIS OF STRUCTURE ... 944–971

Introduction–Importance of Vibration Analysis–Nature of Dynamic Influences–Classification of Vibration Problems–Response of Single Degree Freedom System–Forced Vibration–Support Movement–Vibration Isolation and Transmissibility– Rotating Unbalanced Force.

18. PLASTIC ANALYSIS OF STRUCTURES ... 972–998

Introduction–Stress-Strain Behavior of Metal–Some Useful Terminology–Assumption in Plastic Analysis–Fundamentals of Plastic Analysis–Flexure of Beam–Effect of Axial Force on Fully Plastic Moment–Collapse Mechanisms–Plastic Analysis–Application to Portal Frame–Geometry of Mechanism.

ANSWERS ... 999–1007

LANDMARKS IN THE HISTORY OF STRUCTURAL ENGINEERING ... 1008–1009

BIBLIOGRAPHY ... 1010–1011

INDEX ... 1012–1016

1

BASIS OF STRUCTURAL ANALYSIS

1.1. INTRODUCTION

Engineering structures must carry load with small deformation. Examples of common engineering structures are : buildings, bridges, dams, offshore oil platforms, water and gas tanks, piping systems, nuclear and chemical reactor vessels, ships, air planes, buses and other transportation systems.

From engineering point of view

(*i*) a *Framed structure* is an assembly of members inter connected by joints. Other types of structural systems are shells, domes, dams, walls etc.

(*ii*) *Structural behaviour* is the response of a structure to applied loads and environmental effects (wind, earthquake and temperature changes).

(*iii*) *Structural analysis* is the determination of reactions, member forces, deformation of the structure and deflection of the joints due to applied loads and environmental effects.

1.2. RELATION BETWEEN ANALYSIS AND DESIGN

The primary work of an engineer is to design. But analysis of a structure is required to check whether the design is safe either of a new or old structure. Usually design consists of preliminary design and checking by analysis, redesign with suitable modifications which is repeated a number of times till final design is obtained. At this stage, all the design constraints are fulfilled, that is, stresses and deflections are within permissible limits and cost is usually minimum.

1.3. STRUCTURAL IDEALISATION

Structural analysis is conducted on an analytical (or mathematical) model which is an idealisation of the actual structure. Engineering judgment must be used in defining the idealised structure such that it represents the actual structural behaviour as accurately as is practically possible.

Depending on the problem in hand, structures are idealised using line, plate or brick elements. Line elements are used to model the structural components that have one dimension much greater than other two. Typical examples are truss members, beams, columns etc. Plate elements are used to model structural components such as slabs and shells of which two dimensions are much greater than the third. Brick elements are used to model structures such as thick slabs and machine components where the structural action is truly three dimensional. Assembling the elements in accordance with the geometry of the structure produces a model of the complete structure that can be used to investigate the effect of the applied loading.

Fig. 1.1. (*a*) shows an actual structure.

Fig. 1.1 (*b*) shows the idealisation.

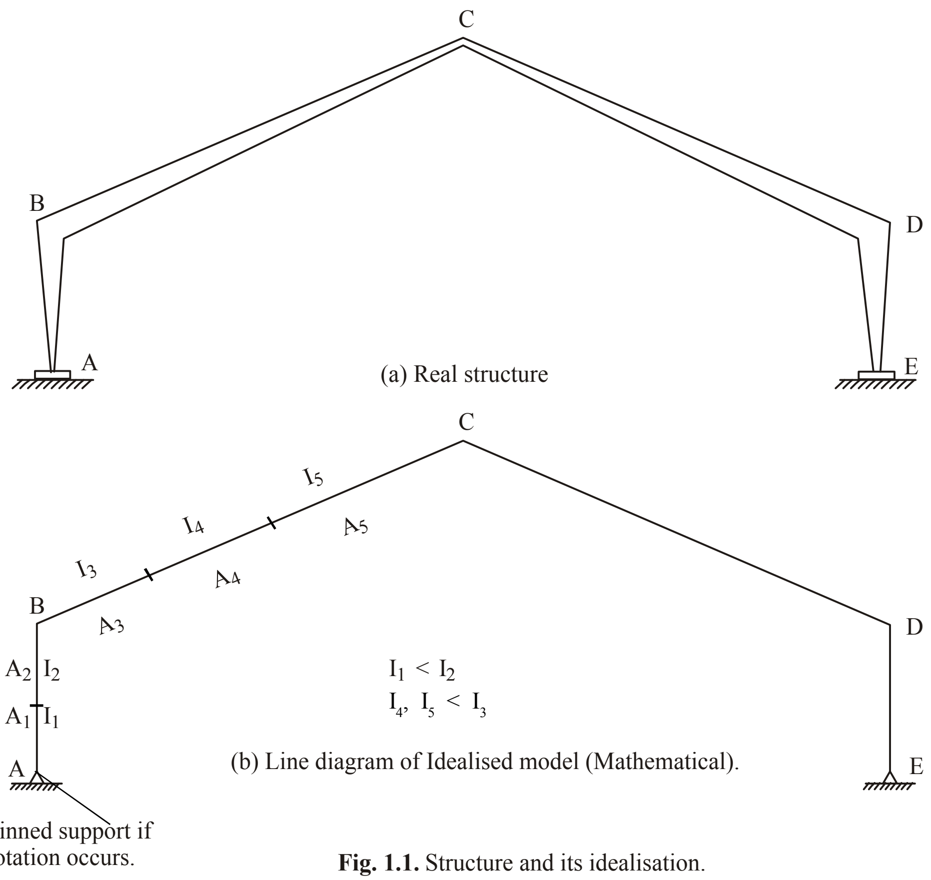


Fig. 1.1. Structure and its idealisation.

The following points can be observed :

(*i*) The actual structure is a welded frame of I section. The section is variable *i.e.* it is thicker at *B* and *D* and thinner at *A, C* and *E*.

(*ii*) It is supported at the ground level by means of concrete foundation. Hence in the idealised model represented by the line diagram we should have variable I and Area, large I and Area near *B* and *D* while small I and Area near the supports *A* and *E*. and mid region *C*. If the supports allow rotations it should be pinned, if no rotation occurs it should be considered as fixed support.

1.4. CLASSIFICATION OF STRUCTURES

Structures can be classified in many different ways. One way of classification is based on dimensions, such as

(*i*) One dimensional, for example, trusses, beams, frames, grids, arch ribs and cable structures. Here one dimension, say length, is very large compared to other two dimensions for the components of the structure.

Some of these structures are shown in Fig. 1.2. (*a*) to (*g*).

(*ii*) Two dimensional or surface structures *e.g.* slab, deep beams, shells or walls. Here length and breadth are large in comparison to thickness. Fig. 1.2 (*i*) and (*j*).

(*iii*) Three dimensional or solid structures where all the three dimensions of length, breadth or height are equally prominent *e.g.* foundations, retaining walls, gravity dams. Fig. 1.2. (*k*) (*l*) (*m*) and (*n*).

They can also be classified from the way they carry load. For example,

(*i*) Beams carry their loads by developing bending moments and shear forces at different sections.

(*ii*) Trusses carry their loads by developing axial forces in the bars.

(*iii*) Frames carry loads by axial force, bending moment and shear.

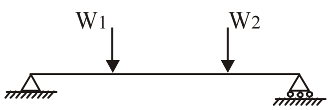
(*iv*) Arches carry load by compression and bending.

(*v*) Cables carry loads by developing axial tension.

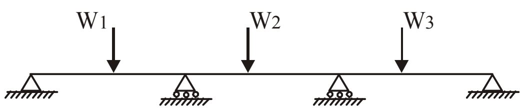
(*vi*) Two way grids are subjected to both bending and twisting

(*vii*) Thin plates are subjected to both bending and twisting.

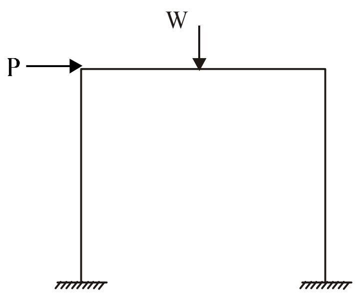
(*viii*) Thin shells transmit forces mainly by in plane or membrane stresses.



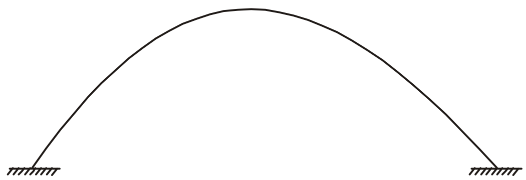
(a) Simple beam.
(One dimensional)



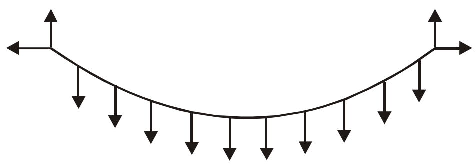
(b) Continuous beam.
(Combination of one dimensional elements)



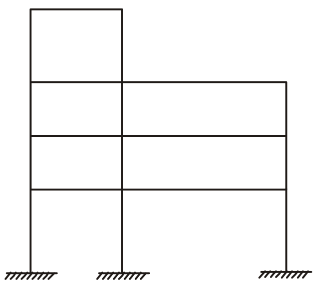
(c) Portal frame (combination of one dimensional elements).



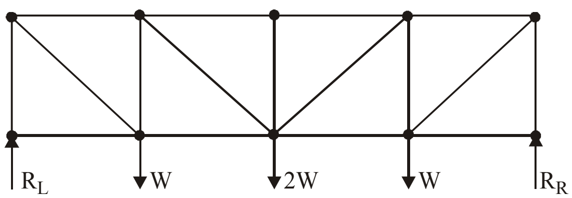
(e) Arch (one dimensional).



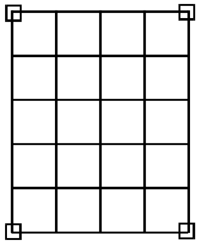
(f) Cable (one dimensional).



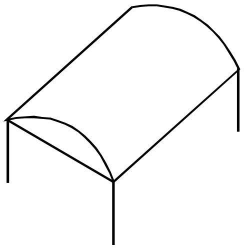
(d) Multi storey plane frame (combination of one dimensional elements)



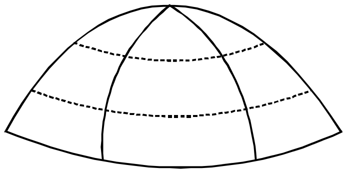
(g) Truss (combination of one dimensional elements)



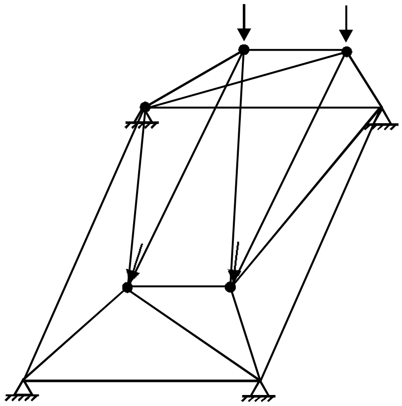
(h) A horizontal grid (Combination of one dimensional elememts).



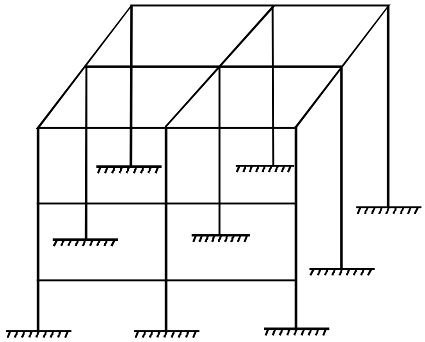
(i) A cylindrical vault (Two dimensional).



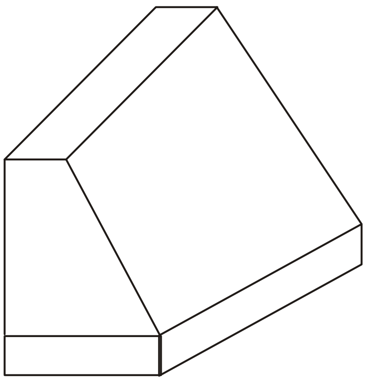
(i) A dome (Two dimensional)



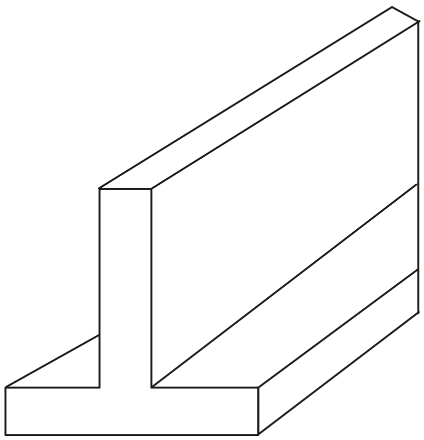
(k) Space Truss. (Three dimensional with one dimensional elements).



(l) Space Frame (Three dimensional with one dimensional elements).



(m) Dam. (Three Dimensional structure).



(n) Retaining Wall (Three dimensional).

1.5. JOINT AND SUPPORT CONDITIONS

(*a*) **Joints.** As already explained, a structure is an assemblage of a number of elements. Joint is connection between these elements. Joints may be (*i*) Rigid or (*ii*) Pinned. Fig. 1.3. (*a*) shows rigid joint. A rigid joint maintains the angles between the centre lines of members framing into the joint. A rigid joint will carry any moment assigned to it and the joint will rotate as a whole and there will be no change in the angle between the members. Fig. 1.3. (*b*) shows a pinned joint.

This joint allows the members joining it independent rotation. Hence angle between the member changes. Since the joint is pinned it cannot resist any moment.

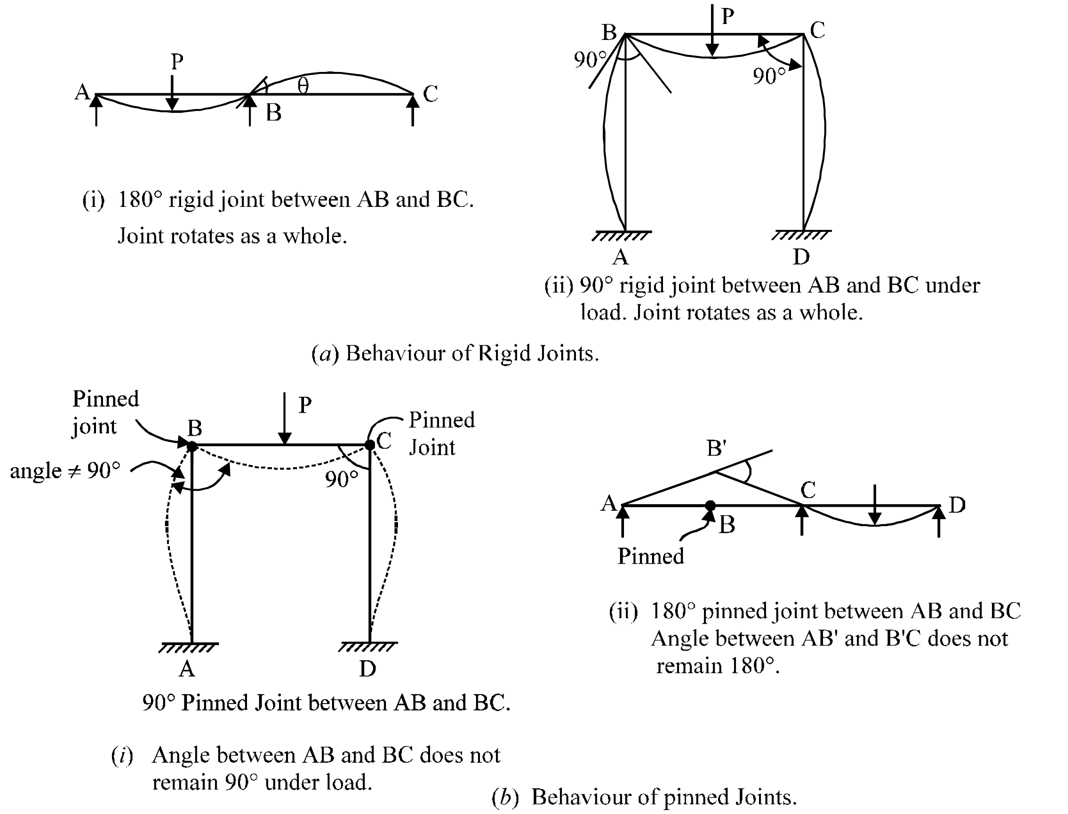
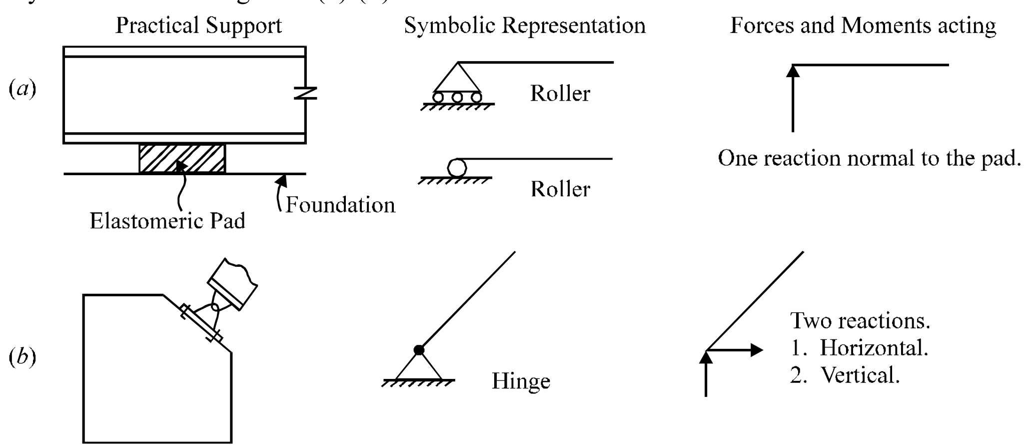
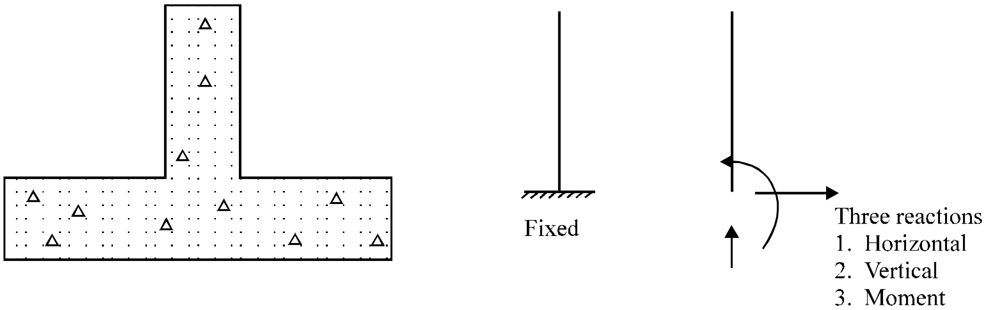


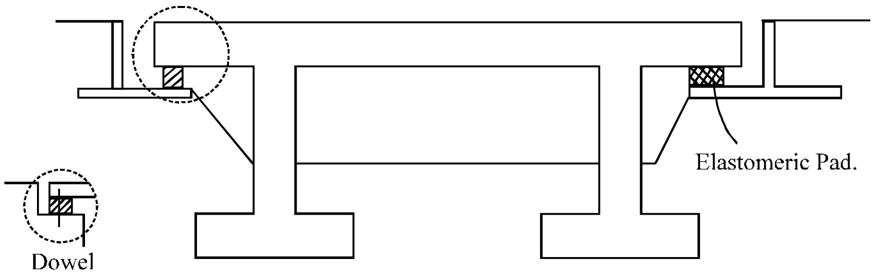
Fig. 1.3. Behaviour of Joints.

(*b*) **Supports.** The loads applied to a structure must be transferred to the ground by the supports, that is, the support will generate reactions to counteract the actions of the loads. The type and number of reactions depend on the support conditions. Support may be roller, hinge or pin and fixed. Fig. 1.4. shows actual supports, how it is symbolically represented and forces and moments which act on the support. The bridge deck constructed as monolithic with the columns is shown in Fig. 1.4. (*d*) (*i*) and the corresponding support conditions that might be taken for analysis is shown in Fig. 1.4. (*d*) (*ii*).



(c)





(i) Actual structure.

(ii) Mathematical Model.

(d) Actual structure and mathematical model of a bridge deck.

Fig. 1.4. Different types of supports (*a*) Roller support (*b*) Hinged support (*c*) Fixed support (*d*) Actual structure and mathematical model of a bridge deck.

1.6. MATERIAL CHARACTERISTICS

Structures are usually made of different materials like wood, concrete or steel. Their stress-strain curves are all different as shown in Fig. 1.5.

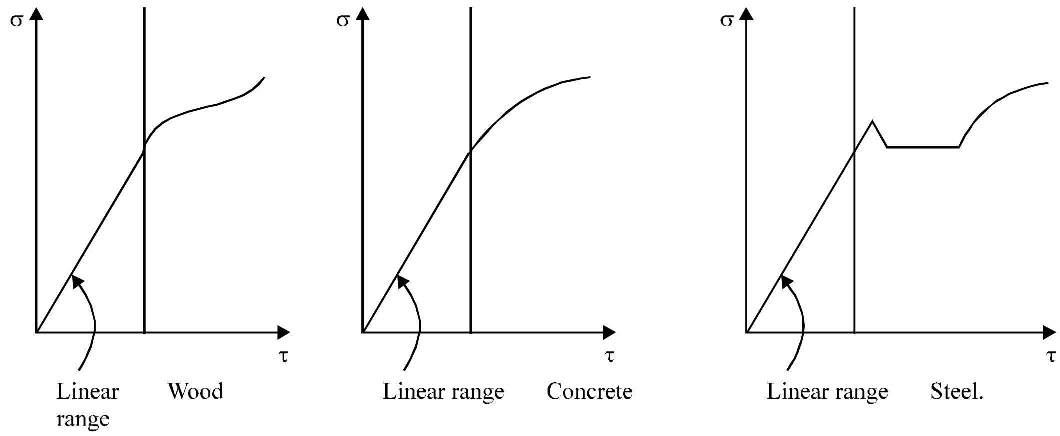


Fig. 1.5. Stress Strain curve of different materials.

Upto some level of load, stress is proportional to strain and *E*, the slope of the stress strain curve is constant and hence stress strain curve is linear. Another characteristic of member is its elasticity. Upon unloading, a material may come back to its original position, then it will be called elastic. When it does not come back to its original position it is termed inelastic, hence considering (*i*) linearity (*ii*) elasticity four combinations of curves are possible.

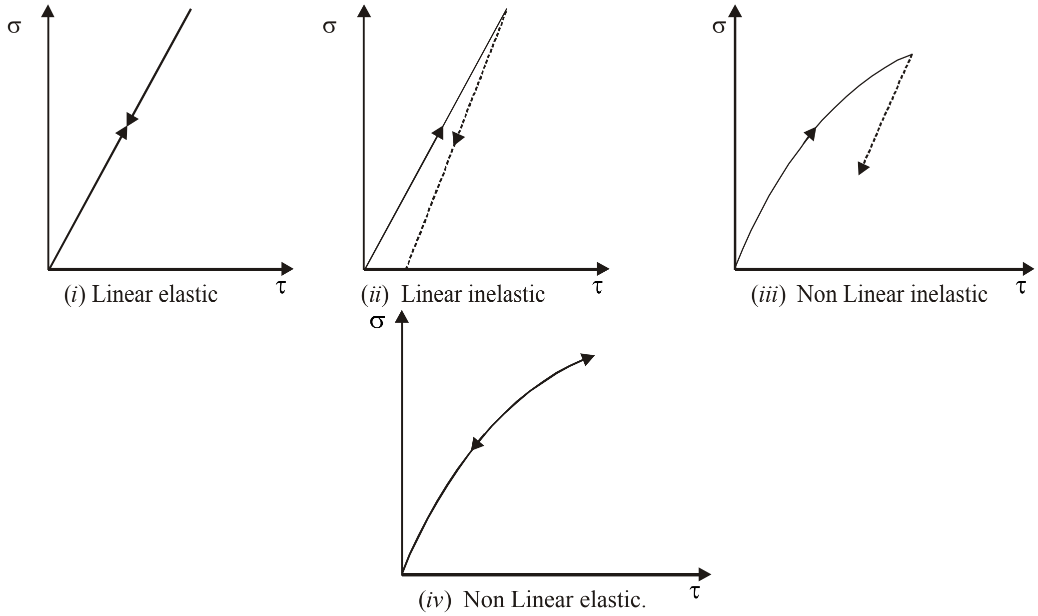


Fig. 1.6 Different types of stress strain curve.

Steel upto proportional limit is linear and elastic; steel beyond its yield point is nonlinear and inelastic. Rubber is nonlinear and elastic.

1.7. SUPER POSITION

The deformations of structures are generally small. In most cases, the geometry of the structure is sensibly unaffected by deformation. In such a case, the conditions of equilibrium and conditions of compatibility can be written down for the undisturbed structure without loss of accuracy. The principle of supper position states that

if	cause	A	produces Effect A
and	cause	B	produces Effect B
then	cause	A +	cause B produce Effect A + Effect B.

Three categories of super position will be considered—

(*i*) Super position of force systems.
(*ii*) Super position of displacements.
(*iii*) Super position of linear elastic structures.

(*i*) **Super position of force systems** : For statically determinate structures the principle of super position is valid with only restriction that distortion of structure is small. This means internal forces and moments can be added together for two different loading systems. Under such conditions super position of forces is valid for statically indeterminate structures also with the additional restriction that material of the structure must obey Hooke's law because the internal forces depend on the deformation of the members.

(*ii*) Super position of displacements : Superposition of displacements is valid both for statically determinate and indeterminate structure provided deflection of the structure is not gross.

The propped cantilever shown in Fig. 1.7. illustrates the principle of superposition.

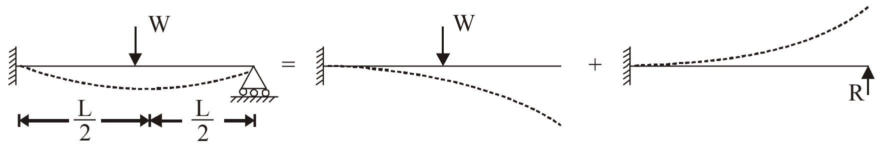


Fig. 1.7 Superposition of displacements.

The diagram shows that if the effect of the propping force (*i.e.* the right hand reaction) is superimposed on the effect of the applied load on the unpropped cantilever then the actual results are obtained. Here propping force and the applied load have bean treated as two independent load systems and then their effects have been superposed. Superposition is not valid when the structural behaviour is not linear. There are two sources of nonlinearity in structures. The *first* is when the displacements are large making small deflection theory inapplicable. The following example explains the situation.

Consider the eccentrically loaded strut of Fig. 1.8

Central deflection $d = e \sec \frac{l}{2}\sqrt{\frac{W}{EI}}$

Here $d_1 = e \sec \frac{l}{2}\sqrt{\frac{W_1}{EI}}$

$d_2 = e \sec \frac{l}{2}\sqrt{\frac{W_2}{EI}}$

and d, due to load of $(W_1 + W_2)$

$= e \sec \frac{l}{2}\sqrt{\frac{(W_1 + W_2)}{EI}}$

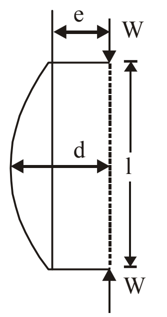
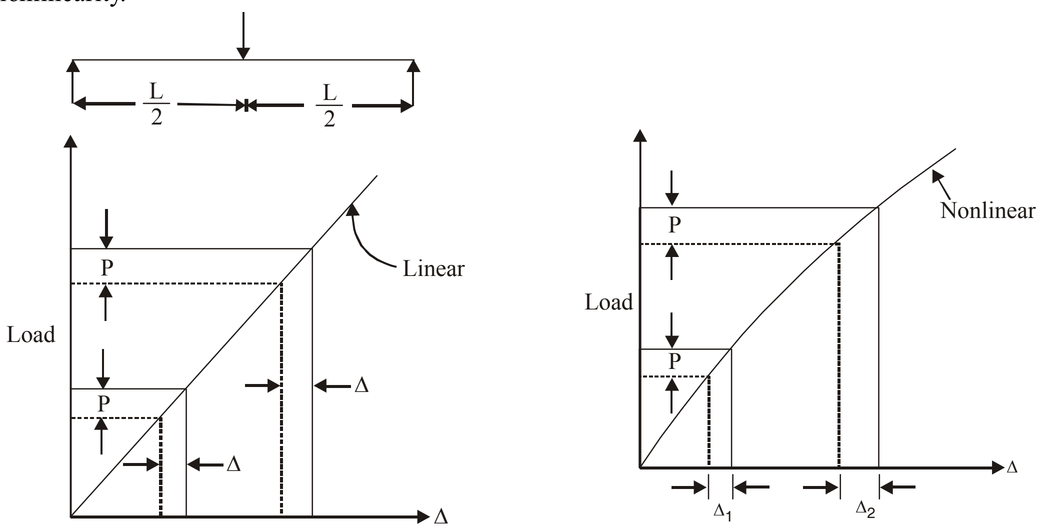


Fig. 1.8 Gross deformation of structure.

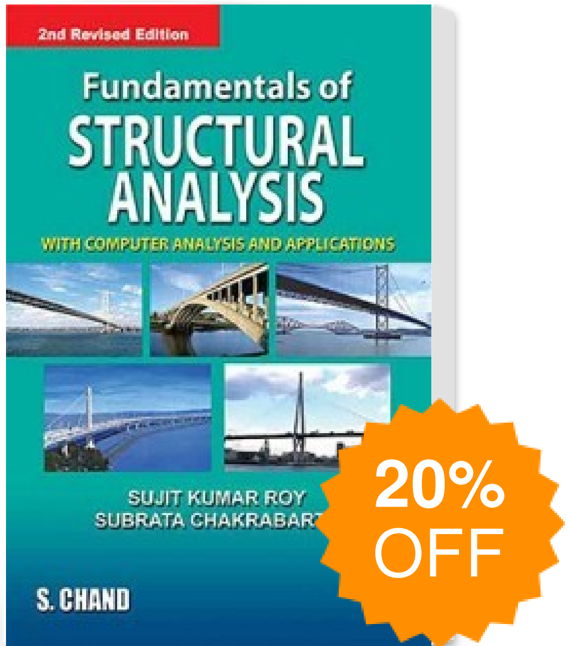
Obviously $d \neq d_1 + d_2$. This is because deformation is gross and conditions of equilibrium are appreciably affected by the distortion of the structure. The *Second* case when method of superposition is not valid occurs when the structural behaviour is nonlinear due to material nonlinearity.



(*a*) Deformation Linear. (*b*) Deformation Nonlinear

Fig. 1.9 Load deformation graphs.

Fundamentals Of Structural Analysis



Publisher : SChand Publications ISBN : 9788121921954 Author : Subrata Chakarabarty

Type the URL : http://www.kopykitab.com/product/10738

Get this eBook