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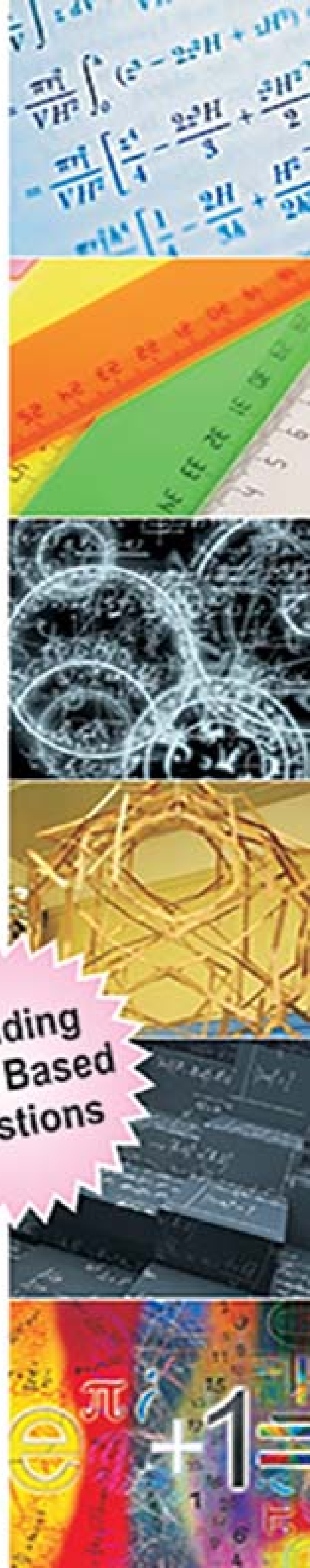
MATHEMATICS

Including Value Based Questions



CLASS XII

Golden MATHEMATICS
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Class XII

Strictly according to new syllabus prescribed by

Central Board of Secondary Education (CBSE) Delhi

and

State Boards of Chhattisgarh, Haryana, Jharkhand, Kerala, Mizoram, Meghalaya, Punjab, Uttrakhand and other States following NCERT Curriculum

By

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DEDICATION

This book is dedicated to all my teachers who taught me and to all my Students who sharpened my abilities and inspired me to learn more.

PREFACE

This book is prepared based on the revised syllabus of CBSE for class XII. A great emphasis has been placed on the fundamental concepts.

The salient features of this book are :

- Important points to remember
- Previous Year's Questions with solutions
- Very important questions with hints and solutions
- Self evaluation tests
- Multiple choice questions with answer key

I hope that this book would serve the purpose of a complete textbook which would be useful to students and teachers.

I am confident that this new book will make learning Mathematics more enjoyable. Still there is ample room for improvement, suggestions and comments from the lovers of Mathematics would be gratefully received.

I express my heart felt thanks to my publisher for his enthusiasm, keen interest and inspiration.

AUTHOR

SYLLABUS

MATHEMATICS (041)
CLASS XII 2012-13

One Paper **Three Hours** **Marks : 100**

Units		Marks
I.	RELATIONS AND FUNCTIONS	10
II.	ALGEBRA	13
III.	CALCULUS	44
IV.	VECTORS AND THREE-DIMENSIONAL GEOMETRY	17
V.	LINEAR PROGRAMMING	06
VI.	PROBABILITY	10
	Total	**100**

The Question Paper will include question(s) based on values to the extent of 5 marks.

UNIT-I : RELATIONS AND FUNCTIONS

1. **Relations and Functions :** **(10) Periods**

   Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.

2. **Inverse Trigonometric Functions :** **(12) Periods**

   Definition, range, domain, principal value branches. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

UNIT-II : ALGEBRA

1. **Matrices :** **(18) Periods**

   Concept, notation, order, equality, types of matrices, zero matrix, transpose of a matrix, symmetric and skew symmetric matrices. Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. **Determinants :** **(20) Periods**

   Determinant of a square matrix (up to 3×3 matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

UNIT-III : CALCULUS

1. **Continuity and Differentiability : (18) Periods**

Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit function. Concept of exponential and logarithmic functions and their derivatives. Logarithmic differentiation. Derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretations.

2. **Applications of Derivatives : (10) Periods**

Applications of derivatives: rate of change, increasing/decreasing functions, tangents & normals, approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. **Integrals : (20) Periods**

Integrations as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts. Only simple integrals of the type :

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}},$$

$$\int \frac{px + q}{ax^2 + bx + c}\, dx, \int \frac{px + q}{\sqrt{ax^2 + bx + c}}\, dx, \int \sqrt{a^2 \pm x^2}\, dx, \int \sqrt{x^2 - a^2}\, dx$$

$$\int \sqrt{ax^2 + bx + c}\, dx, \int (px + q)\sqrt{ax^2 + bx + c}\, dx.$$

Definite integrals as limit of a sum, Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. **Applications of the Integrals : (10) Periods**

Applications in finding the area under simple curves, especially lines, areas of circles/parabolas/ellipses (in standard form only), area between the two above said curves (the region should be clearly identifiable).

5. **Differential Equations : (10) Periods**

Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type :

$\frac{dy}{dx} + py = q$, where p and q are functions of x or constant

$\frac{dx}{dy} + px = q$, where p and q are functions of y or constant

UNIT-IV : VECTORS AND THREE-DIMENSIONAL GEOMETRY

1. **Vectors : (12) Periods**

Vectors and scalars, magnitude and direction of a vector. Direction cosines/ratios of vectors. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Scalar (dot) product of vectors, projection of a vector on a line. Vector (cross) product of vectors.

2. Three-dimensional Geometry : (12) Periods

Direction cosines/ratios of a line joining two points, Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (*i*) two lines, (*ii*) two planes, (*iii*) a line and a plane. Distance of a point from a plane.

UNIT-V : LINEAR PROGRAMMING

Linear Programming : (12) Periods

Introduction, definition of related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimum feasible solutions (upto three non-trivial constraints).

UNIT-VI : PROBABILITY

Probability : (18) Periods

Multiplication theorem on probability. Conditional probability, independent events, total probability, Bayes' theorem, Random variable and its probability distribution, mean and variance of haphazard variable. Repeated independent (Bernoulli) trials and Binomial distribution.

CONTENTS

1

Relations and Functions

IMPORTANT POINTS TO REMEMBER

1. A *relation* R from $A \to B$ is a subset of $A \times B$.

   We write it as $R = \{(a, b) / a \in A, b \in B$ and $aRb\}$.
2. A relation in a set A is a subset of $A \times A$.
3. *Void relation or Empty relation* : $\phi \subseteq A \times A$. This relation is the Empty relation in A.
4. *Universal relation*: $A \times A \subseteq A \times A$ is also relation in A. This relation is the Universal relation in A.

   Example 1. Let $A = \{1, 2, 3\}$. Then $A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$ is the Universal relation.

   Example 2. Let A = {1, 2, 3}, then $R : A \to A = \{(a, b) / a - b = 5\}$ is an Empty relation.
5. *Reflexive relation*: A relation R in a non-empty set A is reflexive if aRa for every $a \in A$ *i.e.*, $(a, a) \in R$ for all $a \in A$.

   Example 1. Let $A = \{1, 2, 3\}$. Then $R = \{(1, 1), (2, 2), (3, 3), (2, 1)\}$ is reflexive because every element of A is related to itself. But $R = \{(1, 1), (1, 2), (3, 3), (2, 1)\}$ is not reflexive.

   Example 2. In the set of lines "is parallel" is reflexive since $l \parallel l$ is true.
6. *Symmetric relation*: A relation in a non-empty A is symmetric if $aRb \Rightarrow bRa$

   i.e., $(a, b) \in R \Rightarrow (b, a) \in R$ for every $a, b \in A$.

   Example 1. Let $A = \{1, 2, 3\}$. Then $R = \{(1, 1), (2, 3), (3, 2), (2, 1), (1, 2)\}$ is symmetric.

   Example 2. In the set of real numbers, the relation "is equal to" is symmetric since $a = b \Rightarrow b = a$.
7. *Transitive relation*: A relation in a set A is said to be transitive if aRb, $bRa \;\Rightarrow\; aRc$

   i.e., $(a, b) \in R, (b, c) \in R \;\Rightarrow\; (a, c) \in R$ for every $a, b, c \in R$.

   Example 1. In the set of real numbers the relation "is equal to" is transitive since $a = b$, $b = c \Rightarrow a = c$.

   Example 2. In the set of lines, "is parallel to" is transitive since $l \parallel m, m \parallel n \;\Rightarrow\; l \parallel n$.
8. *Equivalence relation*: A relation R in a set A is said to an equivalence relation if R is reflexive, symmetric and transitive.

   Example 1. In the set of real numbers the relation "is equal to" is an equivalence relation.

   Example 2. In the set of lines "is parallel to" is an equivalence relation.
9. *Equivalence class*: Let R be an equivalence relation on a non-empty set A. For $a \in A$, the equivalence class of $a \in A$ are the elements of A which are related to $a \in A$ under R.

$[a]$ = equivalence class of $a = \{x/(x, a) \in R\}$

Example : Let $A = \{1, 2, 3, 4\}$.

Consider the equivalence relation $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1)\}$.

The equivalence classes are

$[1] = \{1, 2\}, [3] = \{3\}, [2] = \{2, 1\}, [4] = \{4\}$

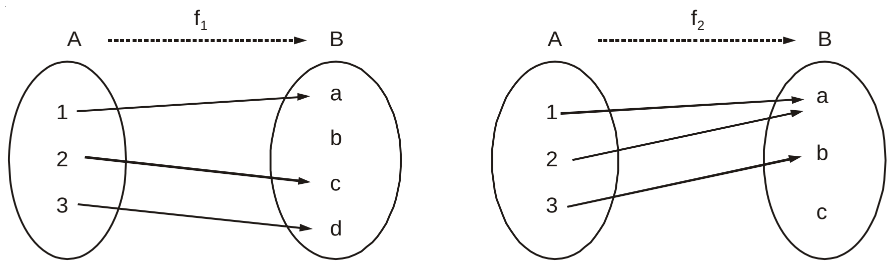
10. *Types of functions:*

(*a*) *One-one function* (*injection*) - *Many-one function* : A function $f : A \to B$ is a one-one function if distinct elements of A has distinct images in B.

To prove f is one-one, we have to show that $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

A function which is not one-one is a many-one function.

Example 1. *Consider the functions* $f_1 : A \to B$ and $f_2 : A \to B$ *defined by the arrow diagrams given below*:



Sol. f_1 is a one-one function where as f_2 is a many-one function.

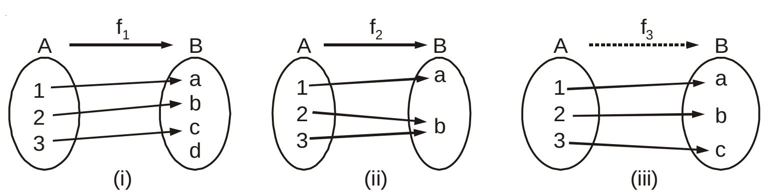
Example 2. $f : R \to R$ *defined by* $f(x) = x^2$. *Check whether the function is one-one or many-one.*

Sol. $f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2$. This need not imply $x_1 = -x_2$ (it can imply $x_1 \neq x_2$ also).

$\therefore$ The function is not one-one. It is a many one function.

(*b*) *Onto function* (*Surjection*) - *Into function*: A function $f : A \to B$ is an onto function or surjection if every element of B is the image of some element of A under f. f is not onto, then f is an into function.

Example 1. *Consider the following functions as shown by the arrow diagrams given below:*



Sol. (*i*) f_1 is an into function because the element d in B is not an image. To be more specific f_1 is a one-one and into function.

(*ii*) f_2 is an onto function because all the elements of B are the images of some elements of A. To be more specific f_2 is an onto and many-one function.

(*iii*) f_3 is one-one and onto function.

To prove $f : A \to B$ *is onto* (*surjection*), *take some element* $y \in B$ (*co-domain*). *If we are able to find an* $x \in A$ (*domain*) such $f(x) = y$, *then* f *is onto.*

Example 1. *$f : R \to R$ defined by $f(x) = x^3 + 2$.*

Sol. Let $y \in R$ (co-domain) and $f(x) = y$.

$\therefore\ x^3 + 2 = y \Rightarrow x^3 = y - 2 \Rightarrow x = (y-2)^{\frac{1}{3}} \in R.$

$\therefore$ Corresponding to each $y \in R$ (co-domain), there exists $x = (y-2)^{\frac{1}{3}} \in R$ (domain) such that $f(x) = y$.

Since y is arbitrary, this is true for every $y \in R$. *i.e.,* co-domain is full of images.

$\therefore f$ is onto.

Example 2. *$f : R \to R$ defined by $f(x) = x^2 + 2$*

Let $y \in R$ (co-domain) and $f(x) = y$.

$\therefore\ x^2 + 2 = y \Rightarrow x^2 = y - 2 \Rightarrow x = \pm\sqrt{y-2}$

If $y < 2$, then x is not a real number.

$\therefore$ All the elements of the co-domain are not images.

$\therefore f$ is not onto.

(*c*) *Bijection:* A function $f : A \to B$ is a bijection if and only if, it is both one-one and onto. To prove a bijection we have to show that f is both one-one and onto.

Example 1. *Prove that the function $f : R \to R$ defined by $f(x) = 2x - 3$ for all $x \in R$ is a bijection.*

Sol. $f(x_1) = f(x_2) \Rightarrow 2x_1 - 3 = 2x_2 - 3 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$

$\therefore f$ is one-one.

Let $y \in R$, and $f(x) = y$

$\therefore\ 2x - 3 = y \Rightarrow x = \frac{y+3}{2} \in R$

i.e., corresponding to each $y \in R$, there exists $\frac{y+3}{2} \in R$ such that $f(x) = y$.

This is true for all $y \in R$.

$\therefore\ f$ is onto.

$\therefore$ We can say that f is a bijection.

Example 2. *Show that the function $f : R \to R$ given by $f(x) = \cos x$ for $x \in R$, is neither one-one nor onto.*

Sol. We know that $f(0) = \cos 0 = 1$, $f(2\pi) = \cos 2\pi = 1$

$\therefore$ 1 is the image of many elements.

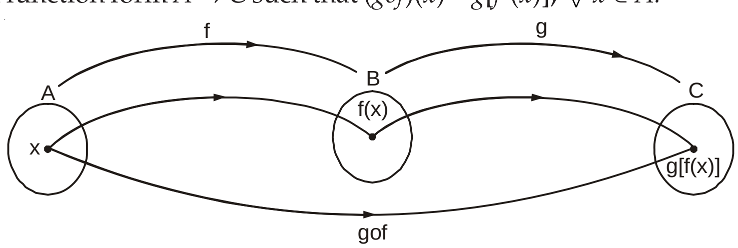
$\therefore\ f$ is many-one.

Since the values of $\cos x$ lies between –1 and 1, it follows that the range of f is not equal to co-domain.

$\therefore f$ is not onto (*i.e.,* into).

11. *Composition of functions:*

Let $f : A \to B$, $g : B \to C$ be two functions. Then the composition of these two functions denoted by gof is a function form $A \to C$ such that $(gof)(x) = g[f(x)]$, $\forall\ x \in A$.



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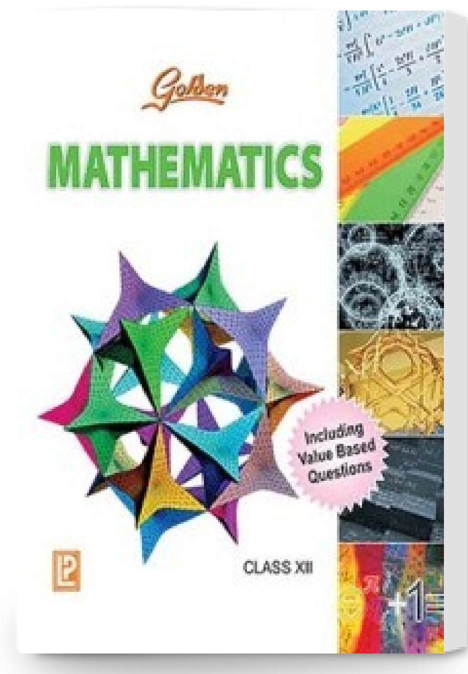

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