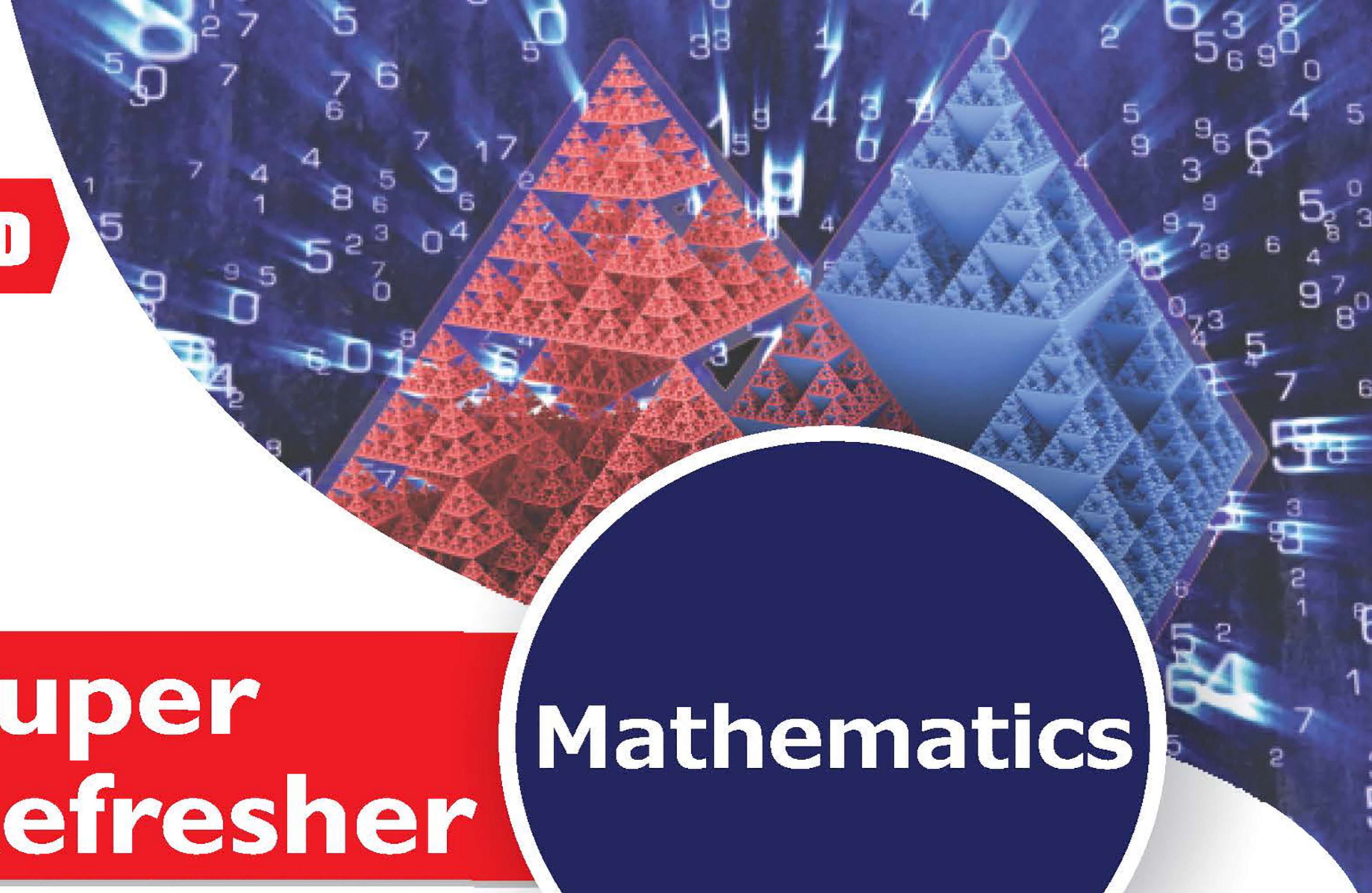
MBD



Super Refresher

Mathematics

9

TERM-I

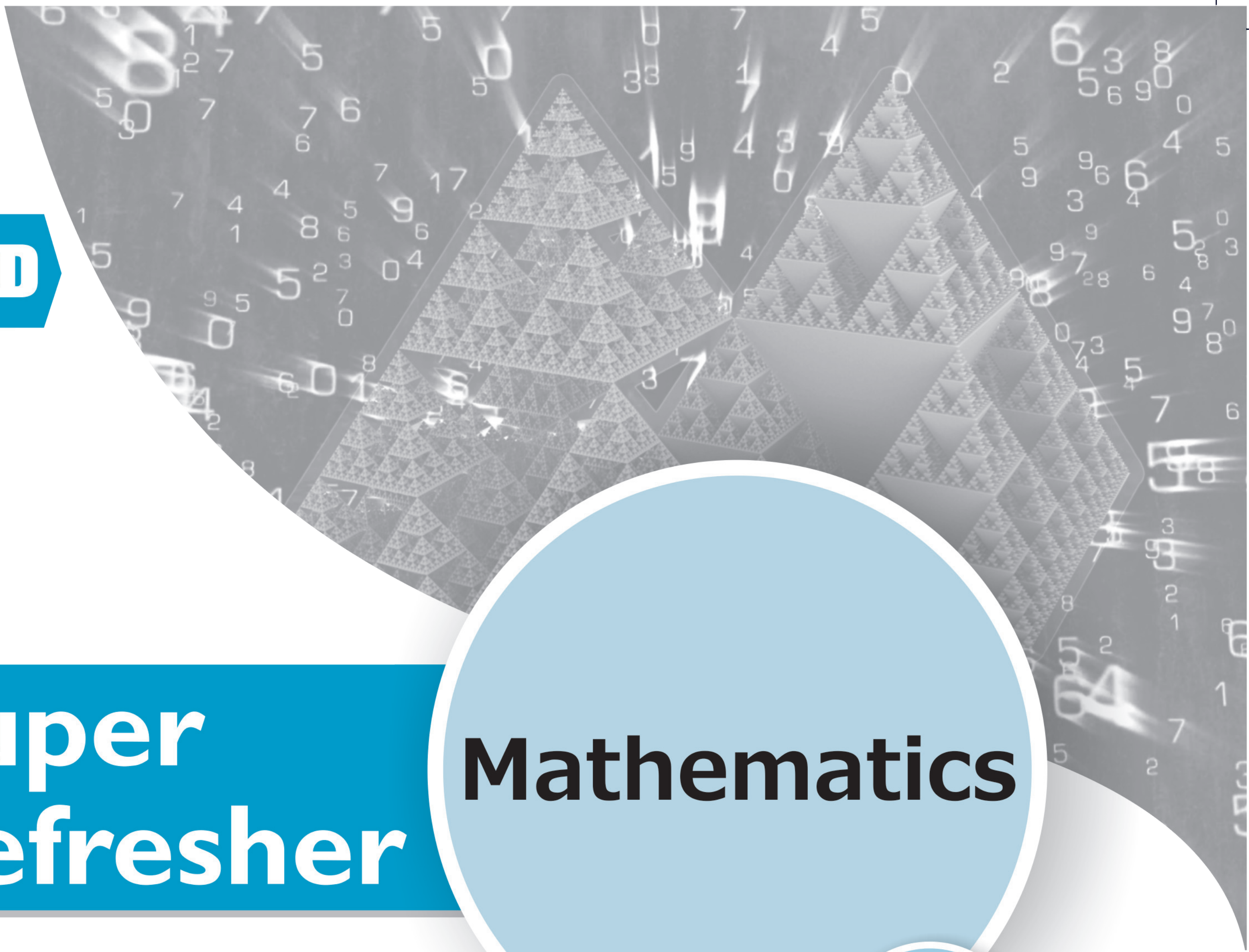
- **NCERT** Textbook Exercises and Exemplar Problems with solutions
- Comprehensive explanation of each chapter
- Large pool of Objective, Short and Long answer type questions

Highlights important information and questions for last minute revision

Includes **HOTS** and **Value Based** questions

Based on the latest syllabus and textbook issued by CBSE/NCERT

MBD



Super Refresher Mathematics 9

TERM-I

By

V. K. Saxena
Principal

Shilpi Ahuja

Edited by:
Sudhansu S. Swain
Vipul Jain

Based on the latest syllabus and textbook issued by **CBSE/NCERT**

MALHOTRA BOOK DEPOT
(Producers of Quality **MBD** Books)
An ISO 9001:2008 Certified Company

Combined Price of Term I & II : ₹ 410.00
Term I: ₹ 180.00
Term II: ₹ 250.00

OUR ADDRESSES IN INDIA

- **New Delhi :** MBD House, Gulab Bhawan, 6, Bahadur Shah Zafar Marg — Ph. 30912330, 30912301, 23318301
- **Mumbai :** A-683, T.T.C. Industrial Area, M.I.D.C. Off. Thane-Belapur Road, Navi Mumbai — Ph. 32996410, 27780821, 8691053365
- **Chennai :** No. 26 B/2 SIDCO Estate, North Phase, Pataravakkam Ambattur Industrial Estate, Ambattur — Ph. 26359376, 26242350
- **Chennai :** Plot No. 3018, Old Y Block, 3rd Street, 12th Main Road, Anna Nagar West — Ph. 23741471
- **Kolkata :** Satyam Building, 46-D, Rafi Ahmed Kidwai Marg — Ph. 22296863, 22161670
- **Jalandhar City :** MBD House, Railway Road — Ph. 2458388, 2457160, 2455663
- **Bengaluru :** 124/31, 1st Main, Industrial Town (Near Chowdeshwari Kalyan Mantap), West of Chord Road, Rajajinagar — Ph. 23103329, 23104667
- **Hyderabad :** 3-4-492, Varun Towers, Barkatpura — Ph. 27564788, 9985820001
- **Ernakulam :** Surabhi Building, South Janatha Road, Palarivattom — Ph. 2338107, 2347371
- **Pune :** Survey No. 44, Behind Matoshree Garden, Kondhwa - Khadi Machine, Pisoli Road, At. Post-Pisoli — Ph. 65271413, 65275071
- **Nagpur :** 'Chandrakor', Plot No. G-15, Aath Rasta Chowk, West High Court Road, Laxmi Nagar — Ph. 2248104, 2248106, 2248649, 2245648
- **Ahmedabad :** Godown No.10, Vedant Prabha Estate, Opp. ONGC Pumping Station, Sarkhej Sanand Road, Sarkhej — Ph. 26890336, 32986505
- **Cuttack :** Badambadi, Link Road — Ph. 2367277, 2367279, 2313013
- **Guwahati :** Chancellor Commercial, Hem Baruah Road, Paan Bazar — Ph. 2131476, 8822857385
- **Lucknow :** 173/15, Dr. B. N. Verma Road, Old 30 Kutchery Road — Ph. 4010992, 4010993
- **Patna :** Ist Floor, Annapurna Complex, Naya Tola — Ph. 2672732, 2686994, 2662472
- **Bhopal :** Plot No. 137, 138, 139, Sector-I, Special Industrial Area, Govindpura — Ph. 2581540, 2601535
- **Jabalpur :** 840, Palash Chamber, Malviya Chowk — Ph. 2405854
- **Goa :** H. No. 932, Plot No. 66, Kranti Nagar (Behind Azad Bhawan), Alto Porvorim, Bardez — Ph. 2413982, 2414394
- **Jaipur :** C-66A, In front of Malpani Hospital, Road No.1, V.K. Industrial Area, Sikar Road — Ph. 4050309, 4020168
- **Raipur :** Behind Aligarh Safe Steel Industries, Vidhan Sabha Road, Avanti Bai Chowk, Lodhi Para Pandri — Ph. 4052529, 2445370
- **Karnal :** Plot No. 203, Sector-3, HSIDC, Near Namaste Chowk, Opp. New World — Ph. 2220006, 2220009
- **Shimla (H.P.) :** C-89, Sector-I, New Shimla-9 — Ph. 2670221,2670816
- **Jammu (J&K) :** Guru Nanak College of Education, Jyoti Chack, Bari Brahmana — Ph. 2467376, 9419104035
- **Ranchi (Jharkhand) :** Shivani Complex, 2nd Floor, Jyoti Sangam Lane, Upper Bazar — Ph. 9431257111
- **Sahibabad (U.P.) :** B-9 & 10, Site IV, Industrial Area — Ph. 3100045, 2896939
- **Dehradun (Uttarakhand) :** Plot No. 37, Bhagirathipuram, Niranjanpur, GMS Road — Ph. 2520360, 2107214

DELHI LOCAL OFFICES:

- Delhi (Shakarpur) : MB 161, Street No. 4 — Ph. 22546557, 22518122
- Delhi (Daryaganj) : MBD House, 4587/15, Opp. Times of India — Ph. 23245676
- Delhi (Patparganj) : Plot No. 225, Industrial Area — Ph. 22149691, 22147073

Contributors

Project Manager: Debasish Subudhi
Composition & Layout: *Dhirender Singh Negi and Team*

Published by: **MALHOTRA BOOK DEPOT**
MBD House, Railway Road, Jalandhar City.

Printed at: **HOLY FAITH INTERNATIONAL (P) LTD.**
B-9 & 10, Site IV, Industrial Area, Shahibabad (U.P.)

SYLLABUS

MATHEMATICS (CLASS–IX)

First Term — Marks : 90

UNITS	MARKS
I. NUMBER SYSTEMS	17
II. ALGEBRA	25
III. GEOMETRY	37
IV. COORDINATE GEOMETRY	06
V. MENSURATION	05
TOTAL (THEORY)	90

UNIT-I : NUMBER SYSTEMS

1. REAL NUMBERS (18) Periods

1. Review of representation of natural numbers, integers, rational numbers on the number line. Representation of terminating/non-terminating recurring decimals on the number line through successive magnification. Rational numbers as recurring/terminating decimals.
2. Examples of non-recurring/non-terminating decimals. Existence of non-rational numbers (irrational numbers) such as $\sqrt{2}, \sqrt{3}$ and their representation on the number line. Explaining that every real number is represented by a unique point on the number line and conversely, viz. every point on the number line represents a unique real number.
3. Existence of $\sqrt{x}$ for a given positive real number x and its representation on the number line with geometric proof.
4. Definition of nth root of a real number.
5. Recall of laws of exponents with integral powers. Rational exponents with positive real bases (to be done by particular cases, allowing learner to arrive at the general laws.)
6. Rationalization (with precise meaning) of real numbers of the type $\frac{1}{a + b\sqrt{x}}$ and $\frac{1}{\sqrt{x}+\sqrt{y}}$ (and their combinations) where x and y are natural number and a and b are integers.

UNIT-II : ALGEBRA

1. POLYNOMIALS (23) Periods

Definition of a polynomial in one variable, with examples and counter examples. Coefficients of a polynomial, terms of a polynomial and zero polynomial. Degree of a polynomial. Constant, linear, quadratic and cubic polynomials. Monomials, binomials, trinomials. Factors and multiples. Zeros of a polynomial. Motivate and state the Remainder Theorem with examples. Statement and proof of the Factor Theorem. Factorization of $ax^2 + bx + c$, $a \neq 0$ where a, b and c are real numbers, and of cubic polynomials using the Factor Theorem.

Recall of algebraic expressions and identities. Verification of identities:

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$, $(x \pm y)^3 = x^3 \pm y^3 \pm 3xy(x \pm y)$, $x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$ and their use in factorization of polynomials.

UNIT-III : GEOMETRY

1. INTRODUCTION TO EUCLID'S GEOMETRY (6) Periods

History – Geometry in India and Euclid's geometry. Euclid's method of formalizing observed phenomenon into rigorous mathematics with definitions, common/obvious notions, axioms/postulates and theorems. The five postulates of Euclid. Equivalent versions of the fifth postulate. Showing the relationship between axiom and theorem, for example:

(Axiom) 1. Given two distinct points, there exists one and only one line through them.

(Theorem) 2. (Prove) Two distinct lines cannot have more than one point in common.

2. LINES AND ANGLES (13) Periods

1. (Motivate) If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and the converse.
2. (Prove) If two lines intersect, vertically opposite angles are equal.
3. (Motivate) Results on corresponding angles, alternate angles, interior angles when a transversal intersects two parallel lines.
4. (Motivate) Lines which are parallel to a given line are parallel.
5. (Prove) The sum of the angles of a triangle is 180°.
6. (Motivate) If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

3. TRIANGLES (20) Periods

1. (Motivate) Two triangles are congruent if any two sides and the included angle of one triangle is equal to any two sides and the included angle of the other triangle (SAS Congruence).
2. (Prove) Two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle (ASA Congruence).
3. (Motivate) Two triangles are congruent if the three sides of one triangle are equal to three sides of the other triangle (SSS Congruence).
4. (Motivate) Two right triangles are congruent if the hypotenuse and a side of one triangle are equal (respectively) to the hypotenuse and a side of the other triangle.
5. (Prove) The angles opposite to equal sides of a triangle are equal.
6. (Motivate) The sides opposite to equal angles of a triangle are equal.
7. (Motivate) Triangle inequalities and relation between 'angle and facing side' inequalities in triangles.

UNIT-IV : COORDINATE GEOMETRY

1. COORDINATE GEOMETRY (6) Periods

The Cartesian plane, coordinates of a point, names and terms associated with the coordinate plane, notations, plotting points in the plane.

UNIT-V : MENSURATION

1. AREAS (4) Periods

Area of a triangle using Heron's formula (without proof) and its application in finding the area of a quadrilateral.

QUESTIONS PAPER DESIGNS
CLASS-IX

MATHEMATICS (CODE NO. 041) Time-3 Hours Marks-90

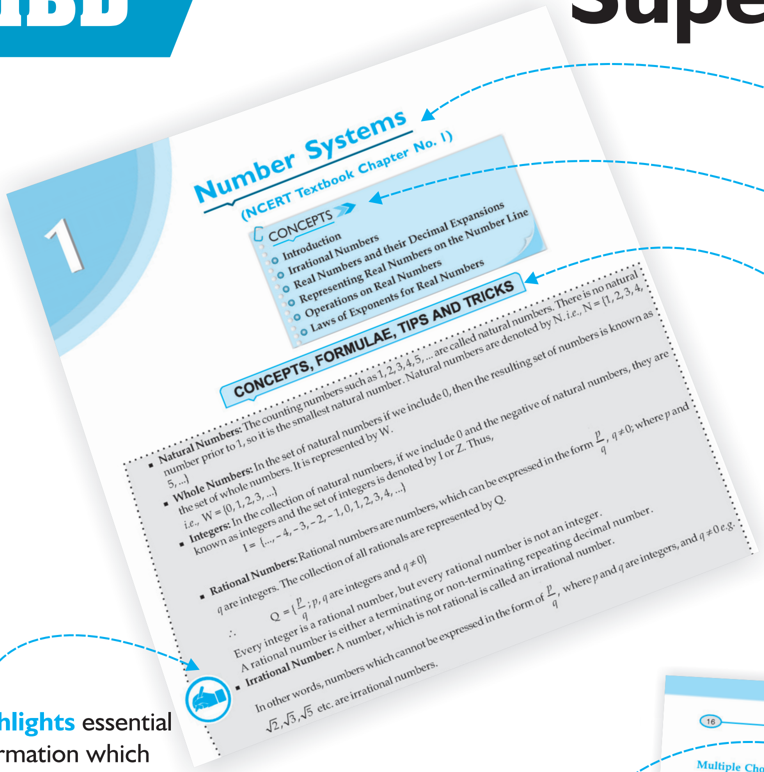
S. No.	Typology of Questions	Very Short Answer (VSA) (1 Mark)	Short Answer-I (SA) (2 Marks)	Short Answer-II (SA) (3 Marks)	Long Answer (LA) (4 Marks)	Total Marks	% Weightage
1	**Remembering-(Knowledge based** Simple recall questions, to know specific facts, terms, concepts, principles, or theories; Identify, define, or recite, information)	1	2	2	3	23	26%
2	**Understanding-(Comprehension**–to be familiar with meaning and to understand conceptually, interpret, compare, contrast, explain, paraphrase, or interpret information)	2	1	1	4	23	26%
3	**Application** (Use abstract information in concrete situation, to apply knowledge to new situations; Use given content to interpret a situation, provide an example, or solve a problem)	1	2	3	2	22	24%
4	**High Order Thinking Skills** (**Analysis & Synthesis**–Classify, compare, contrast, or differentiate between different pieces of information; Organize and/or integrate unique pieces of information from a variety of sources)	–	1	4	–	14	16%
5	**Creating, Evaluation and Multi-Disciplinary**–(Generating new ideas, product or ways of viewing things Appraise, judge, and/or justify the value or worth of a decision or outcome, or to predict outcomes based on values)	–	–	–	2*	8	8%
	Total	4 × 1 = 4	6 × 2 = 12	10 × 3 = 30	11 × 4 = 44	90	100%

Note: The question paper will include a section on Open Text Based Assessment (Questions of 10 marks). The case studies will be supplied to students in advance. These case studies are designed to test the analytical and higher order thinking skills of students.

*One of the LA (4 marks) will to assess the values inherent in the texts.

MBD

Super Refresher

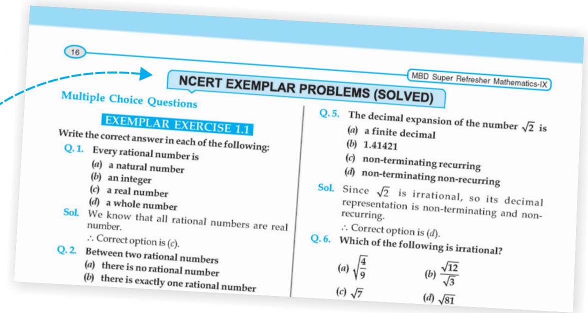


All chapters as per NCERT Syllabus and Textbook

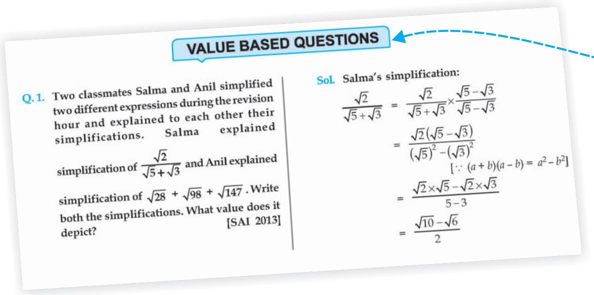
Every chapter divided into Sub-topics

Concepts, Formulae, Tips and Tricks provides a comprehensive summary of the concept

Highlights essential information which must be remembered

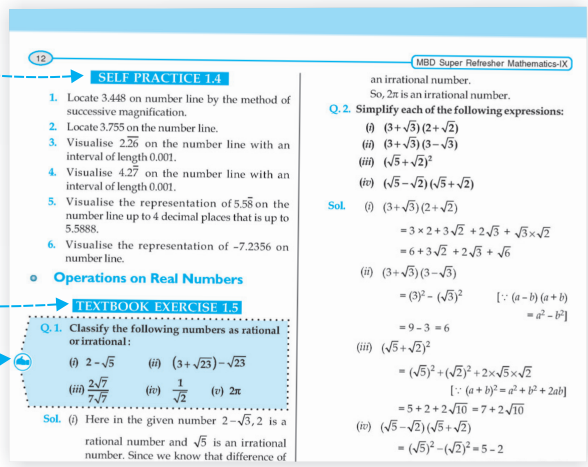


NCERT Exemplar Problems with complete solution to supplement the entire NCERT support material



Value Based Questions to apply mathematical concepts to real life situations with stress on social values

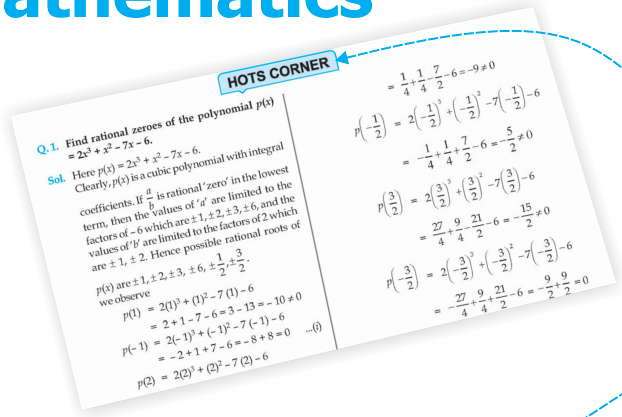
Self Practice questions for consolidation of each concept



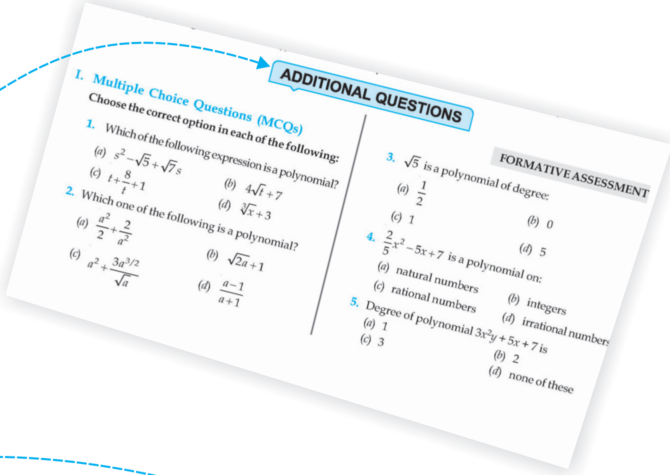
NCERT Textbook Exercises with detailed solution

Important Questions from examination point of view to ensure passing marks

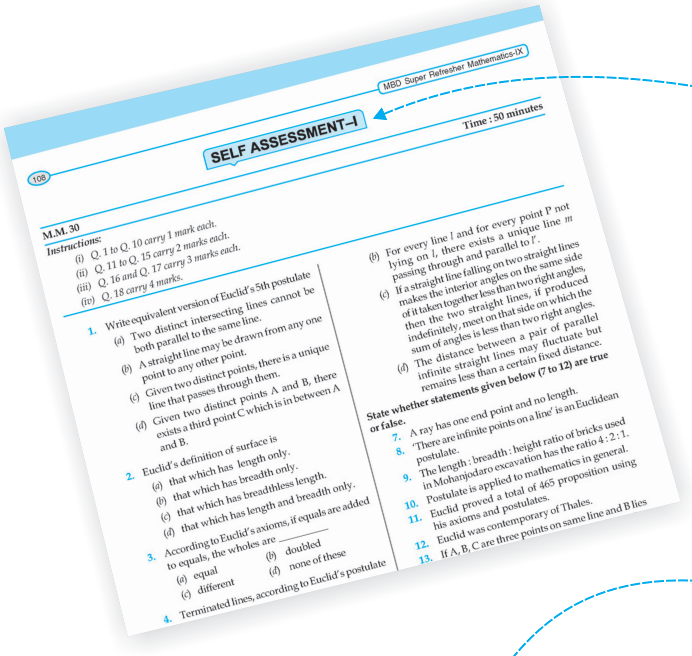
Mathematics



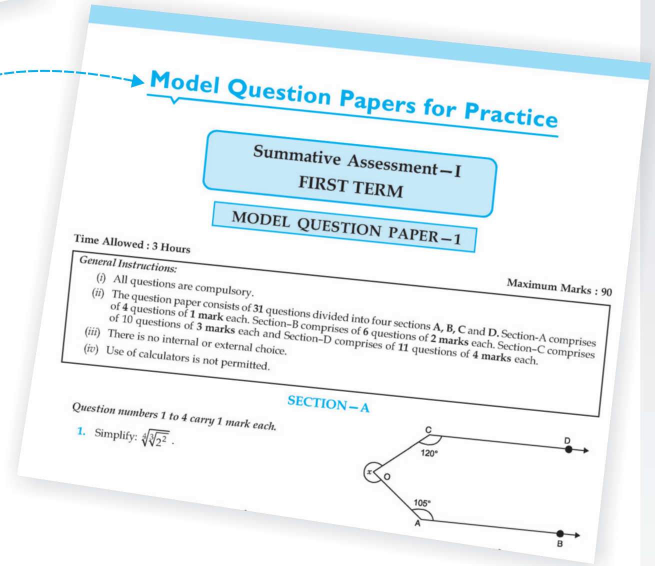
HOTS (Higher Order Thinking Skills)
Questions with answers



Additional Questions with
answers at the end of each chapter



Self Assessment with answers
at the end of each chapter



3 **Model Question Papers**
of 90 marks each

C K

CONTENTS

1

Number Systems

(NCERT Textbook Chapter No. 1)

CONCEPTS

- Introduction
- Irrational Numbers
- Real Numbers and their Decimal Expansions
- Representing Real Numbers on the Number Line
- Operations on Real Numbers
- Laws of Exponents for Real Numbers

CONCEPTS, FORMULAE, TIPS AND TRICKS

- **Natural Numbers:** The counting numbers such as 1, 2, 3, 4, 5, ... are called natural numbers. There is no natural number prior to 1, so it is the smallest natural number. Natural numbers are denoted by N. *i.e.,* N = {1, 2, 3, 4, 5, ...}
- **Whole Numbers:** In the set of natural numbers if we include 0, then the resulting set of numbers is known as the set of whole numbers. It is represented by W.
  i.e., W = {0, 1, 2, 3, ...}
- **Integers:** In the collection of natural numbers, if we include 0 and the negative of natural numbers, they are known as integers and the set of integers is denoted by I or Z. Thus,
  I = {..., – 4, – 3, – 2, – 1, 0, 1, 2, 3, 4, ...}
- **Rational Numbers:** Rational numbers are numbers, which can be expressed in the form $\frac{p}{q}$, $q \neq 0$; where p and q are integers. The collection of all rationals are represented by Q.
  $$\therefore \quad Q = \{\frac{p}{q};\, p, q \text{ are integers and } q \neq 0\}$$
  Every integer is a rational number, but every rational number is not an integer.
  A rational number is either a terminating or non-terminating repeating decimal number.
- **Irrational Number:** A number, which is not rational is called an irrational number.
  In other words, numbers which cannot be expressed in the form of $\frac{p}{q}$, where p and q are integers, and $q \neq 0$ *e.g.* $\sqrt{2}, \sqrt{3}, \sqrt{5}$ etc. are irrational numbers.
  The set of irrational numbers is denoted by S.
  π is ratio of circumference to diameter of a circle and it is an irrational number. $\frac{22}{7}$ or 3.142 is just an approximate value of π.
- **Decimal Representation of a Rational Number:** A rational number is either a terminating decimal or a non-terminating but recurring (repeated) decimal.
  In other words, a terminating decimal or a non-terminating but recurring decimal is a rational number.
  Recurring decimals are also expressed as below:
  (*i*) $0.555 \ldots = 0.\overline{5}$ (*ii*) $0.686868 \ldots = 0.\overline{68}$
  (*iii*) $0.12361236 \ldots = 0.\overline{1236}$ (*iv*) $0.892929292 \ldots = 0.8\overline{92}$

Note: A rational number when expressed in lowest terms having factors 2 or 5 or both in the denominator can be expressed as a terminating decimal otherwise a non-terminating recurring decimal.

- **Decimal Representation of Irrational Number:** The decimal expansion of an irrational number is non-terminating, non-recurring. Moreover, a number whose decimal expansion is non-terminating, non-recurring is irrational number.
- **Insertion of rational numbers between two rationals:** There lies infinite rational numbers between any two rationals.

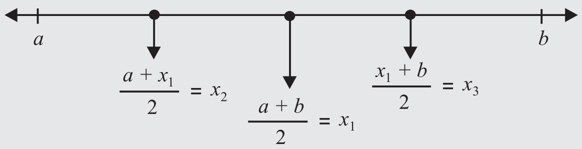
Illustration: A rational number between a and b is given by $\frac{a+b}{2}$.

i.e., $a < \frac{a+b}{2} < b$

Writing $\frac{a+b}{2}$ as x_1.

A rational number between a and x_1 is $\frac{a+x_1}{2}$.

Therefore, we can insert a rational number $\frac{a+x_1}{2} = x_2$ between a and x_1.



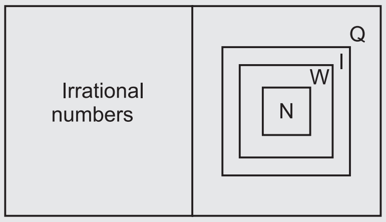
Again between x_1 and b we can insert a rational number $\frac{x_1+b}{2} = x_3$.

Thus, $a < x_2 < x_1 < x_3 < b$.

Repeating the process again and again we can show that **there exists an infinite number of rational numbers $x_2, x_1, x_3, \ldots$ between any two rational numbers a and b so that $a < \ldots < x_2 < x_1 < x_3 < \ldots < b$.**

This is called **denseness property of rational numbers.**

- **Real Numbers:** The collection of all rational numbers and irrational numbers taken together form a collection of real numbers. It can be understood through figure shown below:



Real Numbers

- Every point on number line corresponds to a real number and vice-versa.
- If r_1 and r_2 are any two rational numbers then $r_1 + r_2$, $r_1 - r_2$, $r_1 \times r_2$, $\frac{r_1}{r_2}$ (provided $r_2 \neq 0$) are rational numbers.
- If n is a natural number other than a perfect square then $\sqrt{n}$ is an irrational number.
- If r is a rational number and s is an irrational number, then $r + s$, $r - s$, $r \times s$ and $\frac{r}{s}$ $(s \neq 0)$ are irrational numbers.
- An irrational number between a and b is $\sqrt{ab}$. If ab is not a perfect square.
- For every positive real number x, $\sqrt{x}$ can be represented by a point on the number line by using following steps:
  - (*i*) Draw a ray AX.
  - (*ii*) Mark another point Q such that AQ = x units.
  - (*iii*) Mark point R such that QR = 1 unit.

(*iv*) Find the mid-point of AR and mark it as O.

(*v*) Draw semicircle of radius OR centered at O.

(*vi*) Draw a line perpendicular to AX passing through Q and intersecting the semicircle at S.

(*vii*) With Q as centre and QS as radius draw an arc cutting AX at T. Then, QS = QT = $\sqrt{x}$.

- Another way of representing real numbers on real number line is through *process of successive magnification*. In this method we successively decrease the lengths of the intervals in which given number lies.
- For positive real numbers a and b:

(*i*) $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$ (*ii*) $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

(*iii*) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a - b$ (*iv*) $(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$

(*v*) $(\sqrt{a}+\sqrt{b})^2 = a + 2\sqrt{ab} + b$

- If m and n are rational numbers and a is a positive real number, then

(*i*) $a^m \,.\, a^n = a^{m+n}$ (*ii*) $(a^m)^n = a^{mn}$

(*iii*) $\dfrac{a^m}{a^n} = a^{m-n}$ (*iv*) $a^m b^m = (ab)^m$

(*v*) $a^{-m} = \dfrac{1}{a^m}$ (*vi*) $(a^m)^{1/n} = (a^{1/n})^m = a^{m/n}$

(*vii*) $\dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$

- The principal n^{th} root of positive real number y is denoted by $\sqrt[n]{y}$ or $y^{\frac{1}{n}}$.

The sign $\sqrt{\ }$ is called radical sign for $y > 0$, if $x = \sqrt[q]{y} = y^{\frac{1}{q}}$, then $y^{\frac{1}{q}}$ is the exponential form and $\sqrt[q]{y}$ is the radical form. Here q is the index of radical and is always positive.

- **Rationalising the denominator:** A number is easy to handle if its denominator is a rational number. We generally remove an irrational number from the denominator by certain methods which are explained in the examples ahead:
- **Rationalising Factor (RF):** When product of two irrational numbers is a rational number then each of them is called **Rationalising Factor** of the other.

Illustration: $2\sqrt{7} \times \sqrt{7} = 2(\sqrt{7})^2 = 2 \times 7 = 14$, (a rational number)

$\therefore$ $\sqrt{7}$ is rationalising factor of $2\sqrt{7}$ and vice-versa.

- If a and b are two rational numbers which are not perfect squares, then irrational numbers $\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$ are said to be **conjugate** to each other.
- The product of two conjugate irrational numbers is always a rational number as

$$(\sqrt{a}+\sqrt{b}) \times (\sqrt{a}-\sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$$

$\therefore$ Rationalising factor of: $(\sqrt{a}+\sqrt{b})$ is $(\sqrt{a}-\sqrt{b})$

Similarly, rationalising factor of $(\sqrt{a}-\sqrt{b})$ is $(\sqrt{a}+\sqrt{b})$.

NCERT TEXTBOOK EXERCISE (SOLVED)

Introduction

TEXTBOOK EXERCISE 1.1

Q. 1. Is zero a rational number? Can you write it in the form p/q, where p and q are integers and $q \neq 0$?

Sol. Yes, zero is a rational number and it can be written in the form of $\frac{p}{q}$ as $0 = \frac{0}{1}$, where $p = 0, q = 1 \neq 0$.

Infact q can be any non-zero number *viz.* 2, 3, 4, ...

Q. 2. Find six rational numbers between 3 and 4.

Sol. Let $a = 3$ and $b = 4$

Rational number lying between 3 and 4 is $\frac{a+b}{2}$

$$= \frac{3+4}{2} = \frac{7}{2}$$

Now rational number between 3 and $\frac{7}{2}$

$$\therefore \quad = \frac{3+\frac{7}{2}}{2} = \frac{\frac{6+1}{2}}{2} = \frac{13}{4}$$

Rational number between 4 and $\frac{7}{2}$

$$= \frac{4+\frac{7}{2}}{2} = \frac{\frac{8+7}{2}}{2} = \frac{15}{4}$$

Rational number between $\frac{7}{2}$ and $\frac{13}{4}$

$$= \frac{\frac{7}{2}+\frac{13}{4}}{2} = \frac{\frac{14+13}{4}}{2} = \frac{27}{8}$$

Rational number between $\frac{7}{2}$ and $\frac{15}{4}$

$$= \frac{\frac{7}{2}+\frac{15}{4}}{2} = \frac{\frac{14+15}{4}}{2} = \frac{29}{8}$$

Rational number between $\frac{15}{4}$ and 4

$$= \frac{\frac{15}{4}+4}{2} = \frac{\frac{15+16}{4}}{2} = \frac{31}{8}$$

So, six rational number between 3 and 4 are $\frac{13}{14}, \frac{27}{8}, \frac{7}{2}, \frac{29}{8}, \frac{15}{4}, \frac{31}{8}$.

ALITER:

We want to find six rational numbers between 3 and 4.

Let $a = 3, b = 4$ and total rational numbers = 6

$$\therefore \quad x_n = a + n\left(\frac{b-a}{\text{Total rational numbers + 1}}\right)$$

If $n = 1$

$$\therefore \quad x_1 = 3 + 1\left(\frac{4-3}{6+1}\right) = \frac{22}{7}$$

$n = 2 \Rightarrow x_2 = \frac{23}{7}$

$n = 3 \Rightarrow x_3 = \frac{24}{7}$

$n = 4 \Rightarrow x_4 = \frac{25}{7}$

$n = 5 \Rightarrow x_5 = \frac{26}{7}$

$n = 6 \Rightarrow x_6 = \frac{27}{7}$

$\therefore$ Six rational numbers are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$.

Note: We know to get n rational numbers between rationals 'a' and 'b'. We divide difference between b and a into '$n + 1$' equal parts then n rationals are $x_n = a + n\left(\frac{b-a}{n+1}\right)$

Q. 3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Sol. We know that $\frac{3}{5} = 0.6, \frac{4}{5} = 0.8.$

Hence, five rational numbers are

0.61, 0.62, 0.63, 0.64, 0.65

ALITER:

We know, $\frac{3}{5} = \frac{3 \times 10}{5 \times 10} = \frac{30}{50}$

$\frac{4}{5} = \frac{4 \times 10}{5 \times 10} = \frac{40}{50}$

$\therefore$ Five rational numbers are $\frac{31}{50}, \frac{32}{50}, \frac{33}{50}, \frac{34}{50}, \frac{35}{50}$.

CAUTION: While making denominators same, numerators of both numbers should be sufficiently large to include 5 integers between them.

Q. 4. State whether the following statements are true or false. Give reasons for your answers.

(*i*) Every natural number is a whole number.

(*ii*) Every integer is a whole number.

(*iii*) Every rational number is a whole number.

Sol. (*i*) **True,** since, collection of whole numbers contains all natural numbers.

(*ii*) **False,** as every integer is not a whole number, because negative integers are not whole numbers.

(*iii*) **False,** as every rational number is not a whole number, because rational numbers expressed in fractions like $\frac{5}{7}, \frac{2}{5}, \frac{1}{5}$ etc. are not whole numbers.

SELF PRACTICE 1.1

1. Represent the following rational numbers on number line:

(*i*) $\frac{1}{7}$ (*ii*) $\frac{3}{8}$ (*iii*) $-\frac{8}{9}$ (*iv*) $-\frac{3}{12}$

2. Represent the following rational numbers on number line:

(*i*) $3\frac{5}{7}$ (*ii*) $-2\frac{1}{5}$ (*iii*) $4\frac{3}{8}$ (*iv*) $1\frac{1}{3}$

3. Find a rational number between 3 and 4.
4. Find a rational number between $\frac{1}{2}$ and $\frac{3}{4}$.
5. Find five rational numbers between 3 and 4.
6. Find five rational numbers between $-\frac{2}{3}$ and $-\frac{1}{3}$.
7. Find 16 rational numbers between 2.1 and 2.2.
8. Find 24 rational numbers between 0 and 0.1.
9. State whether the following statements are true or not. Also justify your answer.

(*i*) Some rational numbers are integers.

(*ii*) All integers are natural numbers.

(*iii*) All whole numbers are natural numbers.

(*iv*) All natural numbers need not be whole numbers.

(*v*) 0 is the smallest natural number.

(*vi*) 1 is the smallest natural number.

(*vii*) $-\frac{3}{7}$ is an integer.

(*viii*) 0 is a rational number.

(*ix*) Every negative integer is a rational number.

(*x*) If rational number $\frac{p}{q}$ is not an integer then q can't be ± 1.

(*xi*) There exists an infinite number of integers between two given integers.

(*xii*) $-\frac{3}{8}$ lies to the left of 0 on number line.

(*xiii*) There are infinitely many rational numbers between two given rational numbers.

(*xiv*) If denominator of a rational number is –1, it must be an integer.

(*xv*) Among the collection of whole numbers, integers and natural numbers, collection of integers is the biggest collection.

(*xvi*) –500000000 can't be represented on number line.

(*xvii*) Integers can be positive, negative or zero.

(*xviii*) Every rational number is a whole number.

(*xix*) For every natural number there exist a natural number one more than it.

10. Justify why $\frac{22}{7}$ is rational yet π is not rational.

Irrational Numbers

TEXTBOOK EXERCISE 1.2

Q. 1. State whether the following statements are true or false. Justify your answer:

(*i*) Every irrational number is a real number.

(*ii*) Every point on the number line is of the form $\sqrt{m}$, where *m* is a natural number.

(*iii*) Every real number is an irrational number.

Sol. (*i*) **True.**

Justification: Because set of rational and irrational numbers constitute the set of real numbers. In other words all irrational numbers are real numbers. Hence, the statement 'every irrational number is a real number' is true.

(*ii*) **False.**

Justification: – 3, – 5, – 9 are all real numbers but none of these is square root of any natural number.

(*iii*) **False.**

Justification: 5, 7, 8, 11 are all real numbers but none is irrational.

Q. 2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Sol. No, because $\sqrt{16}$ = 4, which is rational and is therefore not irrational.

Q. 3. Show how $\sqrt{5}$ can be represented on the number line.

Sol. For $\sqrt{5}$

$\because\ 5 = 2^2 + 1^2$

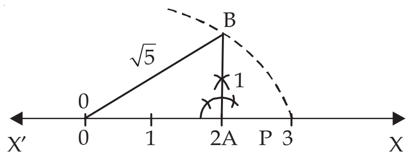
$\therefore$ We can construct $\sqrt{5}$ as the length of hypotenuse of a right triangle whose sides are of lengths 2 and 1 unit.

Let X′OX be a number line on which O represents 0 and A represents 2 units length. Draw a line AB ⊥ OA and mark point B on it so that AB = 1 unit.

Then $OB^2 = OA^2 + AB^2$

$= 2^2 + 1^2 = 4 + 1 = 5$

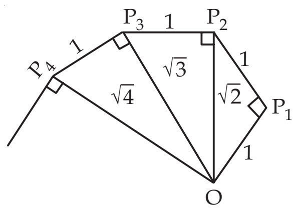
$\Rightarrow\ OB = \sqrt{5}$



Using a compass with centre O and radius OB we mark a point P corresponding to $\sqrt{5}$ on the number line.

Thus P represents the irrational number $\sqrt{5}$.

Q. 4. Classroom activity (Constructing the 'square root spiral'): Take a large sheet of paper and construct the 'square root spiral' in the following fashion. Start with a point O and draw a line segment OP_1 of unit length. Draw a line segment P_1P_2 perpendicular to OP_1 of unit length [see figure]. Now draw a line segment P_2P_3 perpendicular to OP_2. Then draw a line segment P_3P_4 perpendicular to OP_3. Continuing in this manner, you can get the line segment $P_{n-1}P_n$ by drawing a line segment of unit length perpendicular to OP_{n-1}. In this manner, you will have created the points $P_2, P_3, ..., P_n, ...$ and joined them to create a beautiful spiral depicting $\sqrt{2}, \sqrt{3}, \sqrt{4}, ...$



SELF PRACTICE 1.2

1. Classify the following numbers as rational or irrational:

(i) $-\sqrt{2}$ (ii) $\frac{\pi}{3}$

(iii) $(2+\sqrt{3})^3$ (iv) $\sqrt{4}$

(v) 0.34534534512345..... (vi) $2.5\overline{76}$

(vii) $5\sqrt{7}$ (viii) 5.7

(ix) $5 + 2\sqrt{225}$ (x) $\frac{\sqrt{5}}{9}$

(xi) 1.3758758758... (xii) 0.8888...

2. Write two different irrational numbers between the rational number $\frac{4}{7}$ and $\frac{9}{11}$.

3. Find two irrational numbers between 0.616116111611116....... and 0.6835452935.

4. Find two irrational numbers between $\sqrt{3}$ and $\sqrt{5}$.

5. Insert a rational and an irrational number between the numbers 2.301221222122221... and 2.3306.

6. Locate $\sqrt{3}$ on number line.

7. Locate $\sqrt{17}$ on number line.

8. Locate $\sqrt{13}$ on number line.

9. Represent $\sqrt{11}$ on number line.

10. State in each case, whether the given statement is true or false:
    (i) All irrational numbers are rational numbers.
    (ii) π is a rational number.
    (iii) Difference of two irrational numbers is not always irrational.
    (iv) Every real number is always rational or irrational.
    (v) Between 32.1 and 32.3 there are two irrational numbers only.
    (vi) Square root of every positive integer is irrational.
    (vii) Irrational numbers cannot be represented by points on the number line.
    (viii) Product of two irrational numbers may or may not be irrational.

11. Give an example, each of two irrational numbers whose:
    (i) sum is a rational number.
    (ii) sum is an irrational number.
    (iii) difference is a rational number.
    (iv) difference is an irrational number.
    (v) quotient is an irrational number.
    (vi) product is an irrational number.
    (vii) product is a rational number.
    (viii) quotient is a rational number.

MBD CBSE Mathematics Super Refresher Class-9 Term-1