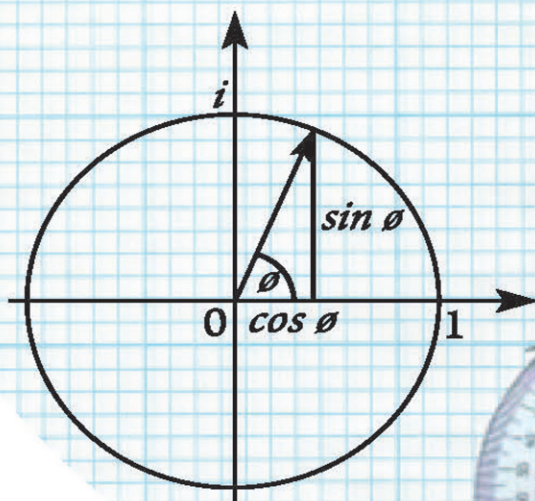


MBD



Mathematics

12

- **NCERT** Textbook Exercises and Exemplar Problems with solutions
- Comprehensive explanation of each chapter
- Large pool of very short, short and Long answer type questions



Highlights important information which must be remembered

Includes **HOTS** and **Value Based** questions

Based on the latest syllabus and textbook(s) issued by CBSE/NCERT

MBD

**Super
Refresher**

Mathematics

12

J.P. Mohindru
&
Bharat Mohindru

Based on the latest syllabus and
textbook(s) issued by **CBSE/NCERT**

MALHOTRA BOOK DEPOT
(Producers of Quality **MBD** Books)
An ISO 9001:2008 Certified Company

Price : ₹ 750.00

OUR ADDRESSES IN INDIA

❑ New Delhi : MBD House, Gulab Bhawan, 6, Bahadur Shah Zafar Marg	Ph. 30912330, 30912301, 23318301
❑ Mumbai : A-683, T.T.C. Industrial Area, M.I.D.C. Off. Thane-Belapur Road, Navi Mumbai	Ph. 32996410, 27780821, 8691053365
❑ Chennai : No. 26 B/2 SIDCO Estate, North Phase, Pataravakkam Ambattur Industrial Estate, Ambattur	Ph. 26359376, 26242350
❑ Chennai : Plot No. 3018, Old Y Block, 3rd Street, 12th Main Road, Anna Nagar West	Ph. 23741471
❑ Kolkata : Satyam Building, 46-D, Rafi Ahmed Kidwai Marg	Ph. 22296863, 22161670
❑ Jalandhar City : MBD House, Railway Road	Ph. 2458388, 2457160, 2455663
❑ Bengaluru : 124/31, 1st Main, Industrial Town (Near Chowdeshwari Kalyan Mantap), West of Chord Road, Rajajinagar	Ph. 23103329, 23104667
❑ Hyderabad : 3-4-492, Varun Towers, Barkatpura	Ph. 27564788, 9985820001
❑ Ernakulam : Surabhi Building, South Janatha Road, Palarivattom	Ph. 2338107, 2347371
❑ Pune : Survey No. 44, Behind Matoshree Garden, Kondhwa - Khadi Machine, Pisoli Road, At. Post-Pisoli	Ph. 65271413, 65275071
❑ Nagpur : Near N.I.T. Swimming Pool, North Ambazari Road, Ambazari Layout	Ph. 2248104, 2248106, 2248649, 2245648
❑ Ahmedabad : Godown No.10, Vedant Prabha Estate, Opp. ONGC Pumping Station, Sarkhej Sanand Road, Sarkhej	Ph. 26890336, 32986505
❑ Cuttack : Badambadi, Link Road	Ph. 2367277, 2367279, 2313013
❑ Guwahati : Chancellor Commercial, Hem Baruah Road, Paan Bazar	Ph. 2131476, 882285385
❑ Lucknow : 173/15, Dr. B. N. Verma Road, Old 30 Kutchery Road	Ph. 4010992, 4010993
❑ Patna : 1st Floor, Annapurna Complex, Naya Tola	Ph. 2672732, 2686994, 2662472
❑ Bhopal : Plot No. 137, 138, 139, Sector-I, Special Industrial Area, Govindpura	Ph. 2581540, 2601535
❑ Jabalpur : 840, Palash Chamber, Malviya Chowk	Ph. 2405854
❑ Goa : H. No. 932, Plot No. 66, Kranti Nagar (Behind Azad Bhawan), Alto Porvorim, Bardez	Ph. 2413982, 2414394
❑ Jaipur : C-66A, In front of Malpani Hospital, Road No.1, V.K. Industrial Area, Sikar Road	Ph. 4050309, 4020168
❑ Raipur : Behind Kailash Provision Store, Ravi Nagar	Ph. 4052529, 2445370
❑ Karnal : Plot No. 203, Sector-3, HSIDC, Near Namaste Chowk, Opp. New World	Ph. 2220006, 2220009
❑ Shimla (H.P.) : C-89, Sector-I, New Shimla-9	Ph. 2670221, 2670618
❑ Jammu (J&K) : MBD Office, 48 Gujjar Colony, C/o Gurjar Desh Charitable Trust, N.H. Bye Pass Road	Ph. 2467376, 9419104035
❑ Ranchi (Jharkhand) : Shivani Complex, 2nd Floor, Jyoti Sangam Lane, Upper Bazar	Ph. 9431257111
❑ Sahibabad (U.P.) : B-9 & 10, Site IV, Industrial Area	Ph. 3100045, 2896939
❑ Dehradun (Uttarakhand) : Plot No. 37, Bhagirathipuram, Niranjapur, GMS Road	Ph. 2520360, 2107214
DELHI LOCAL OFFICES:	
❑ Delhi (Shakarapur) : MB 161, Street No. 4	Ph. 22546557, 22518122
❑ Delhi (Daryaganj) : MBD House, 4587/15, Opp. Times of India	Ph. 23245676
❑ Delhi (Patparganj) : Plot No. 225, Industrial Area	Ph. 22149691, 22147073

MBD BOOKS FOR XII (C.B.S.E.)

❑ MBD Super Refresher English (Core)	❑ MBD Super Refresher Political Science (Contemporary World Politics & Politics in India Since Independence)
❑ MBD Super Refresher Hindi (Elective & Core)	❑ MBD Super Refresher History (Themes in Indian History)
❑ MBD Super Refresher Physics	❑ MBD Super Refresher Geography
❑ MBD Super Refresher Chemistry	❑ MBD Super Refresher Sociology
❑ MBD Super Refresher Biology	❑ MBD Super Refresher Punjabi Guide
❑ MBD Super Refresher Mathematics	❑ MBD Super Refresher Physical Education
❑ MBD Super Refresher Accountancy	
❑ MBD Super Refresher Business Studies	
❑ MBD Super Refresher Economics (Introductory Micro & Macro Economics)	

We are committed to serve students with best of our knowledge and resources. We have taken utmost care and attention while editing and printing this book but we would beg to state that Authors and Publishers should not be held responsible for unintentional mistakes that might have crept in. However, errors brought to our notice, shall be gratefully acknowledged and attended to.

© All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise without the prior written permission of the publisher. Any breach will entail legal action and prosecution without further notice.

Published by: MALHOTRA BOOK DEPOT
MBD House, Railway Road, Jalandhar City.

Printed at: HOLY FAITH INTERNATIONAL (P) LTD.
B-9 & 10, Site-IV, Industrial Area, Sahibabad (U.P.)

SYLLABUS

MATHEMATICS

CLASS–XII

(2016–17)

Paper I

100 Marks
3 Hours

Units		Periods	Marks
I.	Relations and Functions	30	10
II.	Algebra	50	13
III.	Calculus	80	44
IV.	Vectors and Three-Dimensional Geometry	30	17
V.	Linear Programming	20	06
VI.	Probability	30	10
	Total	240	100

Unit 1: Relations and Functions

I. Relations and Functions (15 Periods)

Types of relations : reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.

2. Inverse Trigonometric Functions : (15 Periods)

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

Unit 2: Algebra

I. Matrices : (25 Periods)

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew-symmetric matrices. Operation on matrices: addition and multiplication and multiplication with scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists ; (Here all matrices will have real entries).

2. Determinants : (25 Periods)

Determinant of a square matrix (up to 3×3 matrices), properties of determinants, minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit III: Calculus

1. Continuity and Differentiability : (20 Periods)

Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation..

2. Applications of Derivatives : (10 Periods)

Applications of derivatives : rate of change of bodies, increasing/decreasing functions, tangents and normals, use of derivatives in approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. Integrals : (20 Periods)

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts. Evaluation of simple integrals of the following types and problems based on them:

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$\int \frac{px + q}{ax^2 + bx + c} dx, \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2} dx, \int \sqrt{x^2 - a^2} dx$$

$$\int \sqrt{ax^2 + bx + c} dx, \int (px + q)\sqrt{ax^2 + bx + c} dx$$

Definite integrals as a limit of a sum, Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. Applications of the Integrals : (15 Periods)

Applications in finding the area under simple curves, especially lines, circles/parabolas/ellipses (in standard form only). Area between the two above said curves (the region should be clearly identifiable).

5. Differential Equations : (15 Periods)

Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, solution of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type :

$$\frac{dy}{dx} + py = q, \text{ where } p \text{ and } q \text{ are functions of } x \text{ or constants.}$$

$$\frac{dx}{dy} + px = q, \text{ where } p \text{ and } q \text{ are functions of } y \text{ or constants.}$$

Unit IV: Vectors and Three-Dimensional Geometry.

I. Vectors : (15 Periods)

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and applications of scalar (dot) product of vectors, vector (cross) product of vectors, scalar triple product of vectors.

2. Three-dimensional Geometry : (15 Periods)

Direction cosines and direction ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes, (iii) a line and a plane. Distance of a point from a plane.

Unit V: Linear Programming

I. Linear Programming : (20 Periods)

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded and unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit VI: Probability

I. Probability : (30 Periods)

Conditional probability, multiplication theorem on probability, Independent events, total probability, Baye's theorem, Random variable and its probability distribution, mean and variance of a random variable. Repeated independent (Bernoulli) trials and Binomial distribution.

Prescribed Books:

1. Mathematics Textbook for Class XI NCERT
2. Mathematics Part I - Textbook for Class XII, NCERT
3. Mathematics Part II - Textbook for Class XII, NCERT
4. Mathematics Exemplar Problem for Class XI, NCERT
5. Mathematics Exemplar Problem for Class XII, NCERT

Question Paper Design

CBSE/ACAD/DD(SC)/2016

Dated: 26.07.2016
Circular No. Acad-32/2016

Subject: The question Paper Design in the subject of Mathematics (Class XII) for the Board Examination 2017

The following will be applicable in the subject Mathematics (041) for Class XII for the academic session 2016-17 and Board examination 2017

I. Question Paper Design

S.No.	Typology of Questions	VSA (1 Mark)	SA (2 Marks)	LA-I (4 Marks)	LA-II (6 Marks)	Marks	% Weightage
1	Remembering	2	2	2	1	20	20%
2	Understanding	1	3	4	2	35	35%
3	Application	1	-	3	2	25	25%
4	HOTS	-	3	1	-	10	10%
5	Evaluation	-	-	1 (VBQ)*	1	10	10%
	Total	1×4=4	2×8=16	4×11=44		100	100%

*Value based question of 04 marks

2. Question Wise Break Up

Type of Question	Marks per Question	Total No. of Questions	Total Marks
VSA	1	6	06
SA	2	8	16
LA-I	4	11	44
LA-II	6	6	36
Total		29	100

3. Difficulty level

Difficulty level	WEightage
Easy	20%
Average	60%
Difficult	20%

4. Choice(s):

There will be no overall choice in the question paper. However, 30% internal choices will be given in 4 marks and 6 marks question.

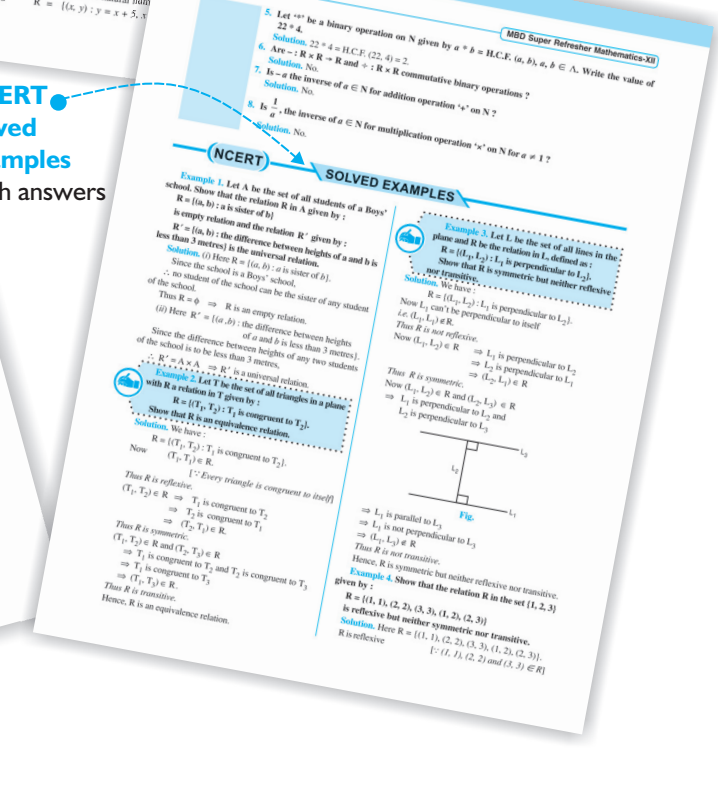
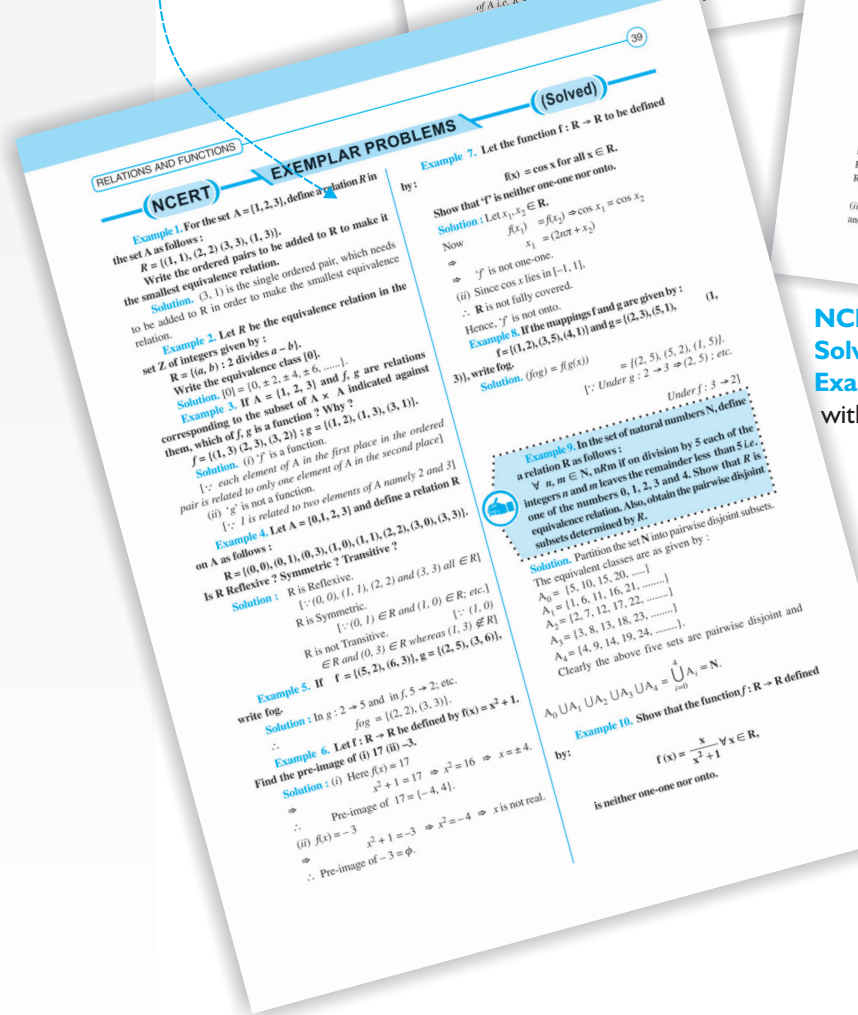
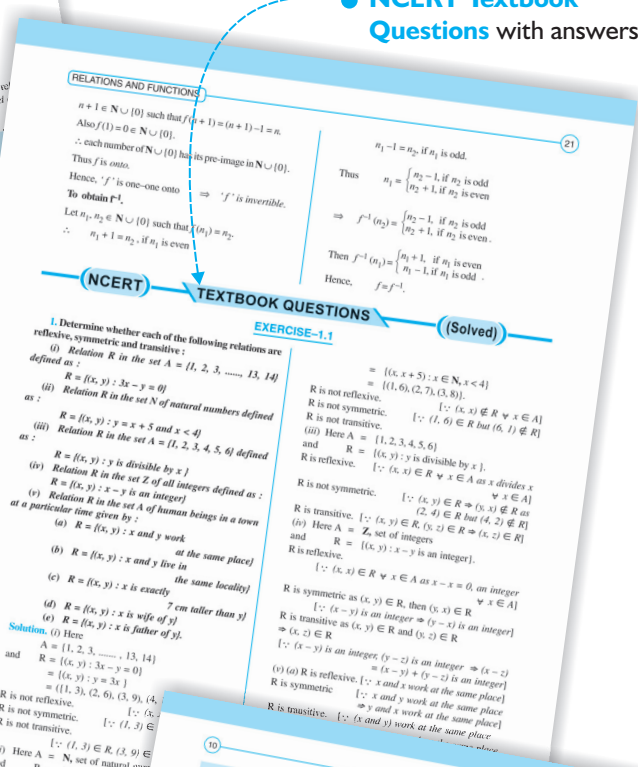
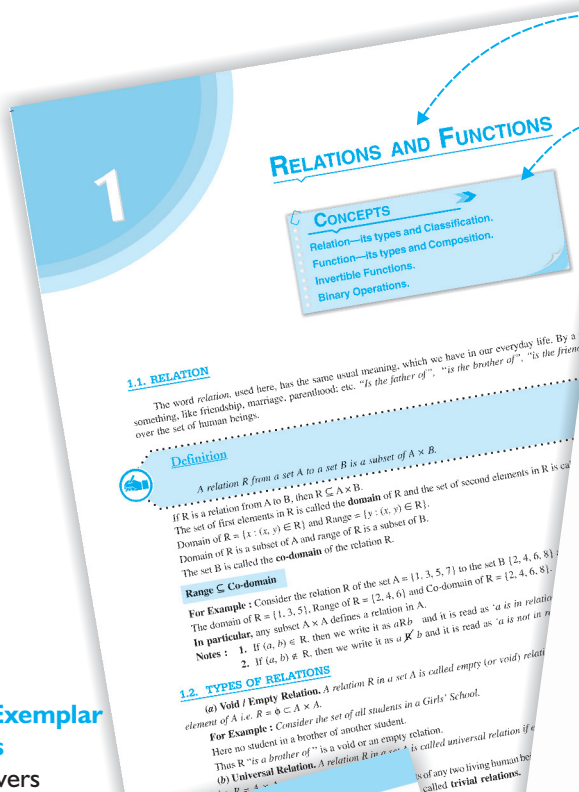
All chapters as per **CBSE** Syllabus and NCERT Textbooks

Every chapter divided into **Sub-topics**

NCERT Textbook Questions with answers

NCERT Exemplar Problems with answers

NCERT Solved Examples with answers



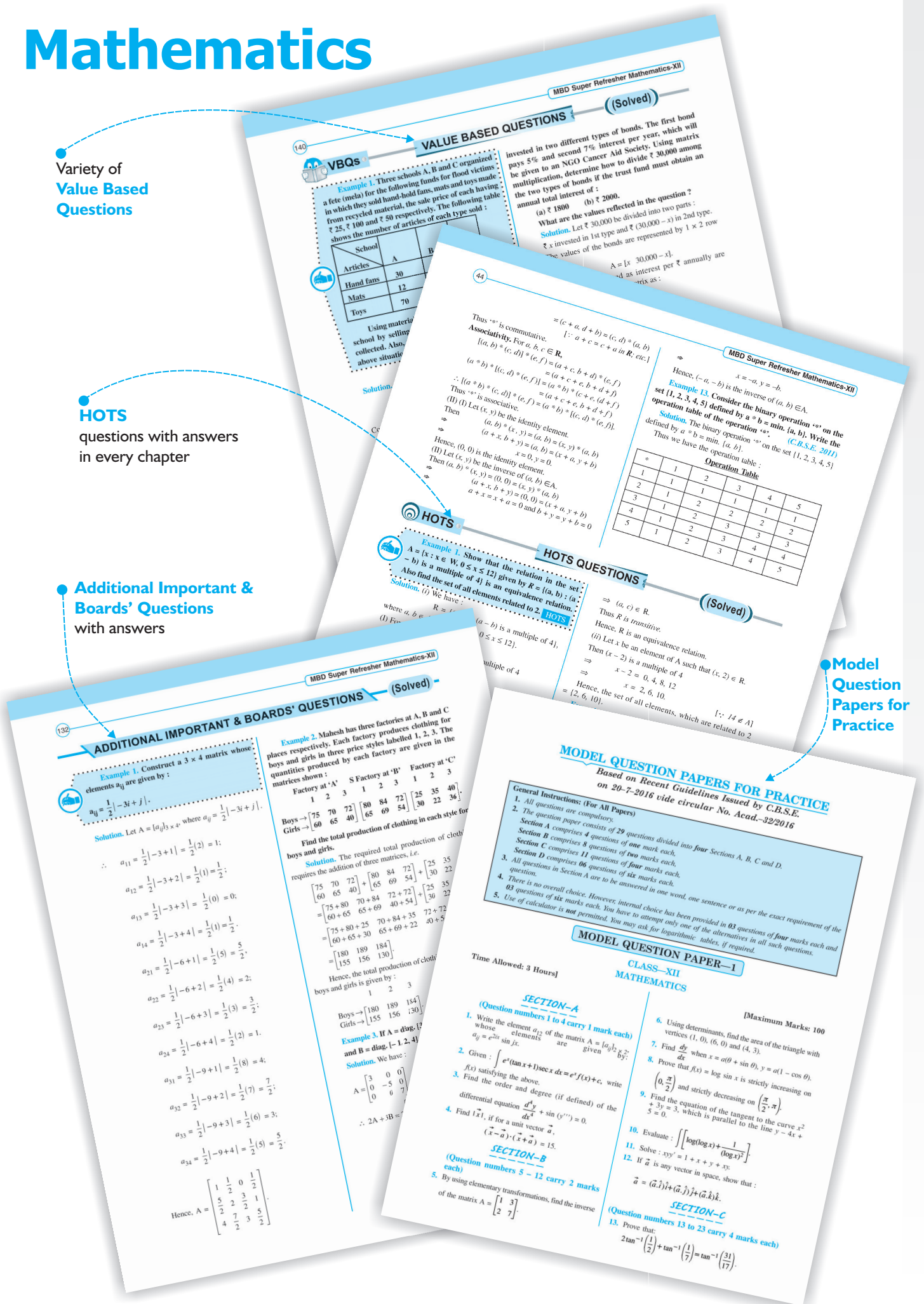
Mathematics

Variety of Value Based Questions

HOTS questions with answers in every chapter

Additional Important & Boards' Questions with answers

Model Question Papers for Practice



MBD Super Refresher Mathematics-XII
VALUE BASED QUESTIONS (Solved)

140 VBQs

Example 1. Three schools A, B and C organized a fete (mela) for the following funds for flood victims in which they sold hand-held fans, mats and toys made from recycled material, the sale price of each having ₹ 25, ₹ 100 and ₹ 50 respectively. The following table shows the number of articles of each type sold:

School	A	B	C
Articles	30	20	10
Hand fans	30	20	10
Mats	12	10	8
Toys	70	60	50

Using matrix multiplication, find the total amount collected by each school by selling the articles. Also, find the total amount collected by all schools together.

invested in two different types of bonds. The first bond pays 5% and second 7% interest per year, which will be given to an NGO Cancer Aid Society. Using matrix multiplication, determine how the trust fund must obtain the two types of bonds if the trust fund must obtain an annual total interest of:

- (a) ₹ 1800 (b) ₹ 2000.

What are the values reflected in the question?

Solution. Let ₹ 30,000 be divided into two parts: ₹ x invested in 1st type and ₹ $(30,000 - x)$ in 2nd type. The values of the bonds are represented by 1×2 row matrix as:

MBD Super Refresher Mathematics-XII
HOTS QUESTIONS (Solved)

Thus \ast is commutative. $\therefore a + c = c + a$ in R ; etc.]
Associativity. For $a, b, c \in R$,
 $[(a \ast b) \ast c] \ast d = (a + c, b + d) \ast d = (a + c + d, b + d + d)$
 $[(a \ast b) \ast (c \ast d)] \ast e = (a + c, b + d) \ast (e, f) = (a + c + e, b + d + f)$
 $(a \ast b) \ast [(c \ast d) \ast e] = (a + c, b + d) \ast (e, f) = (a + c + e, b + d + f)$
 $\therefore [(a \ast b) \ast c] \ast d = (a \ast b) \ast (c \ast d) \ast e$
 $\therefore \ast$ is associative.

(ii) (i) Let (x, y) be the identity element.
 $(a, b) \ast (x, y) = (a, b) = (x + a, y + b)$
 $(a + x, b + y) = (a, b) = (x + a, y + b)$
 $x = 0, y = 0$.
 Hence, $(0, 0)$ is the identity element.
 (ii) Let (x, y) be the inverse of $(a, b) \in A$.
 $(a, b) \ast (x, y) = (0, 0) = (x + a, y + b)$
 $(a + x, b + y) = (0, 0) = (x + a, y + b)$
 $a + x = 0$ and $b + y = 0$
 $x = -a, y = -b$

Hence, $(-a, -b)$ is the inverse of $(a, b) \in A$.
Example 13. Consider the binary operation \ast on the set $\{1, 2, 3, 4, 5\}$ defined by $a \ast b = \min\{a, b\}$. Write the operation table of the operation \ast .
Solution. The binary operation \ast on the set $\{1, 2, 3, 4, 5\}$ defined by $a \ast b = \min\{a, b\}$.
 Thus we have the operation table:

Operation Table:

\ast	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

HOTS

Example 1. Show that the relation in the set $A = \{x : x \in W, 0 \leq x \leq 12\}$ given by $R = \{(a, b) : (a - b) \text{ is a multiple of } 4\}$ is an equivalence relation. Also find the set of all elements related to 2.
Solution. (i) We have $R = \{(a, b) : (a - b) \text{ is a multiple of } 4, 0 \leq x \leq 12\}$.
 (ii) For $(a, c) \in R$,
 $\Rightarrow (a - c) \in R$.
 Thus R is transitive.
 Hence, R is an equivalence relation.
 (iii) Let x be an element of A such that $(x, 2) \in R$.
 $\Rightarrow x - 2$ is a multiple of 4
 $\Rightarrow x - 2 = 0, 4, 8, 12$
 $x = 2, 6, 10$.
 Hence, the set of all elements, which are related to 2 is $\{2, 6, 10\}$.

MBD Super Refresher Mathematics-XII
ADDITIONAL IMPORTANT & BOARDS' QUESTIONS (Solved)

Example 1. Construct a 3×4 matrix whose elements a_{ij} are given by:
 $a_{ij} = \frac{1}{2}(-3i + j)$.

Solution. Let $A = [a_{ij}]_{3 \times 4}$, where $a_{ij} = \frac{1}{2}(-3i + j)$.
 $\therefore a_{11} = \frac{1}{2}(-3 + 1) = \frac{1}{2}(2) = 1$;
 $a_{12} = \frac{1}{2}(-3 + 2) = \frac{1}{2}(1) = \frac{1}{2}$;
 $a_{13} = \frac{1}{2}(-3 + 3) = \frac{1}{2}(0) = 0$;
 $a_{14} = \frac{1}{2}(-3 + 4) = \frac{1}{2}(1) = \frac{1}{2}$;
 $a_{21} = \frac{1}{2}(-6 + 1) = \frac{1}{2}(5) = \frac{5}{2}$;
 $a_{22} = \frac{1}{2}(-6 + 2) = \frac{1}{2}(4) = 2$;
 $a_{23} = \frac{1}{2}(-6 + 3) = \frac{1}{2}(3) = \frac{3}{2}$;
 $a_{24} = \frac{1}{2}(-6 + 4) = \frac{1}{2}(2) = 1$;
 $a_{31} = \frac{1}{2}(-9 + 1) = \frac{1}{2}(8) = 4$;
 $a_{32} = \frac{1}{2}(-9 + 2) = \frac{1}{2}(7) = \frac{7}{2}$;
 $a_{33} = \frac{1}{2}(-9 + 3) = \frac{1}{2}(6) = 3$;
 $a_{34} = \frac{1}{2}(-9 + 4) = \frac{1}{2}(5) = \frac{5}{2}$.

Hence, $A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$.

Example 2. Mahesh has three factories A, B and C respectively. Each factory produces clothing for boys and girls in three price styles labelled 1, 2, 3. The quantities produced by each factory are given in the matrices shown:

	Factory at 'A'			Factory at 'B'			Factory at 'C'		
	1	2	3	1	2	3	1	2	3
Boys \rightarrow	75	70	72	80	84	72	25	35	40
Girls \rightarrow	60	65	40	65	69	54	30	22	36

Find the total production of clothing in each style for boys and girls.

Solution. The required total production of clothing requires the addition of three matrices, i.e.,
 $\begin{bmatrix} 75 & 70 & 72 \\ 60 & 65 & 40 \end{bmatrix} + \begin{bmatrix} 80 & 84 & 72 \\ 65 & 69 & 54 \end{bmatrix} + \begin{bmatrix} 25 & 35 & 40 \\ 30 & 22 & 36 \end{bmatrix}$
 $= \begin{bmatrix} 75+80+25 & 70+84+35 & 72+72+40 \\ 60+65+30 & 65+69+22 & 40+54+36 \end{bmatrix}$
 $= \begin{bmatrix} 180 & 189 & 184 \\ 155 & 156 & 130 \end{bmatrix}$

Hence, the total production of clothing for boys and girls is given by:

Boys \rightarrow	180	189	184
Girls \rightarrow	155	156	130

Example 3. If $A = \text{diag.} \{1, 2, 4\}$ and $B = \text{diag.} \{-1, 2, 4\}$.

Solution. We have:
 $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$
 $B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$
 $\therefore 2A + 3B =$

MODEL QUESTION PAPERS FOR PRACTICE
Based on Recent Guidelines Issued by C.B.S.E. on 20-7-2016 vide circular No. Acad.-32/2016

- General Instructions: (For All Papers)
- All questions are compulsory.
 - The question paper consists of 29 questions divided into four Sections A, B, C and D. Section A comprises 4 questions of one mark each. Section B comprises 8 questions of two marks each. Section C comprises 11 questions of four marks each. Section D comprises 6 questions of six marks each.
 - All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
 - There is no overall choice. However, internal choice has been provided in 03 questions of four marks each and 03 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
 - Use of calculator is not permitted. You may ask for logarithmic tables, if required.

MODEL QUESTION PAPER-1
CLASS-XII
MATHEMATICS

Time Allowed: 3 Hours]

[Maximum Marks: 100

- SECTION-A**
(Question numbers 1 to 4 carry 1 mark each)
- Write the element a_{22} of the matrix $A = [a_{ij}]_{3 \times 2}$, whose elements are given by:
 $a_{ij} = e^{2i} \sin jx$.
 - Given: $\int e^x (\tan x + 1) \sec x \, dx = e^x f(x) + c$, write $f(x)$ satisfying the above.
 - Find the order and degree (if defined) of the differential equation $\frac{d^2y}{dx^2} + \sin(y''') = 0$.
 - Find $\vec{1} \cdot \vec{1}$, if for a unit vector \vec{a} ,
 $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 15$.
- SECTION-B**
(Question numbers 5 - 12 carry 2 marks each)
- By using elementary transformations, find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$.
 - Using determinants, find the area of the triangle with vertices $(1, 0)$, $(6, 0)$ and $(4, 3)$.
 - Find $\frac{dy}{dx}$ when $x = a\theta + \sin \theta$, $y = a(1 - \cos \theta)$.
 - Prove that $f(x) = \log \sin x$ is strictly increasing on $(0, \frac{\pi}{2})$ and strictly decreasing on $(\frac{\pi}{2}, \pi)$.
 - Find the equation of the tangent to the curve $x^2 + 3y = 3$, which is parallel to the line $y - 4x + 5 = 0$.
 - Evaluate: $\int \frac{\log(\log x) + \frac{1}{(\log x)^2}}{x} dx$.
 - Solve: $xyy' = 1 + x + y + xy$.
 - If \vec{a} is any vector in space, show that:
 $\vec{a} = (\vec{a} \cdot \vec{i})\vec{i} + (\vec{a} \cdot \vec{j})\vec{j} + (\vec{a} \cdot \vec{k})\vec{k}$.
- SECTION-C**
(Question numbers 13 to 23 carry 4 marks each)
- Prove that $2 \tan^{-1}(\frac{1}{2}) + \tan^{-1}(\frac{1}{7}) = \tan^{-1}(\frac{31}{17})$.

CONTENTS

1 >>	Relations and Functions	1 – 48
2 >>	Inverse-Trigonometric Functions	49 – 78
3 >>	Matrices	79 – 142
4 >>	Determinants	143 – 222
5 >>	Continuity and Differentiability	223 – 303
6 >>	Application of Derivatives	304 – 394
7 >>	Integrals	395 – 540
8 >>	Applications of the Integrals	541 – 576
9 >>	Differential Equations	577 – 650
10 >>	Vector Algebra	651 – 709
11 >>	Three-Dimensional Geometry	710 – 781
12 >>	Linear Programming	782 – 822
13 >>	Probability	823 – 890
	Model Test Papers for Practice	891 – 905

1

RELATIONS AND FUNCTIONS

CONCEPTS

- Relation—its types and Classification.
- Function—its types and Composition.
- Invertible Functions.
- Binary Operations.

1.1. RELATION

The word *relation*, used here, has the same usual meaning, which we have in our everyday life. By a relation we mean something, like friendship, marriage, parenthood; etc. “*Is the father of*”, “*is the brother of*”, “*is the friend of*” are relations over the set of human beings.

Definition

A relation R from a set A to a set B is a subset of $A \times B$.

If R is a relation from A to B , then $R \subseteq A \times B$.

The set of first elements in R is called the **domain** of R and the set of second elements in R is called the **range** of R .

Domain of $R = \{x : (x, y) \in R\}$ and Range = $\{y : (x, y) \in R\}$.

Domain of R is a subset of A and range of R is a subset of B .

The set B is called the **co-domain** of the relation R .

Range \subseteq Co-domain

For Example : Consider the relation R of the set $A = \{1, 3, 5, 7\}$ to the set $B = \{2, 4, 6, 8\}$ and $R = \{(1, 2), (3, 4), (5, 6)\}$.

The domain of $R = \{1, 3, 5\}$, Range of $R = \{2, 4, 6\}$ and Co-domain of $R = \{2, 4, 6, 8\}$.

In particular, any subset $A \times A$ defines a relation in A .

Notes : 1. If $(a, b) \in R$, then we write it as aRb and it is read as ‘ a is in relation R to b ’.

2. If $(a, b) \notin R$, then we write it as $a \not R b$ and it is read as ‘ a is not in relation R to b ’.

1.2. TYPES OF RELATIONS

(a) **Void / Empty Relation.** A relation R in a set A is called *empty (or void) relation* if no element of A is related to any element of A i.e. $R = \phi \subset A \times A$.

For Example : Consider the set of all students in a Girls’ School.

Here no student in a brother of another student.

Thus R “*is a brother of*” is a void or an empty relation.

(b) **Universal Relation.** A relation R in a set A is called *universal relation* if each element of A is related to every element of A i.e. $R = A \times A$.

For Example : The difference between the heights of any two living human beings is less than 3 meters is an universal relation.

Note. Both (empty and universal) relations are called **trivial relations**.

(c) **Identity Relation.** The relation $I_A = \{(x, x) : x \in A\}$ is called identity relation on A .

For Example : Let $A = \{1, 2, 3, 4\}$.

Then the identity relation on A is given by :

$$I_A = \{(1, 1), (2, 2), (3, 3), (4, 4)\}.$$

(d) **Inverse Relation.**

Let R be a relation from a set A to a set B and let (x, y) be the member of the subset D of $A \times B$ corresponding to the relation R from A to B .

To the relation R from the set A to the set B , there corresponds relation from the set B to the set A , called the inverse of the relation and denoted by R^{-1} such that the subset $B \times A$ corresponding to the relation R^{-1} is :

$$\{(y, x) : (x, y) \in R\} \quad \text{i.e.} \quad yR^{-1}x \Leftrightarrow xRy.$$

Examples : (I) The inverse of the relation “*is the father of*” in the set of all men is the relation “*is the son of.*”

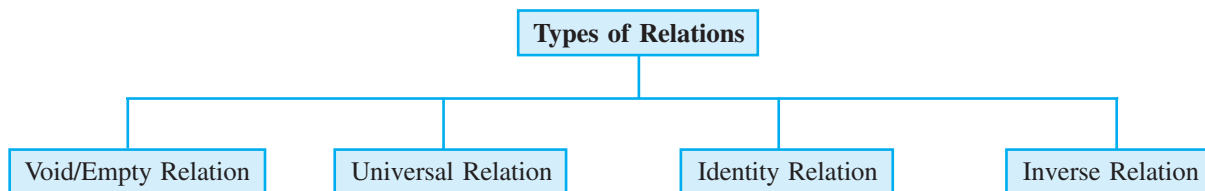
(II) The inverse of the relation “*is less than*” in \mathbf{R} is the relation “*is greater than*” in \mathbf{R} .

KEY POINT

Sometimes the inverse of a relation coincides with the relation itself.

Examples : (I) The inverse of the relation “*is perpendicular to*” in the set of straight lines, is a relation, which coincides with itself.

(II) The inverse of the relation “*is not equal to*” in the set \mathbf{R} is a relation, which coincides with itself.



1.3. CLASSIFICATION OF RELATIONS

(a) **Reflexive Relations.** We introduce the concept of this type of relations by means of **Example :**

Consider the relation “*is less than or equal to*” denoted by ‘ \leq ’ in the set of natural numbers.

Here we have : $2 \leq 2, 3 \leq 3$; and so on.

In general, $x \leq x \quad \forall x \in \mathbf{N}$.

Thus, this relation ‘ \leq ’ is such that each member of the set bears this relation to itself.

A relation in a set is said to be reflexive if it is such that each member bears this relation to itself *i.e.* if $(a, a) \in R$, for every $a \in A$. This relation ‘ \leq ’ in the set of natural numbers is reflexive.

(a) **Reflexive Relations.**

Definition

A relation R in a set A is said to be reflexive if and only if $(a, a) \in R$, for all $a \in A$.

(b) **Symmetric Relations.** We introduce this concept by means of **Example :**

Consider the set of lines in a plane and the relation “*is perpendicular to*” symbolised by ‘ \perp ’.

Let l, m be two straight lines such that $l \perp m$.

Surely when $l \perp m$, then $m \perp l$.

In fact $l \perp m \Rightarrow m \perp l$.

Such a relation is called a *symmetric relation*.

Definition

A relation R in a set A is said to be symmetric if $(a_1, a_2) \in R \Rightarrow (a_2, a_1) \in R$ for all $a_1, a_2 \in A$.

(c) **Transitive Relations.** Again, we introduce this concept by means of **Example** :

Consider the relation “*is a factor of*” in the set of natural numbers.

Let us ask the following question :

Given three numbers a, b, c such that a “*is a factor of*” b and b “*is a factor of*” c . Does it follow that a “*is a factor of*” c ?

The answer to this question is affirmative.

For this, we consider 2, 4, 8.

Here 2 “*is a factor of*” 4 and 4 “*is a factor of*” 8.

And 2 “*is also a factor of*” 8.



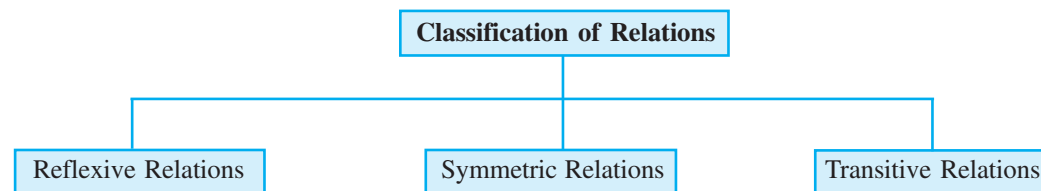
Definition

A relation R in a set A is said to be transitive if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$
 $\Rightarrow (a_1, a_3) \in R$ for all $a_1, a_2, a_3 \in A$.

A relation R is not transitive if there exists even one triplet a_1, a_2, a_3 of members of A such that when we have $(a_1, a_2) \in R$, $(a_2, a_3) \in R$, we do not have $(a_1, a_3) \in R$.

(d) **Anti-symmetric Relations.** A relation R in a set A is said to be anti-symmetric if $(a_1, a_2) \in R$ and $(a_2, a_1) \in R$

$\Rightarrow a_1 = a_2$.



1.4. EQUIVALENCE RELATIONS

A relation R in a set A is said to be an equivalence relation if it is :

(i) reflexive (ii) symmetric and (iii) transitive.

Examples : (I) The relation ‘*is congruent to*’ in the set of all triangles in a plane is an equivalence relation.

(II) The relation ‘*is similar to*’ in the set of all triangles in a plane is an equivalence relation.

(III) The relation ‘*is a divisor of*’ in the set of natural numbers is not an equivalence relation.

In fact, this relation is reflexive and transitive but not symmetric.

1.5. THEOREMS ON EQUIVALENCE RELATIONS

Theorem I. The intersection of two equivalence relations on a set A is an equivalence relation on A .

Proof. Let R and S be two equivalence relations on a set A .

Then either of them is reflexive, symmetric and transitive.

Since $R \subseteq A \times A$; $S \subseteq A \times A \Rightarrow R \cap S \subseteq A \times A$,

$\therefore R \cap S$ is a relation on a set A .

Since R and S are reflexive,

$\therefore (a, a) \in R$ and $(a, a) \in S \quad \forall a \in A$

$\Rightarrow (a, a) \in R \cap S \quad \forall a \in A$.

Thus $R \cap S$ is reflexive.

Now $(a, b) \in (R \cap S)$

$\Rightarrow (a, b) \in R$ and $(a, b) \in S$

$\Rightarrow (b, a) \in R$ and $(b, a) \in S$

$\Rightarrow (b, a) \in (R \cap S)$.

[$\because R$ and S are symmetric]

Thus $R \cap S$ is symmetric.

Again $(a, b) \in (R \cap S)$ and $(b, c) \in (R \cap S)$

$$\Rightarrow (a, b) \in R, (a, b) \in S \text{ and } (b, c) \in R, (b, c) \in S$$

$$\Rightarrow [(a, b) \in R, (b, c) \in R] \text{ and } [(a, b) \in S, (b, c) \in S]$$

$$\Rightarrow (a, c) \in R \text{ and } (a, c) \in S$$

[$\because R$ and S are transitive]

$$\Rightarrow (a, c) \in (R \cap S).$$

Thus $R \cap S$ is transitive.

Hence, $R \cap S$ is an equivalence relation.

Cor. The union of two equivalence relations is not necessarily an equivalence relation.

CHECK YOUR UNDERSTANDING

1. What is the minimum number of ordered pairs to form a non-zero reflexive relation on a set of n elements ?

Solution. n . [\because If $A = \{a_1, a_2, \dots, a_n\}$, then

$\{(a_1, a_1), \dots, (a_n, a_n)\}$ is reflexive of n pairs.]

2. If $R = \{(1, -1), (2, -2), (3, -3)\}$ is relation, then find the range of R .

Solution. Range of $R = \{-1, -2\}$.

3. A relation R on $A = \{1, 2, 3\}$ defined by :

$R = \{(1, 1), (1, 2), (3, 3)\}$ is not symmetric. Why ?

Solution. $(1, 2) \in R$ whereas $(2, 1) \notin R \Rightarrow R$ is not symmetric.

4. State the reason for the relation R , in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$, not to be transitive.

Solution. R is not transitive because $(1, 2) \in R, (2, 1) \in R$ but $(1, 1) \notin R$.

5. If $R = \{(x, y) : x + 2y = 8\}$ is a relation in N , write the range of R .

Solution. Range of $R = \{1, 2, 3\}$.

[\because When $x = 2$, then $y = 3$; when $x = 4$, then $y = 2$; when $x = 6$, then $y = 1$]

6. If R is the relation "greater than" from $A = \{1, 4, 5\}$ to $B = \{1, 2, 4, 5, 6, 7\}$. Write down the elements corresponding to R .

Solution. The elements corresponding to R are given by :

$\{(4, 1), (4, 2), (5, 1), (5, 2), (5, 4)\}$.

7. Give an example of a relation, which is :

(i) Symmetric but neither reflexive nor transitive

(ii) Transitive but neither reflexive nor symmetric.

Solution. Let $A = \{1, 2, 3\}$.

(i) The relation $R = \{(2, 3), (3, 2)\}$ is symmetric but neither reflexive nor transitive.

[$\because (1, 1) \notin R; (2, 3), (3, 2) \in R$ but $(2, 2) \notin R$]

(ii) The relation $R = \{(1, 3), (3, 2), (1, 2)\}$ is transitive but neither reflexive nor symmetric.

[$\because (1, 1) \notin R; (1, 3) \in R$ but $(3, 1) \notin R$]

8. Give an example of a relation, which is :

(i) Reflexive and symmetric but not transitive

(ii) Reflexive and transitive but not symmetric.

Solution. Let $A = \{1, 2, 3\}$.

(i) The relation $\{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2), (1, 2), (2, 1)\}$ is reflexive and symmetric but not transitive.

[$\because (1, 2)$ and $(2, 3) \in R$ but $(1, 3) \notin R$]

(ii) The relation $\{(1, 1), (2, 2), (3, 3), (1, 2)\}$ is reflexive and transitive but not symmetric.

[$\because (1, 2) \in R$ but $(2, 1) \notin R$]

MBD Super Refresher Mathematics For Class 12



Publisher : MBD Group
Publishers

Author : JP Mohindru And
Bharat Mohindru

Type the URL : <http://www.kopykitab.com/product/10013>



Get this eBook