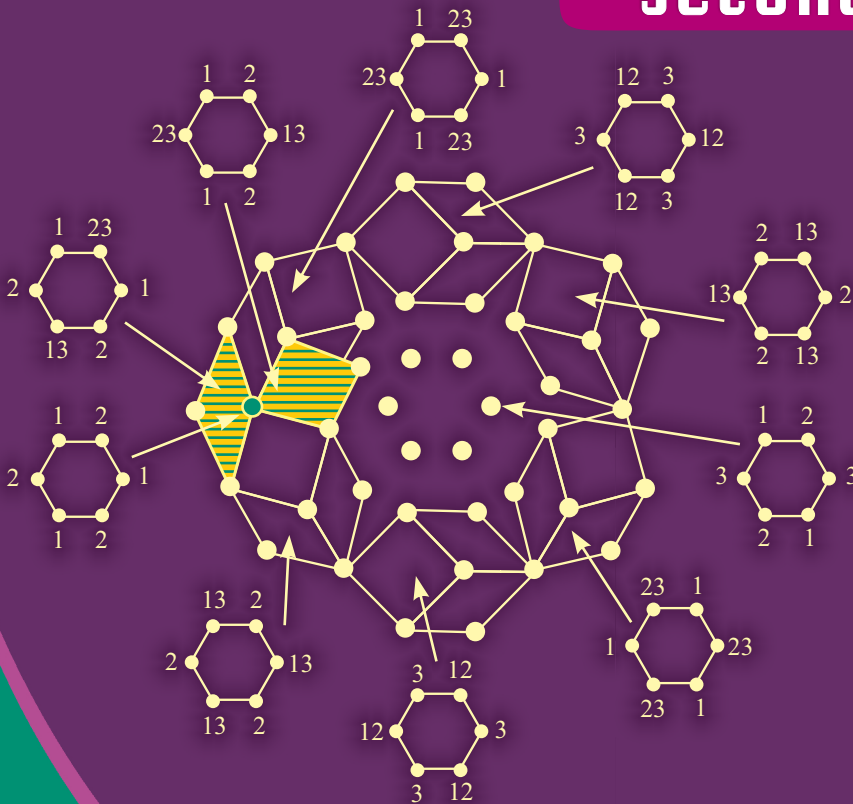


# Discrete Mathematics

Second Edition



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# Discrete Mathematics

SECOND EDITION

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**DISCRETE MATHEMATICS, Second Edition**

N. Chandrasekaran and M. Umaparvathi

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## *Preface*

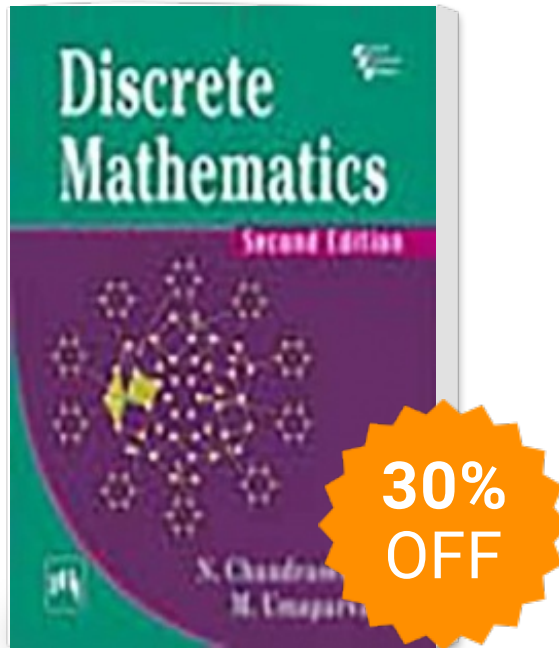
The second edition of Discrete Mathematics is the result of the enthusiastic response that we received from the first edition of this book.

In this edition, we have added a chapter on matrices since it is included in the syllabus for MCA in some universities. This was brought to our attention by some professors. We did not include a chapter on matrices under the impression that MCA students would have studied matrices in their undergraduate courses. While the students of B.E. would have studied matrices in the course on Engineering Mathematics, graduates from other streams would not have studied matrices in their UG course. We thank the professors for bringing this to our notice. We have also incorporated the suggestions made by the professors who are using the textbook, and we thank them for adopting our book.

We thank the editorial and production departments of PHI Learning for bringing out this new edition in a very short time.

**N. Chandrasekaran**  
**M. Umaparvathi**

# Discrete Mathematics



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