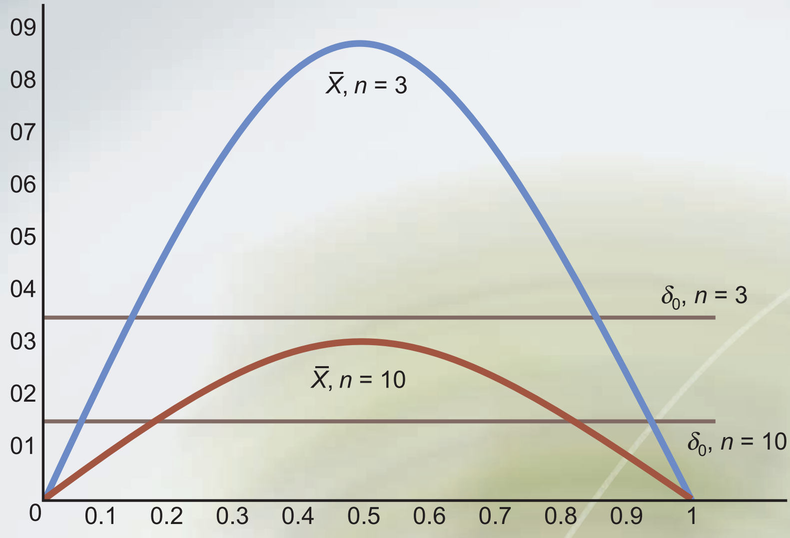

Eastern Economy Edition

STATISTICAL INFERENCE

Theory of Estimation

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PHI

Statistical Inference
Theory of Estimation

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STATISTICAL INFERENCE: Theory of Estimation
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Preface

During our teaching at Master's level, we realized the need of a book for core papers on Statistical Inference namely *Theory of Estimation* and *Testing of Hypotheses*, which could cover most of the syllabi of undergraduate and postgraduate courses prescribed by different universities and competitive examinations. We felt that the students require books solely dedicated to the *Theory of Estimation* and *Testing of Hypotheses*, emphasizing on concept building, detailing of proofs of main theorems, their real life applications in different statistical models and critical and analytical remarks to explain and develop more insight of the subject. In order to serve these academic requirements of the students, we have written a book on *Testing of Hypotheses*, which was published by PHI Learning in the year 2010. In the same sequel, the present book is being released on the *Theory of Estimation* that includes both point and interval estimation. The two books are independent of each other, therefore, can be read or taught in either order.

The text is a full semester course on Theory of Point and Interval Estimation covering the syllabi of Master's level of various Indian universities and of different competitive examinations such as I.A.S., I.S.S., UGC/CSIR-NET etc. This book discusses the Theory of Estimation by using classical approach. Bayesian approach to the Theory of Estimation is also discussed that includes sections on Empirical Bayes, Hierarchical Bayes and Equivariant estimators. The book deals with small sample theory of parametric estimation where optimal estimators and their statistical properties are discussed by using the criteria of unbiasedness, equivariance and minimaxity. The large-sample approach to the theory of point estimation leading to asymptotic optimality theory is also discussed.

The material in each chapter is self-contained and supplemented by numerous solved problems and exercises of varied nature, so selected and framed at different levels of difficulty that it suits the requirements of the students at that level. In addition to it, each chapter provides not only practice problems for students but also many additional results as complementary material to the main text. Thus, the solved problems and exercises that illustrate the applications of different theorems and results discussed form an important part of the book and will prove beneficial for the students.

The book is organized in nine chapters:

Chapter 1 of the book introduces problem of estimation in a statistical model along with certain real life examples and briefly lists basics of mathematical statistics, calculus of integrals and differentiation and fundamentals of large sample theory which are needed to grasp the concepts used in the book.

Chapter 2 introduces the problem of estimation in Decision Theoretic set up as a starting point for a course on Theory of Point Estimation where the data summarization through the principle of sufficiency, Halmos and Savage (1949) factorization theorem to characterize a sufficient statistic and the minimal sufficient statistic that results into greatest reduction in data summarization by introducing levels of data summarization and by partitioning over the sample space are discussed. The chapter also introduces the Basu (1955) theorem that states that complete sufficient statistic is independent of every ancillary statistic. Basu theorem discusses as to how it results into simplification of conditional calculations on its applications. The chapter discusses the Rao (1949) and Blackwell (1947) theorem under the convex loss function that gives an improved estimator by using a sufficient statistic. Numerous examples illustrating the theory are given in the form of solved examples and exercises at the end of the chapter.

Chapter 3 introduces unbiasedness as an impartiality principle of an estimator and shows that how the problem of estimation, i.e. of finding uniformly minimum variance unbiased estimator (UMVUE) is simplified by adopting this principle and the principle of sufficiency. Further, in this chapter two methods of finding UMVUE are discussed, based on the Cramer–Rao lower bound (1946) for variance of an unbiased estimator and by using the property of an UMVUE that it is uncorrelated with every zero estimator. The role of a complete sufficient statistic in finding an UMVUE, known as Lehmann–Scheffe theorem (1950) has been discussed. To illustrate the applications of the theorems and methods of finding UMVUE in various statistical situations, numerous examples and exercises are given at the end of the chapter.

In Chapter 4, the Fisher information (1922) has been defined as a measure of information contained in a sample about a parameter of a statistical model and subsequently it is used under certain regularity conditions to obtain lower bound for the variance of an unbiased estimator which is known as Cramer–Rao (1946) lower bound. Further, relaxing on these regularity conditions, a less sharp lower bound is introduced known as Chapman, Robbins and Kiefer lower bound (1951). Bhattacharya (1946) series of lower bounds has been introduced that approaches to the minimum variance of an unbiased estimator under certain regularity conditions. Further, these lower bounds are obtained. Numerous analytical questions relating to them are explained.

Large sample properties of an estimator such as consistency, consistent asymptotically normality (CAN), best asymptotically normality (BAN), and methods to construct such estimators are discussed in Chapter 5. Also, large sample optimality criterion due to Rao (1963), has also been discussed for judging between such estimators. Numerous examples and exercises, illustrating the applications of the theorems and results of the chapter in various statistical models, are given.

Chapter 6 deals with the conventional methods of estimation and discusses the importance of these methods. In particular, discussion on method of maximum likelihood estimation (MLE) is given in much detail. Fisher's scoring method of finding these estimators and large sample properties of MLE's are the main attraction of this chapter. Numerous illustrations of finding estimators are given at the end of the chapter.

In Chapter 7, the principle of equivariance is discussed as an impartiality criterion by restricting only to such estimators which satisfy certain symmetry requirements corresponding to such symmetry structure present in the statistical model. Pitman estimators for location, scale and location-scale models are derived and are calculated for various statistical models with numerous exercises at the end of the chapter.

Chapter 8 discusses Bayes and minimax estimation under the Decision Theoretic settings as central core of Bayesian estimation. Different types of loss functions and their applicability in different situations are discussed. A number of popular natural conjugate prior distribution are listed for different sampling distributions. A detailed discussion is given on Jeffreys (1961) noninformative priors. Limit of Bayes, generalized Bayes, extended Bayes, empirical Bayes and hierarchical Bayes estimation have also been discussed. A large number of examples and exercises on Bayes and minimax estimation are given at the end of the chapter.

Chapter 9 deals with confidence interval estimation by pivoting the cumulative distribution function. It also explains as to how these are obtained by inverting the acceptance region of a testing problem. This chapter also discusses the construction of credible intervals. Optimality of confidence intervals by considering the criteria of shortest-length, the criterion of minimum probability of false coverage among unbiased confidence interval and the Pratt (1961) and Guenther (1971) minimum expected length criterion are discussed. Numerous examples are given for obtaining optimal confidence interval estimators.

We are thankful to Prof. R.K. Singh, University of Lucknow, for critical remarks that have led to the improvement of the presentation.

Inspite of our best efforts, there might be still some errors and misprints in the presentation. We owe these mistakes and request the readers to kindly bring these to our notice with their comments.

Authors

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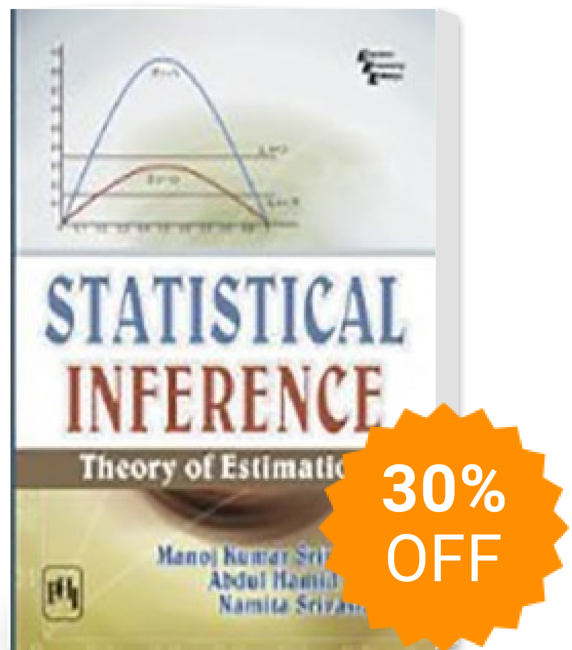

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