Introductory Methods of Numerical Analysis

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Introductory Methods of Numerical Analysis
To
My Grandsons
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Contents

Preface xiii

1. Errors in Numerical Calculations 1–21
   1.1 Introduction 1
       1.1.1 Computer and Numerical Software 3
       1.1.2 Computer Languages 3
       1.1.3 Software Packages 4
   1.2 Mathematical Preliminaries 5
   1.3 Errors and Their Computations 7
   1.4 A General Error Formula 12
   1.5 Error in a Series Approximation 14
   Exercises 19
       Answers to Exercises 21

2. Solution of Algebraic and Transcendental Equations 22–72
   2.1 Introduction 22
   2.2 Bisection Method 23
   2.3 Method of False Position 28
   2.4 Iteration Method 31
   2.5 Newton–Raphson Method 37
   2.6 Ramanujan’s Method 43
   2.7 Secant Method 49
   2.8 Muller’s Method 51
   2.9 Graeffe’s Root-Squaring Method 53
   2.10 Lin–Bairstow’s Method 56
   2.11 Quotient–Difference Method 58
Contents

2.12 Solution to Systems of Nonlinear Equations 62
  2.12.1 Method of Iteration 62
  2.12.2 Newton–Raphson Method 64

Exercises 68
  Answers to Exercises 71

3. Interpolation 73–125
  3.1 Introduction 73
  3.2 Errors in Polynomial Interpolation 74
  3.3 Finite Differences 75
    3.3.1 Forward Differences 75
    3.3.2 Backward Differences 77
    3.3.3 Central Differences 78
    3.3.4 Symbolic Relations and Separation of Symbols 79
  3.4 Detection of Errors by Use of Difference Tables 82
  3.5 Differences of a Polynomial 83
  3.6 Newton’s Formulae for Interpolation 84
  3.7 Central Difference Interpolation Formulae 90
    3.7.1 Gauss’ Central Difference Formulae 90
    3.7.2 Stirling’s Formula 94
    3.7.3 Bessel’s Formula 94
    3.7.4 Everett’s Formula 96
    3.7.5 Relation between Bessel’s and Everett’s Formulae 96
  3.8 Practical Interpolation 97
  3.9 Interpolation with Unevenly Spaced Points 101
    3.9.1 Lagrange’s Interpolation Formula 101
    3.9.2 Error in Lagrange’s Interpolation Formula 107
    3.9.3 Hermite’s Interpolation Formula 108
  3.10 Divided Differences and Their Properties 111
    3.10.1 Newton’s General Interpolation Formula 113
    3.10.2 Interpolation by Iteration 115
  3.11 Inverse Interpolation 116
  3.12 Double Interpolation 118

Exercises 119
  Answers to Exercises 125

4. Least Squares and Fourier Transforms 126–180
  4.1 Introduction 126
  4.2 Least Squares Curve Fitting Procedures 126
    4.2.1 Fitting a Straight Line 127
    4.2.2 Multiple Linear Least Squares 129
    4.2.3 Linearization of Nonlinear Laws 130
    4.2.4 Curve Fitting by Polynomials 133
    4.2.5 Curve Fitting by a Sum of Exponentials 135
4.3 Weighted Least Squares Approximation 138
   4.3.1 Linear Weighted Least Squares Approximation 138
   4.3.2 Nonlinear Weighted Least Squares Approximation 140
4.4 Method of Least Squares for Continuous Functions 140
   4.4.1 Orthogonal Polynomials 143
   4.4.2 Gram–Schmidt Orthogonalization Process 145
4.5 Approximation of Functions 148
   4.5.1 Chebyshev Polynomials 149
   4.5.2 Economization of Power Series 152
4.6 Fourier Approximation 153
   4.6.1 Fourier Transform 156
   4.6.2 Discrete Fourier Transform (DFT) 157
   4.6.3 Fast Fourier Transform (FFT) 161
   4.6.4 Cooley–Tukey Algorithm 161
   4.6.5 Sande–Tukey Algorithm (DIF–FFT) 170
   4.6.6 Computation of the Inverse DFT 174

Exercises 176
  Answers to Exercises 179

5. Spline Functions 181–206
5.1 Introduction 181
   5.1.1 Linear Splines 182
   5.1.2 Quadratic Splines 183
5.2 Cubic Splines 185
   5.2.1 Minimizing Property of Cubic Splines 191
   5.2.2 Error in the Cubic Spline and Its Derivatives 192
5.3 Surface Fitting by Cubic Splines 193
5.4 Cubic B-splines 197
   5.4.1 Representation of B-splines 198
   5.4.2 Least Squares Solution 203
   5.4.3 Applications of B-splines 203

Exercises 204
  Answers to Exercises 206

6. Numerical Differentiation and Integration 207–254
6.1 Introduction 207
6.2 Numerical Differentiation 207
   6.2.1 Errors in Numerical Differentiation 212
   6.2.2 Cubic Splines Method 214
   6.2.3 Differentiation Formulae with Function Values 216
6.3 Maximum and Minimum Values of a Tabulated Function 217
6.4 Numerical Integration 218
   6.4.1 Trapezoidal Rule 219
   6.4.2 Simpson’s 1/3-Rule 221
   6.4.3 Simpson’s 3/8-Rule 222