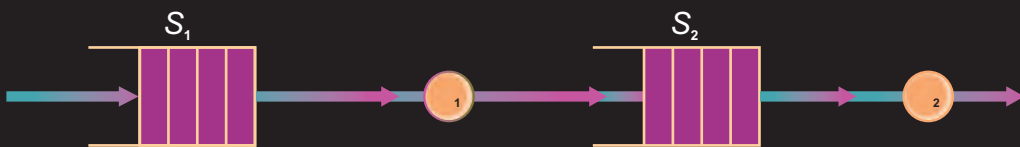


Eastern  
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Edition

S. Palaniammal

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# Probability and Queueing Theory

PHI

# PROBABILITY AND QUEUEING THEORY

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**PROBABILITY AND QUEUEING THEORY**

S. Palaniammal

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*To*  
*My beloved Parents*  
*who made me what I am today*



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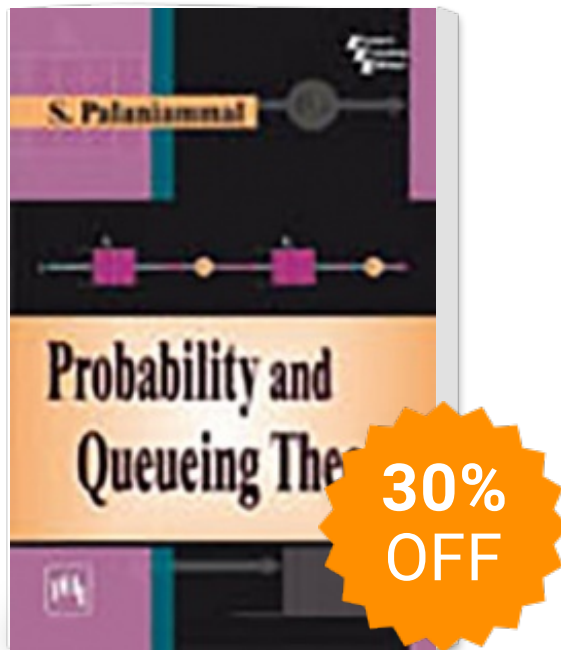
# Preface

Most of the engineering students find the theory of probability very difficult due to inadequate knowledge and understanding of the basic concepts. The aim of this book is to provide a thorough understanding of the fundamental concepts and applications of probability and queueing theory for the undergraduate and postgraduate students of Engineering, in particular, Electronics and Communication Engineering, Computer Science Engineering and Information Technology. This book is an outcome of my long teaching experience. It is written in a simple and an easy-to-understand language. This book covers the syllabi of most Indian universities and, in particular, the entire syllabus of Anna University.

The text is divided into seven chapters. Chapter 1 provides a detailed discussion on probability theory. Chapter 2 presents the concepts of random variables, probability mass and density functions, cumulative distribution function and moments. Chapter 3 discusses discrete and continuous distributions such as binomial, Poisson, geometrical, uniform, exponential, Gamma and Weibull distributions. Chapter 4 analyses the concepts of two-dimensional discrete and continuous random variables, joint probability mass and density function, conditional probability function and cumulative distribution function. It also discusses covariance, correlation, regression, transformation of random variables and central limit theorem.

Chapter 5 deals with random processes such as Poisson, Bernoulli, Sine wave, Ergodic Markov processes and Markov chain. Chapter 6 discusses the basic concepts of queueing theory and both finite and infinite capacity M/M/1, M/M/c and M/G/1 queueing models. Chapter 7 provides in detail the concepts of series queues and open and closed queue networks.

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