

Eastern  
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# Engineering Mathematics

Volume II

A.C. Srivastava • P.K. Srivastava



# Engineering Mathematics

## Volume II

**A.C. Srivastava**

*Formerly*

Professor and Head

Department of Mathematics, Dibrugarh University

Emeritus Scientist (CSIR) and Principal Investigator (UGC, DST)

Lucknow University

**P.K. Srivastava**

Assistant Professor

Department of Applied Science and Humanities

Bundelkhand Institute of Engineering and Technology

Jhansi

**PHI Learning** Private Limited

New Delhi-110001

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**ENGINEERING MATHEMATICS, Volume II**

A.C. Srivastava and P.K. Srivastava

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*To  
the memory of  
Late Mrs. Sharda Srivastava, beloved wife of  
Prof. A.C. Srivastava, whose incessant motivation  
has been the source of inspiration for writing this book*



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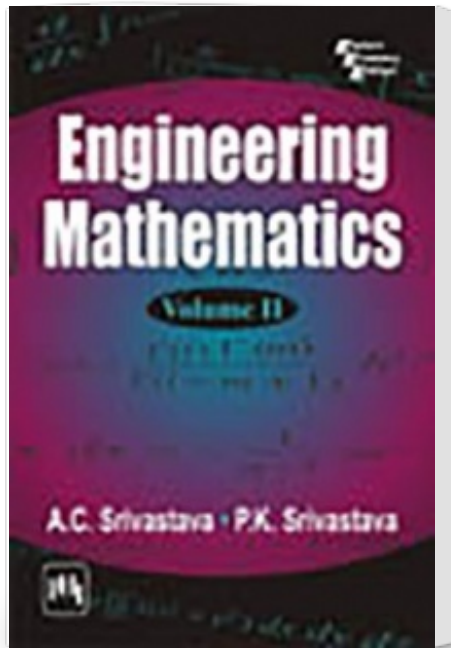
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Author : SRIVASTAVA, A.  
C. , SRIVASTAVA, P. K.

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