

Second Edition (Revised)

**Eastern
Economy
Edition**

MATRIX AND LINEAR ALGEBRA

Aided with MATLAB



Kanti Bhushan Datta

Matrix and Linear Algebra **Aided with MATLAB**

Second Edition

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MATRIX AND LINEAR ALGEBRA—Aided with MATLAB, 2nd ed.
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To my daughter
Somantika
to provide inspiration for strengthening her
skill in mathematics

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Preface

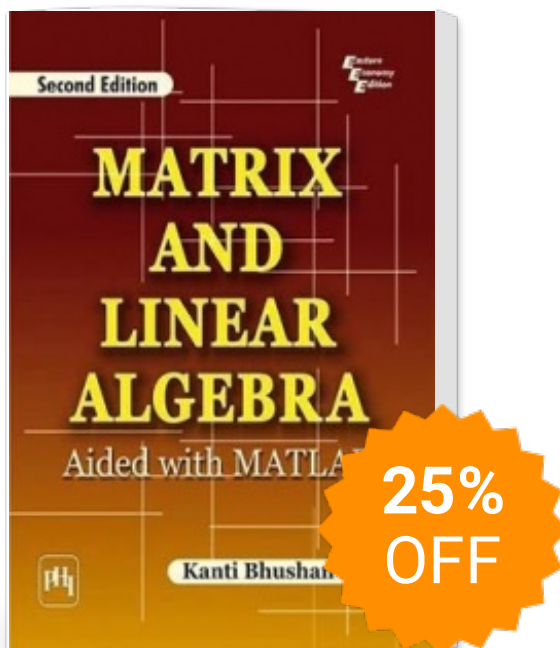
This book survived a seventeen-year acid test of the students, professors and other professionals, for which the author is very much grateful. A revision is thought necessary, being propelled by the motivation of introducing MATLAB for the study of numerical aspect of matrix theory. This may urge the students to solve the different chapter-end problems with a computer, without much computational chore with three p's (pen-paper-pencil). The semester-oriented engineering and science educational curriculum keeps on rolling with such great strides that the average students need ready-to-help books to learn the technicalities of solving problems of diverse nature to tide over the difficult time of an examination. Worked-out examples are, therefore, provided in great abundance, besides a few diagrams illustrating the concepts. A large number of chapter-end problems are incorporated, and answers to all the problems are provided to help the student in self-study. So, the learning of matrix and linear algebra, aided with MATLAB, may turn out to be a pleasant trip to a wonderland with twin lovers.

As, course material, this book can be used in many ways. For an elementary course, one can choose Chapters 1–3, Sections 4.1–4.4; 5.1–5.3, 5.6; 7.1, 7.2, 7.4–7.6, skipping the related linear transformation portions. Last but not least, Chapter 13, the most important part from the application point of view, outlines numerical linear algebra. These topics may form a forty-hour lecture course of one semester supported by homework and tutorials. The remaining chapters and sections may form a second semester advanced course on matrix and linear algebra for those students who are pursuing M.Sc. in Mathematics or Ph.D. programmes.

The present book is a revised edition of the book MATRIX AND LINEAR ALGEBRA and is renamed as MATRIX AND LINEAR ALGEBRA: AIDED WITH MATLAB. A Solutions Manual for all the chapter-end problems is now available for the instructors.

The introduction of MATLAB and how to use it for matrix computation are the major and significant additions to the first edition. Moreover, new sections on square-root of a matrix as

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