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For I & II Semesters of B.Tech.
& Diploma Courses

B.S. Grewal

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Elementary Engineering Mathematics

for I & II Semesters of B. Tech. & Diploma Courses

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Preface to the Fifteenth Edition

The book has been thoroughly revised and a number of sections have been rewritten. Some elementary topics have been included to meet the requirement of beginners in engineering studies deleting some sections of lesser utility. A variety of new illustrative examples and problems selected from various examination papers which sharpen ones skill to use various methods, have been added. The list of 'Objective Type of Questions' appended to each chapter, has also been updated. It is hoped that the book in its revised form will serve a more useful purpose.

The authors take this opportunity to thank fellow professors for their suggestions and patronage of the book. In particular, they are grateful to Prof. B.K. Yadav, Chouksey Engg. College, Bilaspur (C.G.) ; Dr. Jeevragi Phakirappa, RBYM Engg. College, Bellary (Kar.) ; Prof. Pankaj Kumar, Lovely Professional Univ., Phagwara (Pb.) ; Dr. Hemant Kumar Nashine, Disha Inst. of Management & Technology, Raipur (C.G.) ; Prof. Amarapu Ramesh Babu, A.N. Inst. of Science & Technology, Visakhapatnam ; Prof. Pankaj S. Gholap, Dr. D.Y. Patel School of Engg., Lohegaon, Pune; Prof. P.C. Pillai, NSS College of Engg.,(Ker.); Dr. A.P. Burnwal, R.I.T., Koderma (Jh. K.) and Dr. Saroj Panigrahi, J.P. Inst. of Engg. & Tech., Guna (MP).

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New Delhi

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Note : The references given alongside the problems pertain to the various **Engineering Examinations** of the various universities and professional bodies. The abbreviations used for some of these are given below :

<i>Andhra</i>	<i>stands for</i>	Andhra University, Waltair
<i>Anna</i>	”	Anna University, Chennai
<i>Bhopal</i>	”	Rajiv Gandhi Technical University, Bhopal
<i>B.P.T.U.</i>	”	Biju Patnaik Technical University, Rourkela
<i>C.S.V.T.U.</i>	”	Swami Vivekanand Technical University, Chhatisgarh
<i>Coimbatore</i>	”	Bharathiyar University, Coimbatore
<i>Delhi</i>	”	Guru Gobind Singh Indraprastha University, Delhi
<i>D.T.U.</i>	”	Delhi Technical University
<i>J.N.T.U.</i>	”	Jawahar Lal Nehru Technological University, Hyderabad
<i>Kottayam</i>	”	Mahatama Gandhi Memorial University, Kottayam
<i>Kurukshetra</i>	”	National Institute of Technology, Kurukshetra
<i>Madurai</i>	”	Madurai Kamaraj University, Madurai
<i>Marathwada</i>	”	B.A.M. University, Aurangabad
<i>P.T.U.</i>	”	Punjab Technical University, Jalandhar
<i>Rohtak</i>	”	Maharishi Dayanand University, Rohtak
<i>Tirupati</i>	”	Sri Venkateswara University, Tirupati
<i>U.P.T.U.</i>	”	UP Technical University, Lucknow
<i>U.T.U.</i>	”	Uttarakhand Technical University, Dehradun
<i>V.T.U.</i>	”	Visveswaraiah Technological University, Belgaum
<i>W.B.T.U.</i>	”	West Bengal University of Technology, Kolkata

Preliminaries

Algebra

1.1. Progressions

(1) Arithmetic Progression. Numbers $a, a + d, a + 2d, \dots$ are said to be in arithmetic progression (A.P.), where a is its first term and d is the common difference.

(i) Its n th term $T_n = a + \overline{n - 1} d$

(ii) Sum of n terms $S_n = \frac{n}{2} (2a + \overline{n - 1} d)$

Proof. We have $S_n = a + (a + d) \dots + (a + \overline{n - 2} d) + (a + \overline{n - 1} d)$

Reversing the order of terms $S_n = (a + \overline{n - 1} d) + (a + \overline{n - 2} d) + \dots + (a + d) + a$

Adding $2S_n = (2a + \overline{n - 1} d) + (2a + \overline{n - 1} d) + \dots + (2a + \overline{n - 1} d) + (2a + \overline{n - 1} d)$

i.e., $2S_n = n(2a + \overline{n - 1} d) \quad \text{or} \quad S_n = \frac{n}{2} (2a + \overline{n - 1} d)$

(iii) Arithmetic mean between two numbers a and b (A.M.) = $\frac{a + b}{2}$

(2) Harmonic Progression. Numbers $1/a, 1/(a + d), 1/(a + 2d), \dots$ are said to be in Harmonic progression (H.P.) i.e. a sequence is said to be in H.P. if its reciprocals are in A.P. Its n th

term $T_n = \frac{1}{a + (n - 1)d}$.

Example 1.1. Find the first term and the number of terms of an A.P. whose second term is 7.75, 31st term is 0.5 and last term is 6.5.

Sol. If a is the first term and d is the common difference, then

$$T_2 = a + d = 7.75 \quad \dots(i)$$

and $T_{31} = a + 30d = 0.5 \quad \dots(ii)$

Subtracting (i) from (ii), we get

$$29d = 0.5 - 7.75 = -7.25 \quad \text{or} \quad d = -0.25$$

Then (i) gives $a = 7.75 - d = 7.75 - (-0.25) = 8$

Also $T_n = a + \overline{n - 1} d = -6.5$ (given)

or $8 + (n - 1)(-0.25) = -6.5 \quad \text{or} \quad n - 1 = -14.5/(-0.25) = 58.$

Hence, $n = 59$.

Example 1.2. Find the sum of 35 terms of an A.P. whose third term is 1 and 6th term is -11.

Sol. If a is the first term and d the common difference, then

$$T_3 = a + 2d = 1 \quad \text{and} \quad T_6 = a + 5d = -11$$

Solving these equations, we get $d = -4$ and $a = 9$

Hence, sum of 35 terms $= \frac{35}{2} [2 \times 9 + (35 - 1)(-4)] = -2065$.

(3) Geometric Progression. Numbers a, ar, ar^2, \dots are said to be in Geometric Progression (G.P.) where a is its first term and r is the common ratio.

(i) Its n th term $T_n = ar^{n-1}$

(ii) Sum of n terms $S_n = \frac{a(1-r^n)}{1-r}$

Proof. We have, $S_n = a + ar + ar^2 + \dots + ar^{n-1}$

Multiplying by r , we get $rS_n = ar + ar^2 + ar^3 + \dots + ar^n$

Subtracting, $S_n - rS_n = a - ar^n$ or $S_n = \frac{a(1-r^n)}{1-r}$

(iii) Sum to infinity $S_\infty = \frac{a}{1-r}$ where $r < 1$

Since $r^n \rightarrow 0$ for $r < 1$ when $n \rightarrow \infty$

\therefore for $r < 1$, $\text{Lt}_{n \rightarrow \infty} S_n$ (i.e. S_∞) $= \frac{a}{1-r}$.

(iv) Geometric mean between two numbers a and b i.e. $G.M. = \sqrt{ab}$

Since G.M. (= G) between a and b is order that a, G, b are in G.P. i.e.

$$\frac{G}{a} = \frac{b}{G} \quad \text{or} \quad G = \sqrt{ab}$$

Example 1.3. If the common ratio of a G.P. is 3, its last term is 486 and sum of these terms is 728, find its first term.

Sol. Let a be the first term and n the number of terms in the given G.P.

Then $T_n = ar^{n-1} = 486$. or $a(3)^{n-1} = 486 \quad \dots(i) \quad [\because r = 3]$

Also $S_n = \frac{a(1-r^n)}{1-r} = 728$ i.e., $\frac{a(1-3^n)}{1-3} = 728$

or using (i) $a - 3(a3^{n-1}) = -2 \times 728 = -1456$

or $a - 3(486) = -1456$ or $a = 2$.

Example 1.4. Find two numbers whose sum is 120 and the ratio between arithmetic mean and geometric mean is 1.25.

Sol. Let the numbers be a and b so that $a + b = 120 \quad \dots(i)$

Also A.M. $= \frac{1}{2}(a + b) = 60$; G.M. $= \sqrt{ab}$.

$\therefore \frac{\text{A.M.}}{\text{G.M.}} = \frac{60}{\sqrt{ab}} = 1.25$ (given) i.e. $\sqrt{ab} = 60/1.25 = 48$.

or $ab = 2304$, from (i), $a + 2304/a = 120$ or $a^2 - 120a + 2304 = 0$

$\therefore a = \frac{1}{2} [120 + \sqrt{(-120)^2 - 4 \times 2304}] = \frac{1}{2}(120 + 72) = 96$.

(4) Natural numbers. 1, 2, 3, 4, ... are called natural numbers and Σ , pronounced as sigma, denotes the summation sign.

(i) $1 + 2 + 3 + \dots + n$ ($= \Sigma n$) $= \frac{n(n+1)}{2}$

$$(ii) 1^2 + 2^2 + 3^2 + \dots + n^2 (= \Sigma n^2) = \frac{n(n+1)(2n+1)}{6}$$

$$(iii) 1^3 + 2^3 + 3^3 + \dots + n^3 (= \Sigma n^3) = \left[\frac{n(n+1)}{2} \right]^2.$$

Example 1.5. Find the sum of the series

(i) $3.5 + 5.7 + 7.9 + \dots$ to n terms

(ii) $1.3.6 + 2.5.9 + 3.7.12 + \dots$ to n terms

Sol. (i) n th term of the given series = (n th terms of 3, 5, 7 ...) \times (n th term of 5, 7, 9, ...)

$$\begin{aligned} \text{i.e.} \quad T_n &= (3 + \overline{n-2})(5 + \overline{n-2}) = (2n+1)(2n+3) \\ &= 4n^2 + 8n + 3 \end{aligned}$$

$$\therefore S_n = \Sigma (4n^2 + 8n + 3) = 4\Sigma n^2 + 8\Sigma n + 3n$$

$$= 4 \cdot \frac{n(n+1)(2n+1)}{6} + 8 \cdot \frac{n(n+1)}{2} + 3n = \frac{n}{3} [2(2n^2 + 3n + 1) + 12(n+1) + 9]$$

$$= \frac{n}{3}(4n^2 + 18n + 23).$$

$$\begin{aligned} (ii) \text{ Here} \quad T_n &= (n\text{th term of } 1, 2, 3 \dots) (n\text{th term of } 3, 5, 7 \dots) (n\text{th term of } 6, 9, 12, \dots) \\ &= n(2n+1)(3n+3) = 6n^3 + 9n^2 + 3n \end{aligned}$$

$$\therefore S_n = 6 \Sigma n^3 + 9 \Sigma n^2 + 3 \Sigma n = 6 \left[\frac{n(n+1)}{2} \right]^2 + 9 \left[\frac{n(n+1)(2n+1)}{6} \right] + 3 \left[\frac{n(n+1)}{2} \right]$$

$$= \frac{3}{2} (n^3 + 4n^2 + 5n + 3)$$

Example 1.6. Find the sum of the series

(i) $5^2 + 6^2 + 7^2 + \dots + 25^2$

(ii) $3 + 7 + 14 + 24 + 37 + \dots$ to n terms.

Sol. (i) We have

$$\begin{aligned} S &= 5^2 + 6^2 + 7^2 + \dots + 25^2 \\ &= (1^2 + 2^2 + 3^2 + \dots + 25^2) - (1^2 + 2^2 + 3^2 + 4^2) \\ &= (\Sigma n^2)_{n=25} - (\Sigma n^2)_{n=4} \\ &= \frac{25(25+1)(2 \times 25+1)}{6} - \frac{4(4+1)(8+1)}{6} \\ &= \frac{25 \times 26 \times 51}{6} - \frac{4 \times 5 \times 9}{6} = 5495 \end{aligned}$$

$$(ii) \text{ Let} \quad S = 3 + 7 + 14 + 24 + 37 + \dots + T_{n-1} + T_n$$

$$\text{Also} \quad S = 3 + 7 + 14 + 24 + \dots + T_{n-1} + T_n$$

$$\text{Subtracting, we get} \quad 0 = 3 + 4 + 7 + 10 + 13 \dots + (T_n - T_{n-1}) - T_n$$

$$\text{or} \quad T_n = 3 + 4 + 7 + 10 + 13 + \dots + (T_n - T_{n-1})$$

$$= 3 + [4 + 7 + 10 + 13 + (n-1) \text{ terms}] = 3 + \left[\frac{n-1}{2} \{2(4) + (n-1-1)3\} \right]$$

$$= \frac{1}{2} (3n^2 - n + 4)$$

$$\therefore S_n = \frac{1}{2} [3\Sigma n^2 - \Sigma n + 4\Sigma 1] = \frac{1}{2} \left[3 \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 4n \right]$$

$$= \frac{n}{4} (2n^2 + 3n + 1 - n - 1 + 8) = \frac{n}{2} (n^2 + n + 4).$$

1.2. Permutations and Combinations

The process of arranging certain objects in a particular order is called a *permutation* and the process of selecting the objects (without the order of occurrence) is called a *combination*.

For instance the permutations of 2 out of 3 things a, b, c are AB, BA, BC, CB, CA, CB (i.e. 6) while the combinations (groups) of 2 out of 3 things A, B, C are AB, BC, CA (i.e. 3).

Permutations of n things taken r at a time is ${}^n P_r = \frac{n!}{(n-r)!}$

Combinations of n things taken r at a time is ${}^n C_r = \frac{n!}{r!(n-r)!}$

Also ${}^n C_{n-r} = {}^n C_r$; ${}^n C_0 = {}^n C_n = 1$

Example 1.7. (a) If ${}^n C_6 = {}^n C_8$, find ${}^n C_3$. (b) If ${}^4 P_2 = k^4 C_2$, find k .

Sol. (a) We have ${}^n C_6 = {}^n C_8 = {}^n C_{n-8}$ i.e. $6 = n - 8$ or $n = 14$.

Now ${}^n C_3 = {}^{14} C_3 = \frac{14!}{3!(14-3)!} = \frac{14 \cdot 13 \cdot 12}{3 \cdot 2 \cdot 1} = 364$

(b) ${}^4 P_2 = \frac{4!}{2!} = 12$, ${}^4 C_2 = \frac{4!}{2!2!} = 6$

$\therefore {}^4 P_2 = k^4 C_2$ gives $12 = k \cdot 6$ i.e. $k = 2$.

1.3. Logarithms

(1) *Definition.* If a and x are two positive numbers and p is any other number such that $a^p = x$, then p is called the logarithm of x to the base a and is written as $\log_a x$.

For practical purposes, base is taken 10 and logarithms to base 10 is called *common logarithms* and is written as $\log_{10} x$.

Logarithm to the base e is called *natural logarithm* and is simply written as $\log x$.

(2) *Basic Laws:*

(i) $\log(mn) = \log m + \log n$, (ii) $\log(m/n) = \log m - \log n$; (iii) $\log m^n = n \log m$.

(3) *Particular cases:*

(i) $\log_a 1 = 0$, (ii) $\log_a 0 = -\infty$ ($a > 1$), (iii) $\log_a a = 1$.

Problems 1.1

- Find the sum of all odd numbers between 100 and 200.
- If the first term of an A.P. is 2 and last term is 14. Find the common difference if the sum of the series is 128.
- The second term of an A.P. is 2 and seventh term is 22. Find the sum of its first 35 terms.
- Find the sum of the G.P. series:
 - $2, -1/2, 1/8, \dots$ to 12 terms
 - $2 + 6 + 18 + \dots + 4374$
- Find the first term of a G.P. whose common ratio is $-4/5$ and the sum to infinity is $80/9$.
- Find two positive numbers whose difference is 12 and whose A.M. exceeds the G.M. by 2.
- Find the sum to n terms the series:
 - $3.8 + 6.11 + 9.14 + \dots$
 - $1.2.5 + 2.3.6 + 3.4.7 + \dots$
- Find the sum of the following series:
 - $6^2 + 7^2 + 8^2 + \dots + 20^2$
 - $2^3 + 4^3 + 6^3 + \dots + 20^3$
- Find the sum to n terms of the series:
 - $2 + 10 + 30 + 68 + 130 + \dots$
 - $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.10} + \dots$

10. If ${}^nC_r = {}^nC_{r-1}$ and ${}^nP_r = {}^nP_{r+1}$ find the values of n and r .
 11. If ${}^nC_r / {}^nC_{r+1} = 1/2$ and ${}^nC_{r+1} / {}^nC_{r+2} = 2/3$ find n and r .

1.4. Binomial Theorem

(1) When n is a positive integer

$$(a + x)^n = {}^nC_0 a^n x^0 + {}^nC_1 a^{n-1} x^1 + {}^nC_2 a^{n-2} x^2 + \dots + {}^nC_r a^{n-r} x^r + \dots + {}^nC_n a^0 x^n$$

When $a = 1$, $(1 + x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_r x^r + \dots + {}^nC_n x^n$

Expansion of $(1 + x^n)$ has $n + 1$ terms.

(2) *Binomial coefficients.* Coefficients of terms equidistant from the beginning and the end are equal and are known as the *Binomial coefficients*.

i.e., ${}^nC_0 = {}^nC_n, {}^nC_1 = {}^nC_{n-1}, {}^nC_2 = {}^nC_{n-2} \dots [\because {}^nC_r = {}^nC_{n-r}]$

Also sum of the binomial coefficients i.e., ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_n = 2^n$

(3) *Middle term in Binomial expansion*

If n is even, then the middle term is $\left(\frac{n}{2} + 1\right)$ th term

If n is odd, then the middle terms are $\left(\frac{n+1}{2}\right)$ th and $\left(\frac{n+3}{2}\right)$ th terms.

Example 1.8. Using binomial theorem, expand

- (i) $(2x - 1/x)^4$ (ii) $(1 + 2x - 3x^2)$

Sol. (i) $(2x - \frac{1}{x})^4 = {}^4C_0 (2x)^4 \left(-\frac{1}{x}\right)^0 + {}^4C_1 (2x)^3 \left(-\frac{1}{x}\right)^1 + {}^4C_2 (2x)^2 \left(-\frac{1}{x}\right)^2$
 $+ {}^4C_3 (2x)^1 \left(-\frac{1}{x}\right)^3 + {}^4C_4 (2x)^0 \left(-\frac{1}{x}\right)^4$
 $= 1(16x^4) + 4(8x^3) (-1/x) + \frac{4.3}{1.2} (4x^2) (1/x^2) + \frac{4.3}{1.2} (2x) (-1/x^3) + 1(1/x^4)$
 $= 16x^4 - 32x^2 - 24 - 12/x^2 + 1/x^4$

(ii) $(1 + 2x - 3x^2)^5 = (1 + y)^5$ where $y = 2x - 3x^2$
 $= 1 + {}^5C_1 y + {}^5C_2 y^2 + {}^5C_3 y^3 + {}^5C_4 y^4 + {}^5C_5 y^5$
 $= 1 + 5(2x - 3x^2) + \frac{5.4}{1.2} (2x - 3x^2)^2 + \frac{5.4}{1.2} (2x - 3x^2)^3 + 5(2x - 3x^2)^4 + (2x - 3x^2)^5$
 $= 1 + 10x + 25x^2 - 40x^3 - 190x^4 + 92x^5 + 570x^6 - 360x^7 - 675x^8 + 810x^9 - 243x^{10}$

Example 1.9. Evaluate $(3 + \sqrt{2})^5 - (3 - \sqrt{2})^5$.

Sol. We have $(a + x)^n - (a - x)^n = 2[{}^nC_1 a^{n-1} x + {}^nC_3 a^{n-3} x^3 + \dots]$

Putting $a = 3$ and $x = \sqrt{2}$, we get

$$(3 + \sqrt{2})^5 - (3 - \sqrt{2})^5 = 2[{}^5C_1 (3)^4 (\sqrt{2})^1 + {}^5C_3 (3)^2 (\sqrt{2})^3 + {}^5C_5 (3)^0 (\sqrt{2})^5]$$

$$= 2[5(81)\sqrt{2} + 10(9)(2\sqrt{2}) + 5(4\sqrt{2})] = 1210\sqrt{2}$$

Example 1.10. Write the middle terms in the following expansions

- (i) $(x/2 + 7y)^{10}$ (ii) $(3x - x^2/6)^9$

Sol. Here $n = 10$ is an even number, therefore $\left(\frac{10}{2} + 1\right)$ th term i.e., 6th term is the middle term.

Hence, middle term = $T_6 = {}^{10}C_5 (x/2)^{10-5} (7y)^5$

$$= \frac{10!}{5!(10-5)!} \frac{x^5}{32} (7)^5 y^5 = 132\,355.125 x^5 y^5.$$

(ii) Here $n = 9$, therefore $\frac{9+1}{2}$ th and $\frac{9+3}{2}$ th i.e. 5th and 6th are two middle terms.

$$\therefore T_5 = {}^9C_4 (3x)^{9-4} (-x^2/6)^4 = \frac{126 \times (3)^5}{(6)^4} x^{21} = 23.625 x^{21}$$

Example 1.11. Find the term independent of x in the expansion of $\left(\frac{1}{3x} - \frac{3}{2}x^2\right)^6$.

Sol. Let T_{r+1} be the term independent of x .

Now
$$T_{r+1} = {}^6C_r \left(\frac{1}{3x}\right)^{6-r} \left(-\frac{3}{2}x^2\right)^r = {}^6C_r \left(\frac{1}{3}\right)^{6-r} \left(-\frac{3}{2}\right)^r \cdot x^{3r-6}$$

For T_{r+1} to be independent of x , we must have $3r - 6 = 0$ i.e., $r = 2$.

Hence, the term of independent of $x = T_3 = {}^6C_2 \left(\frac{1}{3}\right)^4 \left(-\frac{3}{2}\right)^2 = \frac{5}{12}$.

Problems 1.2

Expand the following expressions:

1. $(x - 3y)^4$

2. $\left(\frac{2}{x} - \frac{x}{2}\right)^5$

3. $\left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right)^6$

4. $\left(1 - \frac{x}{2} + \frac{2}{x}\right)^4$

Evaluate the following:

5. $(\sqrt{5} + 2)^5(\sqrt{5} - 2)^6$

6. $(\sqrt{2} + \sqrt{3})^6 - (\sqrt{2} - \sqrt{3})^6$

Find the middle terms in the following expansions:

7. $\left(2a - \frac{1}{b}\right)^8$

8. $\left(9x + \frac{4}{3}\right)^{10}$

9. $\left(3x - \frac{x^3}{6}\right)^9$

10. $\left(\frac{1}{x^3} - x^4\right)^{11}$

11. Find the term independent of x in the expansion of

(i) $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$

(ii) $\left(\frac{\sqrt{3}}{2x^2} + \sqrt{\frac{x}{3}}\right)^{10}$

1.5. Partial Fractions

(1) We are familiar with the process of combining two or more fractions into a single fraction. For instance,

$$\frac{1}{x+2} + \frac{1}{x-1} + \frac{1}{(x-1)^2} = \frac{2x^2 + 1}{(x+2)(x-1)^2} \quad \dots(i)$$

Now we shall introduce the reader to the reverse process of breaking a given fraction into a sum of simple fractions. These simple fractions are called *partial fractions* of the given

fraction *e.g.*, in (i), the fractions on the left hand side are the partial fractions of the fraction on the right.

A fraction in which the degree of the numerator is less than that of the denominator is called a proper fraction.

(2) Procedure to resolve a given fraction into partial fractions.

Step I. Check whether the given fraction is a proper fraction. If the degree of the numerator is higher or equal to that of the denominator, divide the numerator by the denominator till the remainder is of lower degree than the denominator.

Step II. Factorize the denominator into real factors. These will be either linear or quadratic, and some factors repeated.

Step III. Resolve the proper fraction into sum of partial fractions such that

(i) to a non-repeated linear factor $x - a$ in the denominator corresponds a partial fraction of the form $A/(x - a)$;

(ii) to a repeated linear factor $(x - a)^r$ in the denominator corresponds the sum of r partial fractions of the form $\frac{A_1}{x - a} + \frac{A_2}{(x - a)^2} + \frac{A_3}{(x - a)^3} + \dots + \frac{A_r}{(x - a)^r}$;

(iii) to a non-repeated quadratic factor $(x^2 + ax + b)$ in the denominator, corresponds a partial fraction of the form $\frac{Ax + B}{x^2 + ax + b}$;

(iv) to a repeated quadratic factor $(x^2 + ax + b)^r$ in the denominator, corresponds the sum of r partial fractions of the form $\frac{A_1x + B_1}{x^2 + ax + b} + \frac{A_2x + B_2}{(x^2 + ax + b)^2} + \dots + \frac{A_rx + B_r}{(x^2 + ax + b)^r}$.

Then we have to determine the unknown constants A, A_1, B_1, B_1 etc. as follows:

Step IV. To obtain the partial fraction corresponding to the non-repeated linear factor $x - a$ in the denominator, put $x = a$ everywhere in the given fraction except in the factor $x - a$ itself.

In all other cases, equate the given fraction to a sum of suitable partial fractions in accordance with (i) and (iv) above, having found the partial fractions corresponding to the non-repeated linear factors by the above rule. Then multiply both sides by the denominator of the given fraction and equate the coefficients of like powers of x or substitute convenient numerical values of x on both sides. Finally solve the simplest of the resulting equations to find the unknown constants.

Example 1.12. Resolve into partial fractions $\frac{x^2 + 23x - 18}{(x^2 + x - 2)(2x - 1)}$.

Sol. Since it is a proper fraction, we write it as

$$\frac{x^2 + 23x - 18}{(x - 1)(x + 2)(2x - 1)} = \frac{A}{x - 1} + \frac{B}{x + 2} + \frac{C}{2x - 1}$$

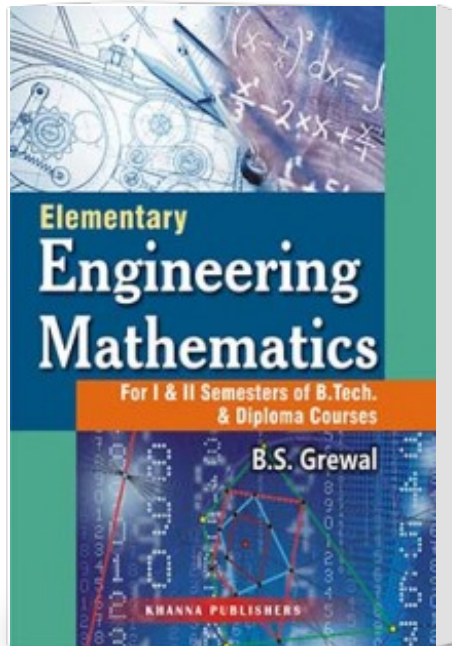
To find A , put $x = 1$ in the L.H.S. except in ' $x - 1$ ' itself. Then

$$A = \frac{1 + 23 - 18}{(1 + 2)(2 - 1)} = 2.$$

To find B , put $x = -2$ in the L.H.S. except in factor ' $x + 2$ ' itself. Then

$$B = \frac{4 - 46 - 18}{(-2 - 1)(-4 - 1)} = -\frac{60}{15} = -4.$$

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