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Chapterwise
Previous Years'

Solved Papers

2015-2000

GATE Physics

Containing 3 Practice Sets

Dr. Vijay Kumar
Dr. Israt Ali

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SOLVED PAPERS
(2015-2000)

GATE
Physics

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Take a Ride of **SUCCESS** in **GATE 2016**

Graduate Aptitude Test in Engineering (GATE) is an all India examination that primarily tests the comprehensive understanding of various undergraduate subjects in Engineering and Technology. The GATE score of a candidate reflects a relative performance of that candidate. The score is used for admissions to post-graduate engineering programmes (e.g., M.E., M.Tech, direct Ph.D.) in Indian higher education institutes with financial assistance provided by MHRD and other Government agencies. The score may also used by Public sector units for employment screening purposes.

Most of the PSUs release their recruitment notifications right after the GATE notification, indicating that candidates have to take GATE to be considered for a job in their organisations.

So making a great score in GATE is very considerable for availing all these opportunities. Keeping the above point of view and after exhaustive research, I have come up with the Solved Papers of **GATE Physics** for the aspirants who are preparing for GATE Exam. This book contains previous years' (2015-2000) GATE Solved Papers with their authentic solutions to help you getting a good score. The chapterwise solved papers in this book will help the students to find the solutions to their problems and to get through the exam without any difficulty. This integrated package of solved papers has been designed to know about the GATE Exam pattern and to put yourself into the real practice.

After going through this solved paper, aspirants can climb the ladder of success and fulfill their dreams.

We are thankful to Arihant Publications (India) Limited for giving us this opportunity to make such book which will help you to get 100% success in GATE exam.

Valuable suggestions are always welcome for further improvement.

Authors

Exploring the **PAGES...**

GATE

Graduate Aptitude Test in Engineering

"Graduate Aptitude Test in Engineering is an all India level examination, conducted and controlled by the Indian Institute of Science in cooperation with seven Indian Institutes of Technology on behalf of the National Coordination Board – GATE, Department of Higher Education, Ministry of Human Resource Development (MHRD), and Government of India. The GATE committee which consists of representatives from the governing institutes is the sole authority for conducting the examination and declaring the results."

GATE Eligibility Criteria

The following categories of candidates are eligible to appear in GATE

- Candidates with Bachelor Degree in Engineering/Technology/Architecture (4 years after 10+2) and those who are in the final of such programme.
- Candidates with Master Degree in any branch of Science/Mathematics/Statistics/ Computer Applications or its equivalent and those who are in the final year of such programme.
- Candidates in the second/third/higher year of the Four-year Integrated Master Degree Programme (Post B.Sc.) in Engineering/Technology, or 4th/5th year of Five-year Integrated Master Degree Programme and Dual Degree Programme in Engineering/ Technology
- Candidates with qualifications obtained through examinations conducted by professional societies recognized by UPSC/AICTE (e.g., AMIE by IE (I), AMICE (I) by the Institute of Civil Engineers (India) - ICE (I)) as equivalent to B.E. / B. Tech. The students who have completed his/ her Bachelor Degree in Engineering (4 years after 10+2) or equivalent of such professional courses are also eligible.

Examination Pattern

The GATE consists of a single paper of 3 hours duration, which contains 65 questions carrying maximum of 100 marks. The question paper is divided into three sections.
First section consists of 25 questions (1 to 25) of ONE MARK each.
Second section consists of 30 questions (26 to 55) of TWO MARKS each.
Third section consists of 10 questions (56 to 65) of ONE MARK (56 to 60) and of TWO MARKS (61 to 65) each.

Multiple Choice and Numerical Answer Type Questions

The question paper consist of multiple choice and numerical answer type questions. In multiple choice type question, each question has four choices for the answer. In numerical answer type questions, there will be no responses to mark at all. To enter a number as your answer, use the virtual numerical keyboard displayed on the monitor.

Zones and Institutes for GATE

The GATE is conducted and controlled through eight zones which are as follow

Zone 1 IISC, Bangalore | **Zone 2** IIT, Bombay | **Zone 3** IIT, Delhi | **Zone 4** IIT, Guwahati
Zone 5 IIT, Kanpur | **Zone 6** IIT, Kharagpur | **Zone 7** IIT, Madras | **Zone 8** IIT, Roorkee

GATE 2015 exam was conducted by IIT, Kanpur

Negative Marking in GATE Exam

Incorrect answer carry negative marks i.e., 0.33 for one mark questions and 0.66 for two marks questions. Also there is no negative marking for questions of numerical answer type.

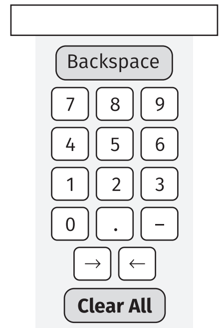
Answering a Question in Online Test

Procedure for answering a multiple choice type questions are as follows

- To select your answer, click on the button of one of the options.
- To deselect your chosen answer, click on the button of the chosen option again or click on the Clear Response button.
- To change your chosen answer, click on the button of another option, and to save your answer, you must click on the Save and Next button.
- To mark the question for review, click on the Mark for Review and Next button.

Procedure for answering a numerical answer type questions are as follows

- To enter a number as your answer, use the virtual numerical keypad displayed on the monitor.
- A fraction (e.g.,-0.3 or -.3) can be entered as an answer with or without '0' before the decimal point.
- To clear your answer, click on the Clear Response button and to save your answer, you MUST click on the Save and Next button.
- To mark the question for review, click on the Mark for Review and Next button.



Virtual Numerical Keypad

If an answer is entered for a question that is Marked for Review, that answer will be considered in the evaluation.

Formula for GATE Score of CE, CS, EC, EE and ME Papers

After the evaluation of the answer normalised marks based on the formula given below was calculated corresponding to the raw marks obtained by a candidate for CE, CS, EC, EE and ME papers. Normalised mark of j^th candidate in i^th session $\overline{M}_{ij}$ in given by

$$\hat{M}_{ij} = \frac{\overline{M}_t^g - M_q^g}{\overline{M}_{ti} - M_{iq}} (M_{ij} - M_{iq}) + M_q^g$$

where,

- M_{ij} = the actual marks obtained by the j^th candidate in i^th session
- $\overline{M}_t^g$ = the average marks of the top 0.1% of the candidates considering all sessions
- M_q^g = the sum of mean and standard deviation marks of the candidates in the paper considering all sessions
- $\overline{M}_{ti}$ = the average marks of the top 0.1% of the candidates in the i^th session
- M_{iq} = the sum of mean marks and standard deviation i^th session

Formula for GATE Score of all Papers

The formula for GATE score of all papers is given below

$$S = S_q + (S_t - S_q)\frac{M - M_q}{\overline{M}_t - M_q}$$

where,

- S = the GATE Score of the candidate,
- M = the marks obtained by the candidate in the paper appeared by the candidate in GATE 2013,
- M_q = the qualifying mark for general category candidates in the paper,
- $\overline{M}_t$ = the mean of marks of top 0.1% or top 10 (whichever is larger) of the candidates who appeared in the paper,
- S_t = 900 is the Score assigned to $\overline{M}_t$,
- S_q = 350 is the Score assigned to M_q.

M_q is usually 25 marks (out of 100) or μ+s, whichever is larger. Here μ is the mean and s is the standard deviation of marks of all the candidates who appeared in the paper.

GATE 2015 Cut-off Marks for Some Branches & Categorywise

BRANCH	GENERAL	SC/ST/PD	OBC (Non-Creamy)
Electronics & Communication Engineering	25.0	16.67	22.50
Electrical Engineering	25.0	16.67	22.50
Mechanical Engineering	32.73	21.82	29.46
Computer Science & IT	25.0	16.67	22.50
Civil Engineering	25.0	16.67	22.50

Paper Codes for GATE

- Aeronautical/Aerospace Engineering (AE)
- Agricultural Engineering (AG)
- Architecture and Planning (AR)
- Biotechnology (BT)
- Civil Engineering (CE)
- Chemical Engineering (CH)
- Computer Science and Information Technology (CS)
- Chemistry (CY)
- Electronics and Communication Engineering (EC)
- Electrical Engineering (EE)
- Geology and Geophysics (GG)
- Instrumentation Engineering (IN)
- Mathematics (MA)
- Mechanical Engineering (ME)
- Mining Engineering (MN)
- Metallurgical Engineering (MT)
- Physics (PH)
- Production and Industrial Engineering (PI)
- Textile Engineering and Fibre Science (TF)
- Engineering Science (XE)
- Life Science (XL)
- Ecology and Evolution (EY)

Recent Changes in GATE

2014

A new paper **Ecology and Evolution** is introduced in GATE 2014.

Examinations for all the 22 papers will be conducted by an ONLINE Computer Based Test (CBT).

GATE 2014 examination was held during forenoon and afternoon session on alternate weekends (Saturday and Sunday) between 1st February 2014 and 2nd March 2014. Examination for some of the papers in GATE 2014 was held in multiple sessions.

Application Fee has been revised again. It is ₹1500 for Male Candidates (General/OBC), ₹ 750 for Women Candidates of any category, ₹1500 for Other Candidates (General/OBC) and ₹750 SC / ST / PwD* Category Candidates

In terms of pattern, only multiple choice questions and numerical answer type questions are asked, there was no coverage of common date and linked answer type questions.

2013

Female candidates are exempted from paying the application fee.

Candidates are required to upload scanned copy of Photograph and Signature.

The Application fee was increased from ₹1000 to ₹1200.

2012

Only final year students and passout students were eligible to write GATE exam.

The application process was made completely online, candidates could view their responses of the ORS and also GATE Office released official solutions for GATE papers.

PLAN Your Study to **KNOCK OUT** the Exam

1. Planning your study after analysing previous years' papers

You must plan your study after taking the analysis of previous years' GATE papers. This will give you the idea that which subject usually has the highest weightage in GATE exam and about the topics asked from the particular subject most. After the analysis of previous years' papers of GATE Physics, you will be able to make the list of topics to be studied on the priority basis. You also will be able to prepare time schedule to study individual topics.

2. Prefer standard books for GATE

Make an extensive search for standard books in the library and go for the best ones. Try to cover complete syllabus within the time you have (5 or 6 months). If it isn't possible, get expertise in the topics you have studied.

3. Group study is a great comrade

Group study is one of the best ways for preparation of GATE. Assign a few sections/topics to your friend and you focus on the remaining. Then have a brief session and discussion and exchange what both of you have studied/gained. This not only saves your time and efforts but also enhances understanding on the topics/concepts.

4. Solve the previous years' GATE papers

Solve previous years' GATE papers to understand what the actual paper would be like. It also brushes up your mind and tells you the weaknesses in the subject knowledge. So try to solve as many test papers as you can. This is the best way to prepare and get through the GATE.

5. Practice Sets is a great platform to check yourself

Attempt all the 65 questions given in a practice set and then check yourself step by step by considering the answer with solutions given along with each practice set. Because, the practice sets are designed based exactly on the latest exam pattern, hence you will find your strongest and weakest points related to all sections i.e., Aptitude, Mathematics and Engineering disciplines in a particular topic. So, keep practicing and secure your success in GATE.

6. Analyse your results

Analysing the results from your solved questions from the practice sets and solved papers is very significant. If you do not analyse, it does not add value to your performance. You should check and find out where you have mistaken and could have scored more. Know your accuracy rates in various topics and prepare a topicwise datasheet to make record of your performance in different solved papers and practice sets.

7. Keep the time for Revision

As time management is an important factor to crack the exam. Hence, give the appropriate time to each subject and complete that subject within your time schedule. After completion of the whole syllabus, you must have time for the quick revision.

TIPS and TACTICS for a Successful Attempt

1. **Read Carefully**
   Make it a habit of always reading the instructions on the front page carefully. Also, before attempting the questions, always read and understand the directions given to attempt a question so that scope for blunders is reduced.

2. **Solve The Easy Questions First**
   Try to bifurcate the questions according to the difficulty level. Always try and attempt the easy questions first as this saves a lot of time. Time management plays a vital part in achieving success.

3. **Strike Off The Wrong Options**
   Try to strike off the wrong options. The options which cannot be the answer even a slightest bit and for which you are cent percent sure, should be separated from the probable answer so that you are able to concentrate on the remaining options and hence you find out the answer easily.

4. **Don't Waste Excessive Time On One Question**
   If you are not able to strike off the wrong options and are unsure of the correct answer then don't waste excessive time as doing so, will lead to a decrease in remaining time and hence other question will suffer.

5. **Use Scribble Pad**
   You can use the scribble pad provided for rough work. You can jot down the points, ideas etc so that least number of mistakes are made while framing the final answer.

6. **Try To Attempt The Leftover Questions Once More**
   After completing all the question go back to the leftover questions and try to give it a shot and solve it once more.

7. **Keep A Time Check**
   During the examination do keep an eye on the time. Try to save time so that you are left with some time to attempt the leftover questions. Try to finish the exam early so that you are left with some amount of time to revise the whole paper once.

8. **Use The Option Of Mark For Review**
   The answer will be considered in the evaluation if an answer is entered for a question that is marked for review.

9. **Use Virtual Numerical Keypad**
   Although, there is no negative marking for numerical answer type questions, but entered a number as your answer if you are sure about that because, these type of questions containing a particular numerical value.

GATE Syllabus for
Physics

Mathematical Physics

Linear vector space; matrices; vector calculus; linear differential equations; elements of complex analysis; Laplace transforms, Fourier analysis, elementary ideas about tensors.

Classical Mechanics

Conservation laws; central forces, Kepler problem and planetary motion; collisions and scattering in laboratory and centre of mass frames; mechanics of system of particles; rigid body dynamics; moment of inertia tensor; noninertial frames and pseudo forces; variational principle; Lagrange's and Hamilton's formalisms; equation of motion, cyclic coordinates, Poisson bracket; periodic motion, small oscillations, normal modes; special theory of relativity – Lorentz transformations, relativistic kinematics, mass-energy equivalence.

Electromagnetic Theory

Solution of electrostatic and magnetostatic problems includingboundary value problems;dielectrics andconductors; Biot-Savart's and Ampere's laws; Faraday's law; Maxwell's equations; scalar and vector potentials; Coulomb and Lorentz gauges; Electromagnetic waves and their reflection, refraction, interference, diffraction and polarization. Poynting vector, Poynting theorem, energy and momentum of electromagnetic waves; radiation from a moving charge.

Quantum Mechanics

Physical basis of quantum mechanics; uncertainty principle; Schrodinger equation; one, two and three dimensional potential problems; particle in a box, harmonic oscillator, hydrogen atom; linear vectors and operators in Hilbert space; angular momentum and spin; addition of angular momenta; time independent perturbation theory; elementary scattering theory.

Thermodynamics and Statistical Physics

Laws of thermodynamics; macrostates and microstates; phase space; probability ensembles; partition function, free energy, calculation of thermodynamic quantities; classical and quantum statistics; degenerate Fermi gas; black body radiation and Planck's distribution law; Bose-Einstein condensation; first and second order phase transitions, critical point.

Atomic and Molecular Physics

Spectra of one- and many-electron atoms; LS and jj coupling; hyperfine structure; Zeeman and Stark effects; electric dipole transitions and selection rules; X-ray spectra; rotational and vibrational spectra of diatomic molecules; electronic transition in diatomic molecules, Franck-Condon principle; Raman effect; NMR and ESR; lasers.

Solid State Physics

Elements of crystallography; diffraction methods for structure determination; bonding in solids; elastic properties of solids; defects in crystals; lattice vibrations and thermal properties of solids; free electron theory; band theory of solids; metals, semiconductors and insulators; transport properties; optical, dielectric and magnetic properties of solids; elements of superconductivity.

Nuclear and Particle Physics

Nuclear radii and charge distributions, nuclear binding energy, Electric and magnetic moments; nuclear models, liquid drop model – semi-empirical mass formula, Fermi gas model of nucleus, nuclear shell model; nuclear force and two nucleon problem; Alpha decay, Beta-decay, electromagnetic transitions in nuclei;Rutherford scattering,nuclear reactions, conservation laws; fission and fusion;particle accelerators and detectors; elementary particles, photons, baryons, mesons and leptons; quark model.

Electronics

Network analysis; semiconductor devices; Bipolar Junction Transistors, Field Effect Transistors, amplifier and oscillator circuits; operational amplifier, negative feedback circuits ,active filters and oscillators; rectifier circuits, regulated power supplies; basic digital logic circuits, sequential circuits, flip-flops, counters, registers, A/D and D/A conversion.

GATE 2015

Solved Paper

Physics

Solved Paper 2015

GATE Physics

Time : 3 hrs **MM :** 100

Read the following instructions carefully

1. This paper consists of 65 questions carrying 100 marks of 3 hrs duration.
2. Questions 1 to 25 carry 1 mark each and questions 26 to 55 carry 2 marks each.
3. Questions 56 to 65 belong to General Aptitude (GA) Type. Questions 56 to 60 carry 1 mark each and questions 61 to 65 carry 2 marks each.
4. Unattempted questions will carry zero marks.
5. For questions 1 to 25 and 56 to 60, 1/3 mark will be deducted for each wrong answer. For questions 26 to 55 and 61 to 65, 2/3 mark will be deducted for each wrong answer.
6. There is no negative marking for numerical answer type questions.
7. Non-programmable type calculator is allowed. Charts, graph sheets and mathematical tables are not allowed in the examination hall.

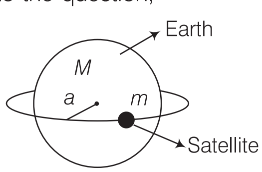
(1 *Mark Questions*)

1. A satellite is moving in a circular orbit around the earth. If T, V and E are its average kinetic, average potential and total energies, respectively, then which one of the following options is correct?

(a) $V = -2T$, $E = -T$ (b) $V = -T$, $E = 0$

(c) $V = -T/2$, $E = T/2$ (d) $V = -3T/2$, $E = -T/2$

Sol. (a) According to the question,



Average kinetic energy of the satellite = T

Average potential energy of the satellite = V

Total energy of the satellite = E

Suppose, mass of the earth = M

Mass of the satellite = m

Radius of the circular orbit = a

For circular motion of the satellite, we can write

$$\frac{GMm}{a^2} = \frac{mv^2}{a}$$

or $$\frac{GM}{a} = v^2$$

or $$v = \sqrt{\frac{GM}{a}}$$

Kinetic energy of the satellite,

$$\text{KE} = T = \frac{1}{2}mv^2 = \frac{1}{2}m \times \frac{GM}{a} = \frac{GMm}{2a}$$

Potential energy of the system (earth + satellite),

$$\text{PE} = V = -\frac{GMm}{a}$$

$\Rightarrow$ $$V = -2T$$

Now, total energy of the system

$$E = T + V = T - 2T = -T$$

2. The Pauli matrices for three spin-$\frac{1}{2}$ particles are σ_1, σ_2 and σ_3, respectively. The dimension of the Hilbert space required to define an operator $\hat{\mathbf{O}} = \sigma_1 \cdot \sigma_2 \times \sigma_3$ is ______ .

Sol. We can write, the Pauli matrices as

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \text{ 2×2 complex matrix}$$

$$\sigma_2 = \sigma_y = \begin{pmatrix} 0 & -1 \\ i & 0 \end{pmatrix}, \text{ 2×2 complex matrix}$$

$$\sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \text{ 2×2 complex matrix}$$

Thus, we can have

$$\sigma_1 \cdot \sigma_2 \equiv 2 \times 2 \qquad \text{[complex matrix]}$$

Similarly, $\hat{\mathbf{O}} = (\sigma_1 \cdot \sigma_2) \times \sigma_3$

$\equiv 2 \times 2$ matrix with four operators

Thus, required dimension is $8(2 \times 4)$.

3. The mean kinetic energy of a nucleon in a nucleus of atomic weight A varies as A^n, where n is ______ (upto two decimal places).

Sol. The mean kinetic energy of a nucleon (–0.67) in a nucleus of atomic weight A is

$$E_{k_{in}}(N, Z) = \frac{3}{10mN} \cdot \frac{\hbar^2}{R_0^2} \left(\frac{9\pi}{4} \right)^{2/3} \cdot \frac{N^{5/3} + Z^{5/3}}{A^{2/3}}$$

$$\Rightarrow \quad E_{k_{in}}(N, Z) \propto \frac{1}{A^{2/3}} \quad \text{or} \quad E_{k_{in}} \propto A^{-2/3}$$

Thus, $\quad n = -\frac{2}{3} \approx -0.67$

4. Let $\mathbf{L}$ and $\mathbf{p}$ be the angular and linear momentum operators, respectively for a particle. The commutator $[L_x, p_y]$ gives

(a) $-i\hbar p_z$ (b) 0

(c) $i\hbar p_x$ (d) $i\hbar p_z$

Sol. (d) Angular momentum,

$$L = \begin{vmatrix} x & y & z \\ r_x & r_y & r_z \\ p_x & p_y & p_z \end{vmatrix}$$

$$L_x = r_y p_z - r_z p_y$$

$$= -i\hbar \left(r_y \frac{\partial}{\partial r_z} - r_z \frac{\partial}{\partial r_y} \right)$$

$$\Rightarrow [L_x, p_y]\psi = -\hbar^2 \left(r_y \frac{\partial}{\partial r_z} - r_z \frac{\partial}{\partial r_y} \right) \frac{\partial}{\partial r_y} \psi$$

$$+ \hbar^2 \frac{\partial}{\partial r_y} \left(r_y \frac{\partial}{\partial r_z} - r_z \frac{\partial}{\partial r_y} \right) \psi$$

$$= -\hbar^2 \left(r_y \frac{\partial}{\partial r_z} \cdot \frac{\partial \psi}{\partial r_y} - r_z \frac{\partial}{\partial r_y} \cdot \frac{\partial \psi}{\partial r_y} - \frac{\partial \psi}{\partial r_z} - r_y \frac{\partial}{\partial r_y} \cdot \frac{\partial \psi}{\partial r_z} + r_z \frac{\partial^2 \psi}{\partial r_y^2} \right)$$

$$= \hbar^2 \frac{\partial \psi}{\partial r_z} = i\hbar \left(-i\hbar \frac{\partial \psi}{\partial r_z} \right)$$

$$= i\hbar \, p_z \psi$$

$$\Rightarrow \quad [L_x, p_y] = i\hbar \, p_z$$

5. The decay $\mu^+ \to e^+ + \gamma$ is forbidden, because it violates

(a) momentum and lepton number conservations

(b) baryon and lepton number conservations

(c) angular momentum conservation

(d) lepton number conservation

Sol. (d) In the standard model, leptonic family numbers would be preserved if neutrinos were massless. Since, neutrino oscillations have been observed, neutrinos do have tiny non-zero mass and conservation law for leptonic family numbers are therefore only approximate. This means that the conservation laws are violated.

6. An operator for a spin-$\frac{1}{2}$ particle is given by $\hat{\mathbf{A}} = \lambda \sigma \cdot \mathbf{B}$, where $\mathbf{B} = \frac{B}{\sqrt{2}}(\hat{\mathbf{x}} + \hat{\mathbf{y}})$, σ denotes Pauli matrices and λ is a constant. The eigenvalues of $\hat{\mathbf{A}}$ are

(a) $\pm \lambda B/\sqrt{2}$ (b) $\pm \lambda B$

(c) $0, \lambda B$ (d) $0, -\lambda B$

Sol. (b) According to the question,

$$\hat{\mathbf{A}} = \lambda \sigma \cdot \mathbf{B} \text{ and } \mathbf{B} = \frac{B}{\sqrt{2}}(\hat{\mathbf{x}} + \hat{\mathbf{y}})$$

We can write, value of Pauli matrices for a spin $\left(-\frac{1}{2}\right)$ particle as

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \text{ and } \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\sigma \cdot \mathbf{B} = \sigma_x B_x + \sigma_y B_y + \sigma_z B_z$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \frac{B}{\sqrt{2}} + \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \cdot \frac{B}{\sqrt{2}}$$

$$= \frac{B}{\sqrt{2}} \cdot \begin{bmatrix} 0 & 1-i \\ 1+i & 0 \end{bmatrix}$$

Now, $\hat{\mathbf{A}} = \lambda \sigma \cdot \mathbf{B}$

$$= \frac{\lambda B}{\sqrt{2}} \cdot \begin{bmatrix} 0 & 1-i \\ 1+i & 0 \end{bmatrix}$$

Suppose eigenvalues of $\hat{\mathbf{A}}$ are denoted by K, then

$$|\hat{\mathbf{A}} - K\hat{\mathbf{I}}| = 0$$

or $$\left|\frac{\lambda B}{\sqrt{2}}\begin{bmatrix} 0 & 1-i \\ 1+i & 0 \end{bmatrix} - K\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0$$

or $$\left|\begin{bmatrix} 0 & \frac{\lambda B}{\sqrt{2}}(1-i) \\ \frac{(1+i)\lambda B}{\sqrt{2}} & 0 \end{bmatrix} - \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}\right| = 0$$

or $$\begin{vmatrix} -K & \frac{\lambda B}{\sqrt{2}}(1-i) \\ \frac{(1+i)\lambda B}{\sqrt{2}} & -K \end{vmatrix} = 0$$

or $$K^2 - \left(\frac{\lambda B}{\sqrt{2}}\right)^2 (1 - i^2) = 0$$

or $$K^2 = \frac{(\lambda B)^2}{2} \times 2 = (\lambda B)^2$$

or $$K = \pm \lambda B$$

7. In an inertial frame S, two events A and B take place at $(ct_A = 0, \mathbf{r}_A = 0)$ and $(ct_B = 0, \mathbf{r}_B = 2\hat{\mathbf{y}})$, respectively. The times at which these events take place in a frame S' moving with a velocity $0.6c\hat{\mathbf{y}}$ with respect to S are given by

(a) $ct'_A = 0, ct'_B = -3/2$ (b) $ct'_A = 0, ct'_B = 0$
(c) $ct'_A = 0, ct'_B = 3/2$ (d) $ct'_A = 0, ct'_B = 1/2$

Sol. (a) For event A,

$$t'_A = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t_A - \frac{v\mathbf{r}_A}{c^2}}{\sqrt{1 - v^2/c^2}} = \frac{0 - 0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$\Rightarrow$ $$ct'_A = 0$$

For event B,

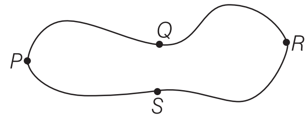
$$t'_B = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{t_B - \frac{v\mathbf{r}_B}{c^2}}{\sqrt{1 - v^2/c^2}} = \frac{0 - (0.6c)(2)/c^2}{\sqrt{1 - 0.36 \times c^2/c^2}}$$

$$= \frac{-(0.6)\left(\frac{2}{c}\right) \times 10}{\sqrt{64}} = \frac{-12}{8c}$$

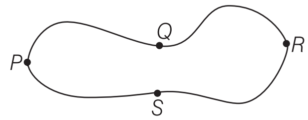
$\Rightarrow$ $$ct'_B = \frac{-3}{2}$$

8. Given that the magnetic flux through the closed loop $PQRSP$ is ϕ. If $\int_P^R \mathbf{A} \cdot \mathbf{dl} = \phi_1$ along PQR, the value of $\int_P^R \mathbf{A} \cdot \mathbf{dl}$ along PSR is



(a) $\phi - \phi_1$ (b) $\phi_1 - \phi$
(c) $-\phi_1$ (d) ϕ_1

Sol. (b) Consider the closed loop $PQRS$ shown below.



It is given that, $\oint_{PQRS} \mathbf{A} \cdot \mathbf{dl} = \phi$

$$\int_{P \to Q}^{R} \mathbf{A} \cdot \mathbf{dl} + \int_{R \to S}^{P} \mathbf{A} \cdot \mathbf{dl} = \phi$$

or $$\phi_1 + \int_{R \to S}^{P} \mathbf{A} \cdot \mathbf{dl} = \phi$$

or $$\int_{R \to S}^{P} \mathbf{A} \cdot \mathbf{dl} = \phi - \phi_1$$

$\Rightarrow$ $$\int_{P \to S}^{R} \mathbf{A} \cdot \mathbf{dl} = -(\phi - \phi_1) = (\phi_1 - \phi)$$

9. If $f(x) = e^{-x^2}$ and $g(x) = |x| e^{-x^2}$, then

(a) f and g are differentiable everywhere
(b) f is differentiable everywhere but g is not
(c) g is differentiable everywhere but f is not
(d) g is discontinuous at $x = 0$

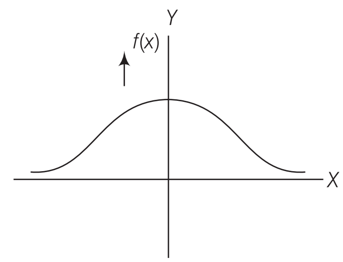
Sol. (b) Given, $f(x) = e^{-x^2}$ and $g(x) = |x| e^{-x^2}$

$$f(x) = e^{-x^2} \text{ for } \forall\, x \in R$$

$$g(x) = xe^{-x^2} \text{ for } x \geq 0$$

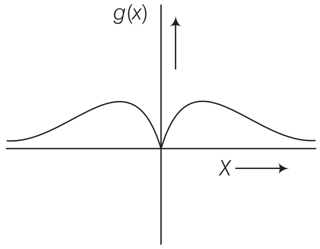
$$= -xe^{-x^2} \text{ for } x < 0$$

Graph of $f(x) = e^{-x^2}$ is shown below.



Due to presence of $|x|$ in $g(x)$.

Corner (at origin) will be formed which makes $g(x)$ non-differentiable at those points.



10. In Bose-Einstein condensates, the particles

(a) have strong interparticle attraction

(b) condense in real space

(c) have overlapping wavefunctions

(d) have large and positive chemical potential

Sol. (c) Bose-Einstein Condensate (BEC) is a state of matter in which separate atoms or subatomic particles when cooled to near absolute zero (0K) coalesce into a single quantum mechanical entity having overlapping wave functions.

11. Consider a system of N non-interacting spin-$\frac{1}{2}$ particles, each having a magnetic moment μ, is in a magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$. If E is the total energy of the system, the number of accessible microstates Ω is given by

(a) $\Omega = \dfrac{N!}{\frac{1}{2}\left(N - \frac{E}{\mu B}\right)!\,\frac{1}{2}\left(N + \frac{E}{\mu B}\right)!}$

(b) $\Omega = \dfrac{\left(N - \frac{E}{\mu B}\right)!}{\left(N + \frac{E}{\mu B}\right)!}$

(c) $\Omega = \frac{1}{2}\left(N - \frac{E}{\mu B}\right)!\,\frac{1}{2}\left(N + \frac{E}{\mu B}\right)!$

(d) $\Omega = \dfrac{N!}{\left(N + \frac{E}{\mu B}\right)!}$

Sol. (a) We can consider half number of particles as parallel to the magnetic field and other half number of particles as antiparallel to the magnetic field.

Number of accessible microstates,

$$\Omega = \frac{N!}{\pi_{j=1}^{2} [(S_1 \uparrow)(N - N_j)\,!]}$$

Here, $N_j = \dfrac{|\mathbf{E}|}{\mu \cdot \mathbf{B}}$

$$N_1 = \frac{E}{\mu B \cos 0^\circ} = \frac{E}{\mu B}$$

$$N_2 = \frac{E}{\mu B \cos 180^\circ} = \frac{E}{-\mu B}$$

$$\Rightarrow \quad \Omega = \frac{N!}{\frac{1}{2}(N - N_1)\,!\;\frac{1}{2}(N - N_2)\,!}$$

$$= \frac{N!}{\frac{1}{2}\left(N - \frac{E}{\mu B}\right)!\;\frac{1}{2}\left(N + \frac{E}{\mu B}\right)}$$

12. For a black body radiation in a cavity, photons are created and annihilated freely as a result of emission and absorption by the walls of the cavity. This is because

(a) the chemical potential of the photons is zero

(b) photons obey Pauli exclusion principle

(c) photons are spin-1 particles

(d) the entropy of the photons is very large

Sol. (a) For a black body, the energy distribution is established by the interaction of the photons with matter, usually the walls of the container. In this course of interaction, the number of photons is not conserved. As a result, the chemical potential of the black body photon gas becomes zero.

13. Consider $w = f(z) = u(x, y) + iv(x, y)$ to be an analytic function in a domain D. Which one of the following options is not correct?

(a) $u(x, y)$ satisfies Laplace equation in D

(b) $v(x, y)$ satisfies Laplace equation in D

(c) $\int_{z_1}^{z_2} f(z)\, dz$ is dependent on the choice of the contour between z_1 and z_2 in D

(d) $f(z)$ can be Taylor expanded in D

Sol. (c) Given function is $f(z) = u(x, y) + iv(x, y)$

Clearly, $f(x)$ is a harmonic function thus $u(x, y)$ and $v(x, y)$ will satisfy Laplace equation.

$f(z)$ cannot be Taylor expanded because $u(x, y)$ and $v(x, y)$ are independent functions.

14. The value of $\int_0^3 t^2 \delta(3t - 6)\, dt$ is _______ (upto one decimal place).

Sol. Let $I = \int_0^3 t^2 \delta(3t - 6) dt = \int_0^3 t^2 \times \delta\{3(t - 2)\} dt$

$$= \frac{1}{3}\int_0^3 t^2 \delta(t - 2) dt \qquad \left[\because \delta(at - b) = \frac{1}{|a|}\delta\left(t - \frac{b}{a}\right)\right]$$

$$= \frac{1}{3}[2]^2 \qquad \left[\because \int_{-\infty}^{\infty} \delta(t - t_0)\phi(t)\lambda t = \phi(t_0)\right]$$

$$= \frac{4}{3} \Rightarrow I = \frac{4}{3} = 1.3$$

15. Which one of the following does not represent an exclusive OR operation for inputs A and B?

(a) $(A + B)\,\overline{AB}$ (b) $A\overline{B} + B\overline{A}$

(c) $(A + B)\,(\overline{A} + \overline{B})$ (d) $(A + B)\,AB$

Sol. (d) (a) Let $y = (A+B)\overline{AB} = (A+B)(\overline{A}+\overline{B})$

$$= A\overline{A} + A\overline{B} + B\overline{A} + B\overline{B}$$

$$= A\overline{B} + \overline{A}B = A \oplus B$$

(b) $y = A\overline{B} + \overline{A}B = A \oplus B$

(c) $y = (A+B)(\overline{A}+\overline{B})$

$$= A\overline{A} + A\overline{B} + B\overline{A} + B\overline{B}$$

$$= A\overline{B} + B\overline{A} = A \oplus B$$

(d) $y = (A+B)AB = AAB + BAB$

$$= AB + BA = AB \neq A \oplus B$$

16. Consider a complex function $f(z) = \dfrac{1}{z\left(z+\dfrac{1}{2}\right)\cos(z\pi)}$.

Which one of the following statements is correct?

(a) $f(z)$ has simple poles at $z = 0$ and $z = -\dfrac{1}{2}$

(b) $f(z)$ has a second order pole at $z = -\dfrac{1}{2}$

(c) $f(z)$ has infinite number of second order poles

(d) $f(z)$ has all simple poles

Sol. (b) Given, $f(z) = \dfrac{1}{z\left(z+\dfrac{1}{2}\right)\cos(z\pi)}$

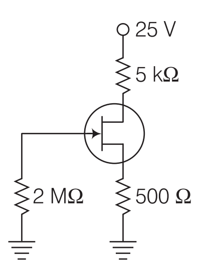
In the denominator, two terms give zeros at $z = -\dfrac{1}{2}$

$$\cos(z\pi) = \cos\left(-\frac{\pi}{2}\right) = 0$$

$$z + \frac{1}{2} = -\frac{1}{2} + \frac{1}{2} = 0$$

Thus, $f(z)$ has second order pole at $z = -\dfrac{1}{2}$.

17. In the given circuit, the voltage across the source resistor is 1 V. The drain voltage (in V) is _______ .



Sol. According to the given condition,

$$V_{DS} \geq V_{GS} - V_p$$

Thus, JFET is in saturation region.

Now, we can write

$$I_D \times 500 = 1\text{V}$$

$$\Rightarrow \quad I_D = 2 \text{ mA}$$

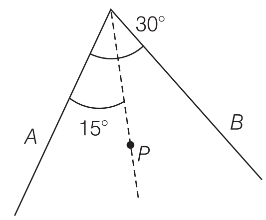
Similarly, $V_{DD} = (I_D \times 5 \text{ k}\Omega) + V_{DG}$

or $\quad 25 = (2 \text{ mA})(5 \text{ k}\Omega) + V_{DG}$

or $\quad V_{DG} = 25 - 10 = 15 \text{ V}$

18. A point charge is placed between two semi-infinite conducting plates which are inclined at an angle of 30° with respect to each other. The number of image observed is _______ .

Sol. Suppose, a point P is placed between two semi-infinite conducting plates A and B, as shown in the figure.



The system is equivalent to image formation by two plane mirrors inclined at an angle θ and the object is placed symmetrically.

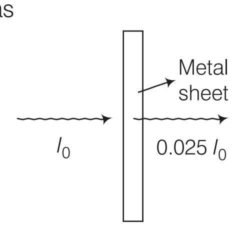
Thus, number of images observed

$$n = \frac{360°}{\theta} - 1$$

or $$n = \frac{360°}{30°} - 1 = 12 - 1 = 11$$

19. A beam of X-ray of intensity I_0 is incident normally on a metal sheet of thickness 2 mm. The intensity of the transmitted beam is $0.025 I_0$. The linear absorption coefficient of the metal sheet (in per m) is _______ (upto one decimal place)

Sol. The intensity of transmitted beam through the metal sheet can be written as



$$I = I_0 e^{-\alpha z}$$

where, α = absorption coefficient

z = thickness of the sheet.

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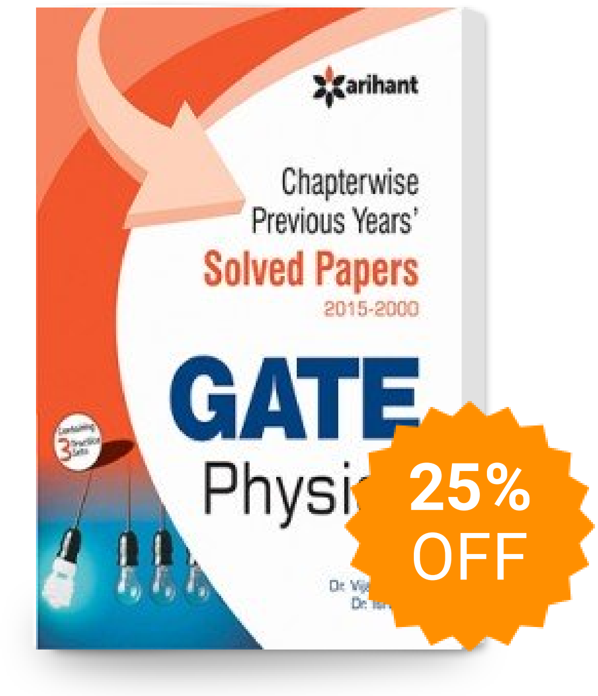

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