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DELHI SUBORDINATE SERVICES SELECTION BOARD

P.G.T.

RECRUITMENT EXAM.

Mathematics

Editorial Board
PRATIYOGITA DARPAN

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Phone : 4053333, 2530966, 2531101

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E-mail : care@upkar.in, **Website :** www.upkar.in

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1-8-1/B, R.R. Complex (Near Sundaraiah Park, Adjacent to Manasa Enclave Gate),
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ALGEBRA

1 THEORY OF EQUATIONS & SYMMETRIC FUNCTIONS OF THE ROOTS

1. Polynomial equation—A polynomial which is equal to zero is called a polynomial equation.

For example—$2x + 5 = 0$, $x^2 - 2x + 5 = 0$, $2x^3 - 5x^2 + 1 = 0$ etc. are polynomial equations.

2. Root of a polynomial equation—If $f(x) = 0$ is a polynomial equation and $f(\alpha) = 0$, then α is called a root of the polynomial equation $f(x) = 0$.

3. Factor Theorem—If α is a root of equation $f(x) = 0$, then the polynomial $f(x)$ is exactly divisible by $x - \alpha$ (*i. e.*, remainder is zero).

For example—$x^2 - 5x + 6 = 0$ is divisible by $x - 2$ because 2 is the root of the given equation.

4. Theorem—Every equation $f(x) = 0$ of n^{th} degree has exactly n roots.

5. Multiplicity of a Root—If α is a root of the polynomial equation $f(x) = 0$, then α is called a root of multiplicity r if $(x - \alpha)^r$ divides $f(x)$.

For example—In the equation

$(x + 1)^4 (x - 2)^3 (2x - 6) = 0$ the roots $-1, 2, 3$ are of multiplicity 4, 3 and 1 respectively.

6. Complex roots of equations with real coefficients—If the equation $f(x) = 0$ with real constant coefficients has a complex root $\alpha + i\beta$ $(\alpha, \beta \in R, \beta \neq 0)$, then the complex conjugate $\alpha - i\beta$ would also be a root of the polynomial equation $f(x) = 0$.

Example—Let the equation is—

$$x^2 - 4x + 13 = 0.$$

Here, the coefficients are all real numbers.

$\therefore$ Roots of the equation are

$$x = \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$= 2 \pm \sqrt{-9} = 2 \pm 3i$$

$\therefore \quad x = 2 + 3i$ and $x = 2 - 3i$

$\therefore$ The roots are complex conjugate of each other.

Example—Let the equation be

$x^2 - (1 - 2i)x - 2i = 0$.

The roots of this equation are $1, 2i$ which are not conjugate pair because all the coefficients of the given equation are not real.

7. Irrational roots of equations with rational coefficients—If the equation $f(x) = 0$ with rational coefficients has an irrational root $\alpha + \sqrt{\beta}$ $(\alpha, \beta \in Q, \beta > 0)$ and is not a perfect square, then $\alpha - \sqrt{\beta}$ would also be a root of the polynomial equation $f(x) = 0$

Example—Consider the polynomial equation

$$x^2 - 4x + 1 = 0$$

Here, the coefficients are all rational numbers

$\therefore$ The roots of the equation are

$$x = \frac{4 \pm \sqrt{16 - 4}}{2}$$

$$= \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}.$$

$\therefore \quad x = 2 + \sqrt{3}, x = 2 - \sqrt{3}.$

The irrational roots have occurred in pair.

Example—Consider the equation

$$x^2 - \left(2 + \sqrt{3}\right)x + 2\sqrt{3} = 0.$$

The roots of the equation are $2, \sqrt{3}$

Here, the roots $2, \sqrt{3}$ are not in conjugate pair because all the coefficients of the given equation are not integer.

8. Quadratic Equation—Consider the equation

$$ax^2 + bx + c = 0, \forall\ a, b, c \in R$$

$\therefore$ The roots of this equation are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $D = b^2 - 4ac$ is called the *discriminant* of the equation.

The roots are real and unequal, if $D > 0$

The roots are real and equal if, $D = 0$

The roots are complex and unequal, if $D < 0$

9. Relation between roots and coefficients of an equation—

Let $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ be the n roots of the equation

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n = 0,$$

then we have the following relations

Sum of the roots taken one at a time $= \Sigma\alpha_1$

i.e., $\alpha_1 + \alpha_2 + \ldots + \alpha_n = -\dfrac{a_1}{a_0}$

Sum of the roots taken two at a time $= \Sigma\alpha_1\alpha_2$

i.e., $\alpha_1\alpha_2 + \alpha_2\alpha_3 + \ldots = \dfrac{a_2}{a_0}$

Sum of the roots taken three at a time

$= \Sigma\alpha_1\alpha_2\alpha_3$

i.e., $\alpha_1\alpha_2\alpha_3 + \alpha_2\alpha_3\alpha_4 + \ldots = -\dfrac{a_3}{a_0}$

- -

- -

Product of the n roots

$$= \alpha_1\alpha_2\alpha_3\ldots\alpha_n = \frac{(-1)^n a_n}{a_0}$$

The expressions $\Sigma\alpha_1, \Sigma\alpha_1\alpha_2, \Sigma\alpha_1\alpha_2\alpha_3, \ldots \alpha_1\alpha_2\alpha_3\ldots\alpha_n$ are called the elementary symmetric functions of $\alpha_1, \alpha_2, \ldots, \alpha_n$.

10. (i) Relation for quadratic equations—Let α, β be two roots of the quadratic equation $a_0x^2 + a_1x + a_2 = 0$, then

$$\alpha + \beta = -\frac{a_1}{a_0}$$

$$\alpha\beta = \frac{a_2}{a_0}.$$

(ii) Relation for cubic equations—Let α, β, γ be three roots of the cubic equation $a_0x^3 + a_1x^2 + a_2x + a_3 = 0$, then

$$\alpha + \beta + \gamma = -\frac{a_1}{a_0}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{a_2}{a_0}$$

$$\alpha\beta\gamma = -\frac{a_3}{a_0}$$

(iii) Relation for bi-quadratic equations—Let $\alpha, \beta, \gamma, \delta$ be the four roots of the bi-quadratic equation

$a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$, then—

$$\alpha + \beta + \gamma + \delta = -\frac{a_1}{a_0}$$

$$\alpha\beta + \beta\gamma + \gamma\delta + \delta\alpha + \alpha\gamma + \beta\delta = \frac{a_2}{a_0}$$

$$\alpha\beta\gamma + \beta\gamma\delta + \gamma\delta\alpha + \delta\alpha\beta = -\frac{a_3}{a_0}$$

$$\alpha\beta\gamma\delta = \frac{a_4}{a_0}$$

ILLUSTRATIONS

Example 1. If the ratio of the roots of the equation $ax^2 + bx + c = 0$ is r, then $\dfrac{(r+1)^2}{r}$ is equal to—

(A) $\dfrac{a^2}{bc}$ (B) $\dfrac{b^2}{ca}$

(C) $\dfrac{c^2}{ab}$ (D) $\dfrac{1}{abc}$

Solution : Let the roots be α, β, then

$\alpha + \beta = -b/a$ and $\alpha\beta = c/a$.

Given that $\dfrac{\alpha}{\beta} = r$

$$\therefore \quad \frac{(r+1)^2}{r} = \frac{\left(\frac{\alpha}{\beta} + 1\right)^2}{\frac{\alpha}{\beta}}$$

$$= \frac{(\alpha + \beta)^2}{\alpha\beta} = \frac{(-b/a)^2}{c/a} = \frac{b^2}{ac}$$

$\therefore$ The correct answer is (B).

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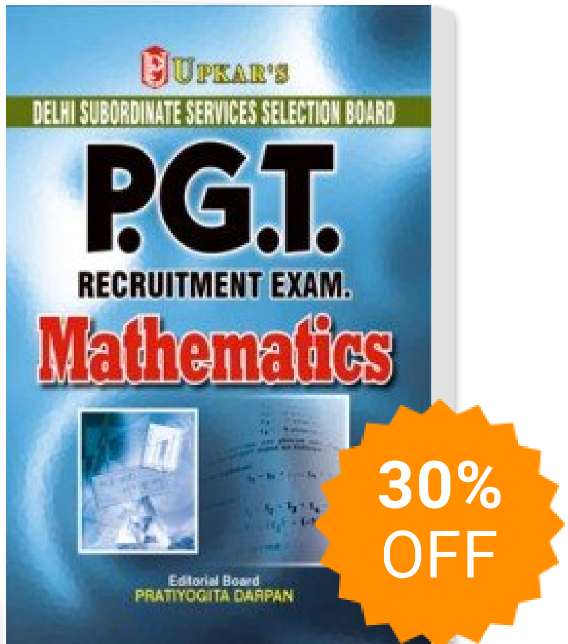

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