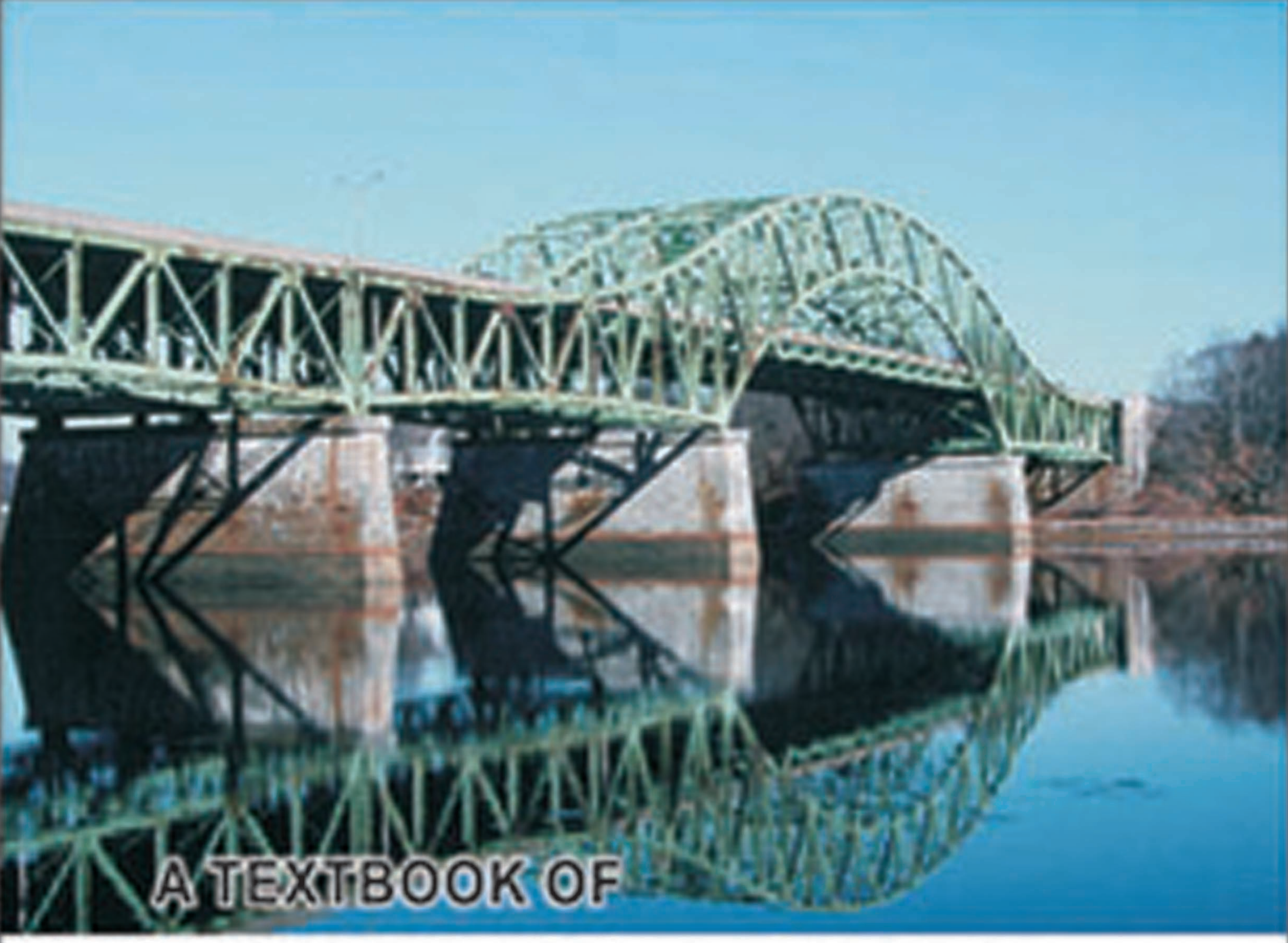

A TEXTBOOK OF

ENGINEERING MATHEMATICS

(For Cochin University of Science and Technology, Kerala)

SEMESTER-IV

N.P. Bali

Dr. Remadevi.S

A TEXTBOOK OF ENGINEERING MATHEMATICS

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ENGINEERING MATHEMATICS

For

B.Tech., (*IV Semester*)
Cochin University of Science and Technology, Kerala
(Strictly according to the latest revised syllabus)

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PREFACE TO THE FIRST EDITION

This book is a part of the original book **"A Textbook of Engineering Mathematics"** by N.P. Bali running into eighth edition and very well received by the students and teachers of all Indian Universities. The rapid sale of the eighth edition bears testimony to the overwhelming response. We thank them for the appreciation. Some new topics that are required according the syllabus are added.

The present form of the book is divided into four modules and covers the entire portion in the B.Tech., fourth semester Cochin University Engineering Mathematics-III syllabus. Problems taken from Cochin University question papers are included with solutions in each module.

There is no dearth of books on Engineering Mathematics but the students find it difficult to solve most of the problems in the exercise in the absence of adequate number of solved examples. An outstanding and distinguishing feature of the book is the large number of typical solved examples followed by well-graded problems.

We have endeavoured to present the fundamental concepts in a comprehensive and lucid manner. We are indebted to all authors, Indian and Foreign, whose works we have frequently consulted.

All efforts have been made to keep the book free from errors. Answers to all exercise have been rechecked. All suggestions for improvement will be highly appreciated and gratefully acknowledged.

—Authors

SYLLABUS

FOR B.TECH., IV-SEMESTER
COCHIN UNIVERSITY OF SCIENCE AND TECHNOLOGY, KERALA
CE/CS/EB/EC/EE/EI/IT/ME/SE 401 ENGINEERING MATHEMATICS–III

Module I

Complex Analytic Functions and Conformal Mapping: Curves and regions in the complex plane, complex functions, limit, derivative, analytic function, Cauchy-Riemann equations, Elementary complex functions such as powers, exponential function, logarithmic, trigonometric and hyperbolic functions.

Conformal Mapping: Linear fractional transformations, mapping by elementary functions like z^2, e^z, sin z, cos z, sinh z, and cosh z, $z + 1/z$.

Module II

Complex Integration: Line integral, Cauchy's integral theorem, Cauchy's integral formula, Taylor's series, Laurent's series, residue theorem, evaluation of real integrals using integration around unit circle, around the semi circle, integrating contours having poles, on the real axis.

Module III

Partial Differential Equations: Formation of partial differential equations. Solutions of equations of the form $F(p, q) = 0$, $F(x, p, q) = 0$, $F(y, p, q) = 0$, $F(z, p, q) = 0$, $F_1(x, p) = F_2(y, q)$, Lagrange's form $Pp + Qq = R$. Linear homogeneous partial differential equations with constant coefficients.

Module IV

Vibrating String: One dimensional wave equation, D'Alembert's solution, solution by the method of separation of variables, one dimensional heat equation, solution of the equation by the method of separation of variables. Solutions of Laplace's equation over a rectangular region and a circular region by the method of separation of variables.

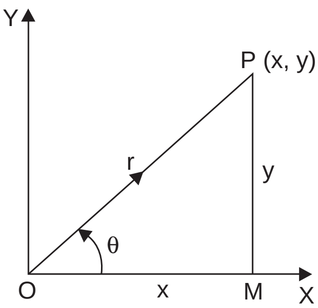
1
Functions of a Complex Variable

1.1. INTRODUCTION

A complex number z is an ordered pair (x, y) of real numbers and is written as

$$z = x + iy, \quad \text{where } i = \sqrt{-1}.$$

The real numbers x and y are called the real and imaginary parts of z. In the Argand's diagram, the complex number z is represented by the point P(x, y). If (r, θ) are the polar coordinates of P, then $r = \sqrt{x^2 + y^2}$ is called the modulus of z and is denoted by $|z|$. Also $\theta = \tan^{-1} \dfrac{y}{x}$ is called the argument of z and is denoted by arg. z. Every non-zero complex number z can be expressed as



$$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

If $z = x + iy$, then the complex number $x - iy$ is called the conjugate of the complex number z and is denoted by $\bar{z}$.

Clearly, $\quad |\bar{z}| = |z|, |z|^2 = z\bar{z},$

$$R(z) = \frac{z + \bar{z}}{2}, \quad I(z) = \frac{z - \bar{z}}{2i}.$$

1.2. FUNCTION OF A COMPLEX VARIABLE

If x and y are real variables, then $z = x + iy$ is called a **complex variable.** If corresponding to each value of a complex variable z $(= x + iy)$ in a given region R, there correspond one or more values of another complex variable w $(= u + iv)$, then w is called a function of the complex variable z and is denoted by

$$w = f(z) = u + iv$$

For example, if $\quad w = z^2, \quad$ where $z = x + iy$ and $w = f(z) = u + iv$

then $\quad u + iv = (x + iy)^2 = (x^2 - y^2) + i(2xy)$

$\Rightarrow \quad u = x^2 - y^2 \quad \text{and} \quad v = 2xy$

Thus u and v, the real and imaginary parts of w, are functions of the real variables x and y.

$\therefore \quad w = f(z) = u(x, y) + iv(x, y)$

If to each value of z, there corresponds one and only one value of w, then w is called a *single-valued function* of z. If to each value of z, there correspond more than one values of w, then w is called a *multi-valued function of z*.

To represent $w = f(z)$ graphically, we take two Argand diagrams : one to represent the point z and the other to represent w. The former diagram is called the *xoy*-plane or the z-plane and the latter *uov*-plane or the w-plane.

1.3. LIMIT OF f(z)

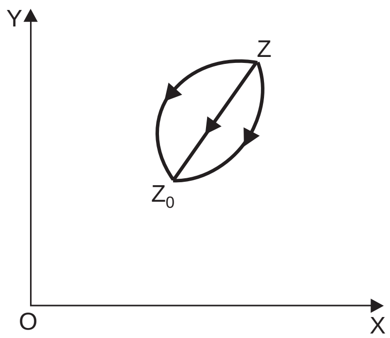
A function $f(z)$ tends to the limit l as z tends to z_0 *along any path,* if to each positive arbitrary number ε, however small, there corresponds a positive number δ, such that

$$| f(z) - l | < \varepsilon \quad \text{whenever} \quad 0 < | z - z_0 | < \delta$$

i.e., $$l - \varepsilon < f(z) < l + \varepsilon$$

whenever $$z_0 - \delta < z < z_0 + \delta, z \neq z_0$$

and we write $\underset{z \to z_0}{\text{Lt}} f(z) = l.$



Note. In real variables, $x \to x_0$ implies that x approaches x_0 along the number line, either from left or from right. In complex variables, $z \to z_0$ implies that z approaches z_0 along any path, straight or curved, since the two points representing z and z_0 in a complex plane can be joined by an infinite number of curves.

1.4. CONTINUITY OF f(z)

A single-valued function $f(z)$ is said to be continuous at a point $z = z_0$ if $\underset{z \to z_0}{\text{Lt}} f(z) = f(z_0)$.

A function $f(z)$ is said to be continuous in a region R of the z-plane if it is continuous at every point of the region.

1.5. DERIVATIVE OF f(z)

Let $w = f(z)$ be a single-valued function of the variable $z(= x + iy)$, then the derivative or differential co-efficient of $w = f(z)$ is defined as

$$\frac{dw}{dz} = f'(z) = \underset{\delta z \to 0}{\text{Lt}} \frac{f(z + \delta z) - f(z)}{\delta z}$$

provided the limit exists, independent of the manner in which $\delta z \to 0$.

1.6. ANALYTIC FUNCTION

If a single-valued function $f(z)$ possesses a unique derivative at every point of a region R, then $f(z)$ is called an **analytic function** or a **regular function** or a **holomorphic function** of z in R.

A point where the function ceases to be analytic is called a **singular point.**

1.7. NECESSARY AND SUFFICIENT CONDITIONS FOR f(z) TO BE ANALYTIC

The necessary and sufficient conditions for the function

$$w = f(z) = u(x, y) + iv(x, y)$$

to be analytic in a region R, are

(*i*) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ *are continuous functions of x and y in the region R.*

(*ii*) $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}, \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$.

The conditions in (*ii*) are known as **Cauchy-Riemann equations** or briefly **C-R equations.**

Proof. (*a*) **Necessary Condition.** Let $w = f(z) = u(x, y) + iv(x, y)$ be analytic in a region R, then $\dfrac{dw}{dz} = f'(z)$ exists uniquely at every point of that region.

Let δx and δy be the increments in x and y respectively. Let δu, δv and δz be the corresponding increments in u, v and z respectively. Then,

$$f'(z) = \underset{\delta z \to 0}{\text{Lt}} \frac{f(z+\delta z) - f(z)}{\delta z} = \underset{\delta z \to 0}{\text{Lt}} \frac{(u+\delta u) + i(v+\delta v) - (u+iv)}{\delta z}$$

$$= \underset{\delta z \to 0}{\text{Lt}} \left(\frac{\delta u}{\delta z} + i\frac{\delta v}{\delta z} \right) \quad \text{...(1)}$$

Since the function $w = f(z)$ is analytic in the region R, the limit (1) must exist independent of the manner in which $\delta z \to 0$, *i.e.,* along whichever path δx and $\delta y \to 0$.

First, let $\delta z \to 0$ along a line parallel to x-axis so that $\delta y = 0$ and $\delta z = \delta x$.

[since $z = x + iy$, $z + \delta z = (x + \delta x) + i(y + \delta y)$ and $\delta z = \delta x + i\delta y$]

$$\therefore \quad \text{From (1),} \quad f'(z) = \underset{\delta x \to 0}{\text{Lt}} \left(\frac{\delta u}{\delta x} + i\frac{\delta v}{\delta x} \right) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} \quad \text{...(2)}$$

Now, let $\delta z \to 0$ along a line parallel to y-axis so that $\delta x = 0$ and $\delta z = i\,\delta y$.

$$\therefore \quad \text{From (1),} \quad f'(z) = \underset{\delta y \to 0}{\text{Lt}} \left(\frac{\delta u}{i\delta y} + i\frac{\delta v}{i\delta y} \right) = \frac{1}{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$$

$$= \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y} \quad \text{...(3)} \quad \Big| \because \frac{1}{i} = -i$$

From (2) and (3), we have $\dfrac{\partial u}{\partial x} + i\dfrac{\partial v}{\partial x} = \dfrac{\partial v}{\partial y} - i\dfrac{\partial u}{\partial y}$

Equating the real and imaginary parts, $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$.

Hence the necessary condition for f(z) to be analytic is that the C-R equations must be satisfied.

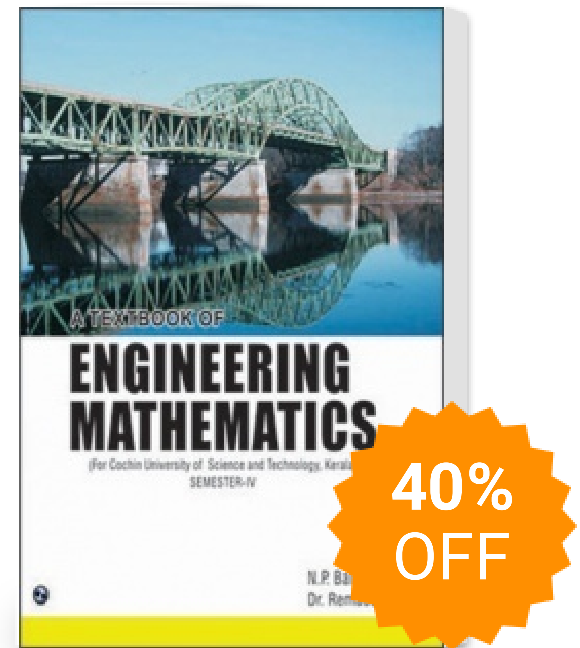
(*b*) **Sufficient Condition.** Let $f(z) = u + iv$ be a single-valued function possessing partial derivatives $\dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}, \dfrac{\partial v}{\partial x}, \dfrac{\partial v}{\partial y}$ at each point of a region R and satisfying C-R equations.

i.e., $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$.

We shall show that $f(z)$ is analytic, *i.e.,* $f'(z)$ exists at every point of the region R.

By Taylor's theorem for functions of two variables, we have, on omitting second and higher degree terms of δx and δy.

A Textbook of Engineering Mathematics (CUST, KERALA) Sem IV



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