Field Theory

Notes

FIELD THEORY

Sub Code : EC44 | **IA Marks : 25**
Hrs/Week : 04 | **Exam Hrs. : 03**
Total Hrs. : 52 | **Exam Marks : 100**

1. Electric Fields — 18 hours
a. Coulomb's law and Electric field intensity
b. Electric flux density, Gauss law and divergence
c. Energy and potential
d. Conductors, dielectrics and capacitance
e. Poisson's and Laplace's equations

2. Magnetic fields — 14 hours
a. The steady magnetic field
b. Magnetic forces, materials and inductance

3. Time varying fields and Maxwell's equations — 5 hours

4. Electromagnetic waves — 15 hours

Text Books :

William H Hayt Jr and John A Buck, "Engineering Electromagnetics", Tata McGraw-Hill, 6th Edition, 2001

Reference books :

John Krauss and Daniel A Fleisch, "Electromagnetics with Application", McGraw-Hill, 5th Edition, 1999
Guru and Hiziroglu, Electromagnetics Field theory fundamentals, Thomson Asia Pvt. Ltd I Edition, 2001
Joseph Edminster, "Electromagnetics", Schaum Outline Series, McGraw-Hill
Edward C Jordan and Keith G Balmain, "Electromagnetic Waves and Radiating Systems", Prentice-Hall of India, II Edition, 1968, Reprint 2002.
David K Cheng, "Field and Wave Electromagnetics", Pearson Education Ais II Edition, 1989, Indian Repr-01

Introduction to Field Theory

The behavior of a physical device subjected to electric field can be studied either by Field approach or by Circuit approach. The Circuit approach uses discrete circuit parameters like RLCM, voltage and current sources. At higher frequencies (MHz or GHz) parameters would no longer be discrete. They may become **non linear** also depending on material property and strength of v and i associated. This makes circuit approach to be difficult and may not give very accurate results.

Thus at high frequencies, Field approach is necessary to get a better understanding of performance of the device.

FIELD THEORY

The 'Vector approach' provides better insight into the various aspects of Electromagnetic phenomenon. Vector analysis is therefore an essential tool for the study of Field Theory.

The 'Vector Analysis' comprises of 'Vector Algebra' and 'Vector Calculus'.

Any physical quantity may be 'Scalar quantity' or 'Vector quantity'. A 'Scalar quantity' is specified by magnitude only while for a 'Vector quantity' requires both magnitude and direction to be specified.

Examples :

Scalar quantity : Mass, Time, Charge, Density, Potential, Energy etc.,
Represented by alphabets – A, B, q, t etc

Vector quantity : Electric field, force, velocity, acceleration, weight etc., represented by alphabets with arrow on top.

$\vec{A},\ \vec{B},\ \vec{E},\ \vec{B}$ etc.,

Vector algebra : If $\vec{A},\ \vec{B},\ \vec{C}$ are vectors and m, n are scalars then

(1) Addition

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$ Commutative law

$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$ Associative law

(2) Subtraction

$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

(3) Multiplication by a scalar

$m\vec{A} = \vec{A}m$ Commutative law

$m(n\vec{A}) = n(m\vec{A})$ Associative law

$(m+n)\vec{A} = m\vec{A} = n\vec{A}$ Distributive law

$m(\vec{A}+\vec{B}) = m\vec{A} + m\vec{B}$ Distributive law

A 'vector' is represented graphically by a directed line segment.

A ‘Unit vector’ is a vector of unit magnitude and directed along ‘that vector’.
$\hat{a}_A$ is a Unit vector along the direction of $\vec{A}$.

Thus, the graphical representation of $\vec{A}$ and $\hat{a}_A$ are

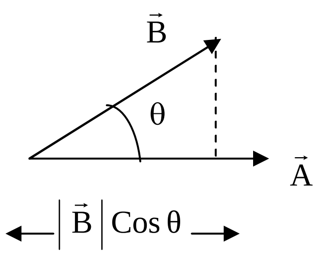
Vector $\vec{A}$ Unit vector $\hat{a}_A$

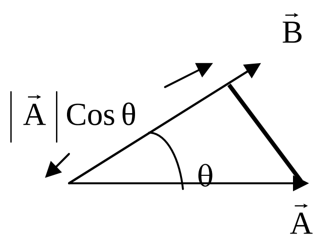
Also $\hat{a}_A = \vec{A} \,/\, |\vec{A}|$ or $\vec{A} = \hat{a}_A\, |\vec{A}|$

<u>Product of two or more vectors</u> :

(1) Dot Product (.)

$$\vec{A}\,.\,\vec{B} = |\vec{A}|\,(|\vec{B}|\,\text{COS}\;\theta \;\; \text{OR} \;\; \{\,|\vec{A}|\,\text{COS}\;\theta\}\,|\vec{B}|,\; 0 \le \theta \le \pi$$





A . B = B . A   (A Scalar quantity)

(2) CROSS PRODUCT (X)

$C = A \times B = |A|\,|B|\;\text{SIN}\,\theta\;\hat{n}$

Ex.,

where ’θ ’is angle between $\vec{A}$ and $\vec{B}$ $(0 \le \theta \le \pi)$

and $\hat{n}$ is unit vector perpendicular to plane of $\vec{A}$ and $\vec{B}$

directed such that $\vec{A}\,\vec{B}\,\vec{C}$ form a right handed system of vectors

$\vec{A} \times \vec{B} = -\,\vec{B} \times \vec{A}$

$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

CO-ORDINATE SYSTEMS :

For an explicit representation of a vector quantity, a 'co-ordinate system' is essential.

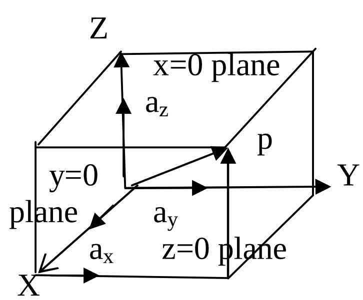
Different systems used :

Sl.No.	System	Co-ordinate variables	Unit vectors
1.	Rectangular	x, y, z	a_x , a_y , a_z
2.	Cylindrical	ρ, ϕ, z	a_ρ , a_ϕ , a_z
3.	Spherical	r, θ, ϕ	a_r , a_θ , a_ϕ

These are 'ORTHOGONAL' i.e., unit vectors in such system of co-ordinates are mutually perpendicular in the right circular way.

i.e., $\overrightarrow{x\,y\,z}$, $\overrightarrow{° ¤ z}$, $\overrightarrow{r ¯ ¤}$

RECTANGULAR CO-ORDINATE SYSTEM :



$$a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$$
$$a_x \times a_y = a_z$$
$$a_y \times a_z = a_x$$
$$a_z \times a_x = a_y$$

a_z is in direction of 'advance' of a right circular screw as it is turned from a_x to a_y

Co-ordinate variable 'x' is intersection of planes OYX and OXZ i.e, z = 0 & y = 0

Location of point P :

If the point P is at a distance of r from O, then

If the components of r along X, Y, Z are x, y, z then

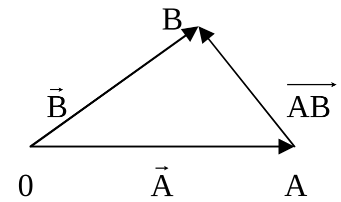
$$\vec{r} = x\, a_x + y\, a_y + z\, a_z = |r|\, a_r$$

Equation of Vector $\overrightarrow{AB}$:

If $\overrightarrow{OA} = \vec{A} = A_x\ a_x + A_y\ a_y + A_z\ a_z$

and $\overrightarrow{OB} = \vec{B} = B_x\ a_x + B_y\ a_y + B_z\ a_z$ then

$\vec{A} + \overrightarrow{AB} = \vec{B}$ or $\overrightarrow{AB} = \vec{B} - \vec{A}$



where A_s , A_y & A_z are components of A along X, Y an

and B_s , B_y & B_z are components of B along X, Y and Z

Dot and Cross Products :

$\vec{A}.\vec{B} = (A_x\ a_x + A_y\ a_y + A_z\ a_z) . (B_x\ a_x + B_y\ a_y + B_z\ a_z) = A_x\ B_x + A_y\ B_y + A_z\ C_z$

$\vec{A} \times \vec{B} = (A_x\ a_x + A_y\ a_y + A_z\ a_z) \times (B_x\ a_x + B_y\ a_y + B_z\ a_z)$

Taking 'Cross products' term by term and grouping, we get

$$\vec{A} \times \vec{B} = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{A}.(\vec{B} \times \vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

If $\vec{A}, \vec{B}$ and $\vec{C}$ are non zero vectors,

(i) $\vec{A}.\vec{B} = 0$ then $\cos\theta = 0$ i.e., $\theta = 90^0$ A and B are perpendicular

$\vec{A} \times \vec{B} = 0$ then $\sin\theta = 0$ $\quad \theta = 0$ A and B are parallel

(ii) $\vec{A}.(\vec{B} \times \vec{C})$ represents the volume of a parallelopoid of sides $\vec{A}, \vec{B}$ and $\vec{C}$

Unit Vector along $\overrightarrow{AB}$

$$a_{AB} = \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|}$$

where

Vector length $\overrightarrow{AB} = \left|\overrightarrow{AB}\right| = \sqrt{(\overrightarrow{AB} . \overrightarrow{AB})}$

Differential length, surface and volume elements in rectangular co-ordinate systems

$$\vec{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$$

$$d\vec{r} = \frac{\partial \vec{r}}{\partial x}\,dx + \frac{\partial \vec{r}}{\partial y}\,dy + \frac{\partial \vec{r}}{\partial z}\,dz$$

$$d\vec{r} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$$

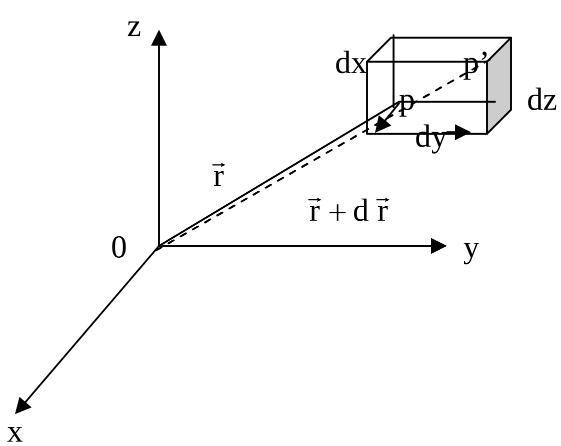
Differential length $\left| d\vec{r} \right| = [\,dx^2 + dy^2 + dz^2\,]^{1/2}$ -----1

Differential surface element, $d\vec{s}$

1. $\perp^r$ to z : dxdy $\hat{a}_z$
2. $\perp^r$ to z : dxdy $\hat{a}_z$ ------ 2
3. $\perp^r$ to z : dxdy $\hat{a}_z$

Differential Volume element

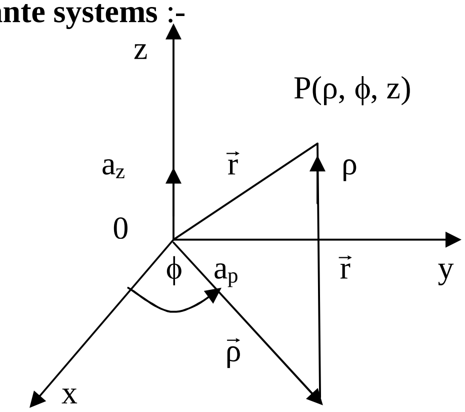
dv = $d_x\, d_y\, d_z$ ------ 3



Other Co-ordinate systems :-

Depending on the geometry of problem it is easier if we use the appropriate co-ordinate system than to use the Cartesian co-ordinate system always. For problems having cylindrical symmetry cylindrical co-ordinate system is to be used while for applications having spherical symmetry spherical co-ordinate system is preferred.

Cylindrical Co-ordiante systems :-



$x = \rho \cos\phi$
$y = \rho \sin\phi$
$z = z$

$\rho = \sqrt{x^2 + y^2}$

$\varphi = \tan^{-1} y / x$

$z = z$

$$\vec{r} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$$

$$\vec{r} = \rho\,\text{Cos}\,¤\,\hat{a}_x + \rho\,\text{Sin}\,¤\,\hat{a}_y + z\,\hat{a}_z$$

$$d\vec{r} = \frac{\partial \vec{r}}{\partial \rho}\,d\rho + \frac{\partial \vec{r}}{\partial ¤}\,d¤ + \frac{\partial \vec{r}}{\partial z}\,dz \qquad \text{------}1$$

$$\frac{\partial \vec{r}}{\partial \rho} = \text{Cos}\,¤\,\hat{a}_x + \text{Sin}\,¤\,\hat{a}_y = \left|\frac{\partial \vec{r}}{\partial \rho}\right|\hat{a}_{◦} = h_\rho\;\hat{a}_\rho \quad ; \quad h_\rho = \left|\frac{\partial \vec{r}}{\partial \rho}\right| = 1$$

$$\frac{\partial \vec{r}}{\partial ¤} = -\rho\,\text{Sin}\,¤\,\hat{a}_x + \rho\,\text{Cos}\,¤\,\hat{a}_y = \left|\frac{\partial \vec{r}}{\partial ¤}\right|\hat{a}_{¤} = \rho\,\hat{a}_{¤}\;;\; h_{¤} = \left|\frac{\partial \vec{r}}{\partial ¤}\right| = \rho$$

$$\frac{\partial \vec{r}}{\partial z} = \hat{a}_z \qquad\qquad h_z = \left|\frac{\partial \vec{r}}{\partial z}\right| = 1$$

Thus unit vectors in (ρ, ϕ, z) systems can be expressed in (x,y,z) system as

$$a_\rho = \text{Cos}\,¤\,a_x + \text{Sin}\,¤\,a_y \qquad a_x = \text{Cos}\,¤\,a_{◦} - \text{Sin}\,¤\,a_{¤}$$

$$a_{◦} = -\text{Sin}\,¤\,a_x + \text{Cos}\,¤\,a_y \qquad a_y = \text{Sin}\,¤\,a_{◦} + \text{Cos}\,¤\,a_{◦}$$

$$a_z = a_z \quad ; \quad a_{◦}\,,\ a_{¤}\ \text{ and }\ a_z\ \text{ are orthogonal}$$

$$\text{Further},\ d\vec{r} = d\rho\,\hat{a}_{◦} + \rho\,d¤\,\hat{a}_{¤} + dz\,\hat{a}_z \qquad \text{------}2$$

$$\text{and}\ \left|d\vec{r}\right|^2 = d\rho^2 + (\rho\,d¤)^2 + (dz)^2$$

Differential areas :

$$ds\,\hat{a}_z = (d\,\rho)\;(\rho\,d¤)\,.\,\hat{a}_z$$

$$ds\,\hat{a}_{◦} = (dz)\,(\rho\,d¤)\,.\,\hat{a}_{◦} \qquad \text{-------}3$$

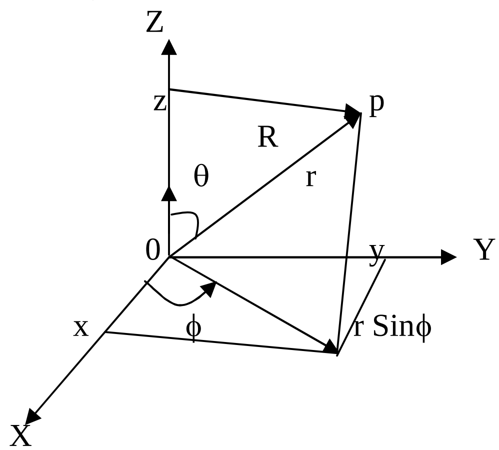
$$ds\,\hat{a}_{¤} = (d\,\rho\,dz)\;\hat{a}_{¤}$$

Differential volume :

$$d² = (d\,\rho)\,(\rho\,d¤)\,(dz)$$

$$\text{or}\quad d² = \rho\,d\rho\,d¤\,dz \qquad \text{-----}\ 4$$

Spherical Co-ordinate Systems :-



X = r Sin θ Cos ϕ
Y = r Sin θ Sin ϕ
Z = r Cos θ

$$\vec{R} = r \sin\theta \cos\phi \, \hat{a}_x + r \sin\theta \sin\phi \, \hat{a}_y + r \cos\theta \, \hat{a}_z$$

$$\hat{a}_r = \frac{\partial \vec{R}}{\partial r} \Big/ \left| \frac{\partial \vec{R}}{\partial r} \right| = \sin\theta \cos\phi \, \hat{a}_x + \sin\theta \sin\phi \, \hat{a}_y + \cos\theta \, \hat{a}_z$$

$$\hat{a}_\theta = \frac{\partial \vec{R}}{\partial \theta} \Big/ \left| \frac{\partial \vec{R}}{\partial \theta} \right| = \cos\theta \cos\phi \, \hat{a}_x + \cos\theta \sin\phi \, \hat{a}_y - \sin\theta \, \hat{a}_z$$

$$\hat{a}_\phi = \frac{\partial \vec{R}}{\partial \phi} \Big/ \left| \frac{\partial \vec{R}}{\partial \phi} \right| = - \sin\phi \, \hat{a}_x + \cos\phi \, \hat{a}_y$$

$$d\vec{R} = \frac{\partial \vec{R}}{\partial r} dr + \frac{\partial \vec{R}}{\partial \theta} d\theta + \frac{\partial \vec{R}}{\partial \phi} d\phi$$

$$d\vec{R} = dr \, \hat{a}_r + r \, d\theta \, \hat{a}_\theta + r \sin\theta \, d\phi \, \hat{a}_\phi$$

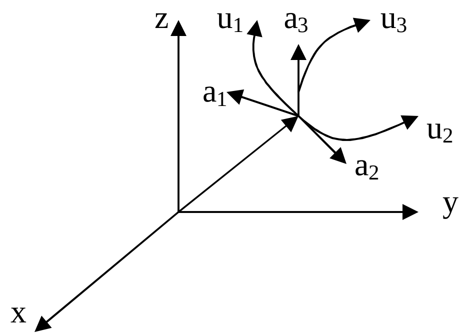
$$d S_r = r^2 \sin\theta \, d\theta \, d\phi$$

$$d S_\theta = r^2 \sin\theta \, dr \, d\phi$$

$$d S_\phi = r \, dr \, d\theta$$

$$d v = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

General Orthogonal Curvilinear Co-ordinates :-



Co-ordinate Variables : (u_1 , u_2, u_3) ;
Here
u_1 is Intersection of surfaces $u_2 = C$ & $u_3 = C$
u_2 is Intersection of surfaces $u_1 = C$ & $u_3 = C$
u_3 is Intersection of surfaces $u_1 = C$ & $u_2 = C$

$\hat{a}_1, \hat{a}_2, \hat{a}_3$ are ubnit vectors tangential to u_1, u_2 & u_3

System is Orthogonal if $\hat{a}_1 . \hat{a}_2 = 0 , \hat{a}_2 . \hat{a}_3 = 0 \;\&\; \hat{a}_3 . \hat{a}_1 = 0$

If $\vec{R} = x\,\hat{a}_x + y\,\hat{a}_y + z\,\hat{a}_z$ & x, y, z are functions of u_1, u_2 & u_3

then $d\,R = \dfrac{\partial \vec{R}}{\partial u_1} du_1 + \dfrac{\partial \vec{R}}{\partial u_2} du_2 + \dfrac{\partial \vec{R}}{\partial u_3} du_3$

$= h_1\, du_1\, \hat{a}_1 + h_2\, du_2\, \hat{a}_2 + h_3\, du_3\, \hat{a}_3$

where h_1, h_2, h_3 are scale factors ;

$$h_1 = \left|\frac{\partial \vec{R}}{\partial u_1}\right| , \; h_2 = \left|\frac{\partial \vec{R}}{\partial u_2}\right| , \; h_3 = \left|\frac{\partial \vec{R}}{\partial u_3}\right|$$

Co-ordinate Variables, unit Vectors and Scale factors in different systems

Systems	Co-ordinate Variables			Unit Vector			Scale factors		
General	u_1	u_2	u_3	a_1	a_2	a_3	h_1	h_2	h_3
Rectangular	x	y	z	a_x	a_y	a_z	1	1	1
Cylindrical	ρ	ϕ	z	a_ρ	a_ϕ	a_z	1	ρ	1
Spherical	r	θ	ϕ	a_r	a_θ	a_ϕ	1	r	r sin θ

Transformation equations (x,y,z interms of cylindrical and spherical co-ordinate system variables)

Cylindrical : $x = \rho \cos\phi$, $y = \rho \sin\phi$, $z = z$; $\rho \geq 0,\ 0 \leq \phi \leq 2\pi$ $-\infty < z < \infty$

Spherical

$x = r \sin\theta \cos\phi$, $y = r \sin\theta \cos\phi$, $z = r \sin\theta$

$r \geq 0$, $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$

$$\nabla V = \frac{1}{h_1}\frac{\partial v}{\partial u_1}\hat{a}_1 + \frac{1}{h_2}\frac{\partial v}{\partial u_2}\hat{a}_2 + \frac{1}{h_3}\frac{\partial v}{\partial u_3}\hat{a}_3$$

$$\nabla . \vec{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}(h_2 h_3 A_1) + \frac{\partial}{\partial u_2}(h_1 h_3 A_2) + \frac{\partial}{\partial u_3}(h_1 h_2 A_3)\right]$$

$$\nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1 \hat{a}_1 & h_2 \hat{a}_2 & h_3 \hat{a}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix}$$

where $V = V(u_1, u_2, u_3)$ a Scalar field

& $\vec{A} = A_1 \hat{a}_1 + A_2 \hat{a}_2 + A_3 \hat{a}_3$ is a Vector field where $A_1 = A_1 (u_1, u_2, u_3)$

$A_2 = A_2 (u_1, u_2, u_3)$ and $A_3 = A_3 (u_1, u_2, u_3)$

Vector Transformation from Rectangular to Spherical :

Rectangular : $A_R = A_x \ \hat{a}_x + A_y \ \hat{a}_y + A_z \ \hat{a}_z$

Spherical : $A_S = (\vec{A}_R \ \hat{a}_r) \ \hat{a}_r + (\vec{A}_R \ \hat{a}_\theta) \ \hat{a}_\theta + (\vec{A}_R . \hat{a}_\phi) \hat{a}_\phi$

$$= A_r \ \hat{a}_r + A_\theta \ \hat{a}_\theta + A_\phi \ \hat{a}_\phi$$

where A_r , A_θ , A_ϕ are related to A_x , A_y , A_z as

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \hat{a}_x . \hat{a}_r & \hat{a}_y . \hat{a}_r & \hat{a}_z . \hat{a}_r \\ \hat{a}_x . \hat{a}_\theta & \hat{a}_y . \hat{a}_\theta & \hat{a}_z . \hat{a}_\theta \\ \hat{a}_x . \hat{a}_\phi & \hat{a}_y . \hat{a}_\phi & \hat{a}_z . \hat{a}_\phi \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Field Theory

A ‘field’ is a region where any object experiences a force. The study of performance in the presence of Electric field $(\vec{E})$, Magnetic field (ϕ) is the essence of EM Theory.

P1 : Obtain the equation for the line between the points P(1,2,3) and Q (2,-2,1)

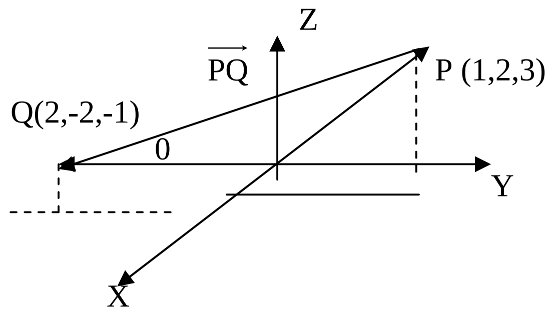
$$\overrightarrow{PQ} = a_x - 4a_y - 2a_z$$

P2 : Obtain unit vector from the origin to G (2, -2, 1)

Problems on Vector Analysis

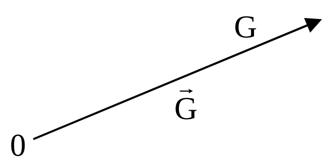
Examples :-

1. **Obtain the vector equation for the line $\overrightarrow{PQ}$ between the points P (1,2,3)m and Q (2, -2, 1) m**



$$\text{The vector } \overrightarrow{PQ} = (x_q - x_p)\,\hat{a}_x + (y_q - y_p)\,\hat{a}_y + (z_q - z_p)\,\hat{a}_z$$
$$= (2-1)\,\hat{a}_x + (-2-2)\,\hat{a}_y + (-1-3)\,\hat{a}_z$$
$$= (\hat{a}_x - 4\,\hat{a}_y - 2\,\hat{a}_z)$$

2. **Obtain unit vector from origin to G (2,-2,-1)**



$$\text{The vector } \vec{G} = (x_g - 0)\,\hat{a}_x + (y_g - 0)\,\hat{a}_y + (z_g - 0)\,\hat{a}_z$$
$$= (2\,\hat{a}_x - 2\,\hat{a}_y - \hat{a}_z)$$

$$\text{The unit vector, } \hat{a}_g = \frac{\vec{G}}{|\vec{G}|}$$

$$|\vec{G}| = \sqrt{2^2 + (-2)^2 + (-1)^2} = 3$$

$$\therefore \hat{a}_g = (0.667\,\hat{a}_x - 0.667\,\hat{a}_y - 0.333\,\hat{a}_z)$$

3. **Given**

$$\vec{A} = 2\,\hat{a}_x - 3\,\hat{a}_y + \hat{a}_z$$
$$\vec{B} = -4\,\hat{a}_x - 2\,\hat{a}_y + 5\hat{a}_z$$

find (1) $\vec{A}\cdot\vec{B}$ and (2) $\vec{A} \times \vec{B}$

Solution :

(1) $\vec{A}\cdot\vec{B} = (2\,a_x - 3\,a_y + a_z)\cdot(-4\,a_x - 2\,a_y + 5\,a_z)$

$= -8 + 6 + 5 = 3$

Since $a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 0$ and $a_x\, a_y = a_y\, a_z = a_z\, a_x = 0$

(2) $\vec{A} \times \vec{B} = \begin{vmatrix} a_x & a_y & a_z \\ 2 & -3 & 1 \\ -4 & -2 & 5 \end{vmatrix} = (-13\,a_x \;\; -14\,a_y - 16\,a_z)$

4. **Find the distance between A(2, ®/6, 0) and B = (1, ®/2, 2)**

Soln : The points are given in Cylindrical Co-ordinate (ρ,ϕ, z). To find the distance between two points, the co-ordinates are to be in Cartesian (rectangular). The corresponding rectangular co-ordinates are (ρ Cosϕ, ρ Sinϕ, z)

$$\therefore\ \vec{A} = 2\,\text{Cos}\,\frac{®}{6}\,\hat{a}_x + 2\,\text{Sin}\,\frac{®}{6}\,\hat{a}_y = 1.73\,\hat{a}_x + \hat{a}_y$$

$$\&\ \vec{B} = \text{Cos}\,\frac{®}{6}\,\hat{a}_x + \text{Sin}\,\frac{®}{2}\,\hat{a}_y + 2\,\hat{a}_z = \hat{a}_y + 2\,\hat{a}_z$$

$$\therefore \overrightarrow{AB} = (B_x - A_x)\,\hat{a}_x + (B_y - A_y)\,\hat{a}_y + (B_z - A_z)\,\hat{a}_z$$

$$= -1.73\,\hat{a}_x + (1-1)\,\hat{a}_y + (2-0)\,\hat{a}_z$$

$$= -1.73\,\hat{a}_x + 2\,\hat{a}_z$$

$$\therefore (\overrightarrow{AB}) = \sqrt{1.73^2 + 2^2} = 2.64$$

5. **Find the distance between A(1, ®/4, 0) and B = (1, 3®/4, ®)**

Soln : The specified co-ordinates (r, θ, ϕ) are spherical. Writing in rectangular, they are (r Sin θ Cos ϕ, r Sin θ Sin ϕ, r Cos θ).

Therefore, A & B in rectangular co-ordinates,

$$\vec{A} = (1\,\text{Sin}\,\frac{®}{4}\,\text{Cos}\,0\,\hat{a}_x + 1\,\text{Sin}\,\frac{®}{4}\,\text{Sin}\,0\,\hat{a}_y + 1\,\text{Cos}\,\frac{®}{4}\,\hat{a}_z)$$

$$= (0.707\,\hat{a}_x + 0.707\,\hat{a}_y)$$

$$\vec{B} = (\text{Sin}\,\frac{3®}{4}\,\text{Cos}\,®\,\hat{a}_x + \text{Sin}\,\frac{3®}{4}\,\text{Sin}\,®\,\hat{a}_y + \text{Cos}\,\frac{3®}{4}\,\hat{a}_z)$$

$$= (0.707\,\hat{a}_x - 0.707\,\hat{a}_y)$$

$$\overrightarrow{AB} = (B_x - A_x)\,\hat{a}_x + (B_y - A_y)\,\hat{a}_y + (B_z - A_z)\,\hat{a}_z$$

$$= -1.414\,\hat{a}_x + (-0.707)\,\hat{a}_y + (-0.707)\,\hat{a}_z$$

$$\left|\overrightarrow{AB}\right| = (\overrightarrow{AB}\,.\,\overrightarrow{AB})^{1/2}$$

$$= (2 + 0.5 + 0.5)^{1/2} = 1.732$$

6. Find a unit vector along $\overrightarrow{AB}$ in Problem 5 above.

$$\hat{a}_{AB} = \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = [\,-1.414\,a_x + (-0.707)\,a_y + (-0.707)\,a_z]\,\frac{1}{1.732}$$

$$= (-0.816\,\hat{a}_x - 0.408\,\hat{a}_y - 0.408\,\hat{a}_z)$$

7. Transform $\vec{F} = (10\,\hat{a}_x - 8\,\hat{a}_y + 6\,\hat{a}_z)$ into $\vec{F}$ in Cylindrical Co-ordinates.

Soln :

$$\vec{F}_{Cyl} = (\vec{F}\,.\,\hat{a}_p)\,\hat{a}_p + (\vec{F}\,.\,\hat{a}_{¤})\,\hat{a}_{¤} + (\vec{F}\,.\,\hat{a}_z)\,\hat{a}_z$$