

Linear Control Systems

with MATLAB Applications

B.S. Manke



KHANNA PUBLISHERS

LINEAR CONTROL SYSTEMS

with MATLAB Applications

B.S. MANKE

*Formerly Prof. of Electrical Engineering,
Maulana Azad National Institute of Technology
Bhopal (M.P.)*



KHANNA PUBLISHERS

4575/15, ONKAR HOUSE, OPP. HAPPY SCHOOL
DARYAGANJ, NEW DELHI-110002

Phone : 011-2324 30 42 ; Fax : 011-2324 30 43

Published by :

R.C. Khanna
for KHANNA PUBLISHERS
2-B, Nath Market, Nai Sarak
Delhi-110006 (India).

All Rights Reserved

[This book or part thereof cannot be translated or reproduced in any form (except for review or criticism) without the written permission of the Author and the Publishers.]

ISBN No. : 978-81-7409-310-3

Eleventh Edition : 2012

Fifth Reprint : 2013

Price : ₹ 320.00

Typesetted at : Excellent Graphics, Delhi.

Printed at : New A.S. Offset Press.

*To my wife
Sulbha
and sons
Manish, Shailesh.*



Preface to the First Edition

This book has been written to explain the basic principles of **Linear Control Systems** and an effort is made to present the subject in a simple and sequential manner to enable the students to acquire a good grasp of fundamentals of the subject.

The text presented covers the course content of the subject **Linear Control Systems** of Indian Universities and is meant for pre-final/final year students of electrical, electronics and mechanical engineering.

The material given in this book has been thoroughly class tested by the author while teaching the subject of control systems at undergraduate level for the past several years.

This book is divided in 9 chapters. The first four chapters give the basic concepts of the subject from the view point of control system representation. Chapter 5 presents the modelling of control systems and the respective mathematical models derived therein. The time response and steady state analysis is given in Chapter 6. Necessary derivations have been derived from the first principles. The stability analysis is described in Chapter 7. The methods of ascertaining stability: Routh-Hurwitz criterion, Nyquist criterion, Bode plot and root locus plot have been explained step by step in a simplified manner to make the explanation easily understandable. The compensation methods and introduction to state space analysis is described in chapters 8 and 9 respectively.

Suitable illustrative examples as well as solved examples have been incorporated in the text to make the subject clear and interesting. A list of references is given at the end.

Selective unsolved problems have been included at the end of each chapter to help the student to judge himself whether he has gained sufficient workable knowledge of basic principles involved. Answers to odd numbered problems being given in Appendix I.

The salient feature of this book is the inclusion of objective type multiple choice questions given in Appendix II covering the entire text which would be of great help for the students preparing for competitive examinations.

The author hopes that this book will serve the purpose of introducing basic principles of **Linear Control Systems** to undergraduate students for whom it is written.

The author would welcome any comments and suggestions to further improve the usefulness of this book.

The author acknowledges his indebtedness to Miss Saroj Rangnekar, Asst. Prof. in Elect. Engg., Maulana Azad College of Technology, Bhopal who thoroughly checked the manuscript and made useful suggestions.

Bhopal
October, 1987

B.S. Manke

Preface to the Tenth Edition

The text written in the book deals with the concepts of feed-back control theory. The first five chapters stress on the fundamental concepts regarding representation and modelling of a control system. The subsequent chapters deal with the time response analysis, stability analysis, compensation method, state variable approach, and sampled data/discrete data systems.

Each chapter contains solved examples to support the theory developed. Unsolved problems have been included as an exercise.

The answers to graphical solutions may slightly deviate due to graphical errors.

The chapter on computer solutions to control problems gives the use of MATLAB* software. The examples on various topics in the text have been solved using MATLAB software. This verifies the answers obtained using analytical solution.

Appendices given at the end of the book include :

Appendix I : Answers to Selected Problems

Appendix II : A Set of Objective Questions

Appendix III : Short Answer Type Questions

Appendix IV : List of Key Formulae, Charts and Calculation Tables

The author wishes to acknowledge the outcome of discussions with Dr. D.M. Deshpande, Prof. M.A.N.I.T., Bhopal towards the revision of this edition.

The author is thankful to Shri Vineet Khanna of Khanna Publishers, Delhi for bringing out this edition on time and presentable manner.

B-309, Sarvadharam Colony,
Kolar Road, Bhopal

B.S. Manke

*MATLAB is registered trade mark of Mathwork, Inc.

Contents

1. INTRODUCTION	1–15
1.1 An Example of Control Action	...
1.2 Open-Loop Control System	...
1.3 Closed-Loop Control System	...
1.4 Use of Laplace Transformation in Control Systems	...
1.5 Laplace Transform	...
1.5.1 Derivation of Laplace Transform	...
1.5.2 Basic Laplace Transform Theorems	...
1.6 Solved Examples	...
<i>Problems</i>	...
	15
2. TRANSFER FUNCTION	16–24
2.1 Poles and Zeros of a Transfer Function	...
2.2 Transfer Function and its Relationship with Impulse Response	...
2.3 Procedure for Determining the Transfer Function of a Control System	...
2.4 Solved Examples	...
<i>Problems</i>	...
	24
3. BLOCK DIAGRAMS	25–61
3.1 Block Diagram Reduction	...
3.2 Solved Examples	...
<i>Problems</i>	...
	58
4. SIGNAL FLOW GRAPHS	62–78
4.1 Rules for Drawing Signal Flow Graphs	...
4.2 Mason's Gain Formula	...
4.3 Drawing Signal Flow Graph from a Given Block Diagram	...
4.4 Solved Examples	...
<i>Problem</i>	...
	78
5. MODELLING A CONTROL SYSTEM	79–126
5.1 Electrical Networks	...
5.2 Mechanical Systems	...
5.2.1 Translation Mechanical Systems	...
5.2.2 Rotational Mechanical System	...
	84

<i>Chapter</i>	<i>Pages</i>
5.3 Hydraulic System	86
5.4 Pneumatic System	88
5.5 Thermal System	89
5.6 Servo Motors	90
5.7 Generators	97
5.8 Error Detectors	102
5.9 Solved Examples	106
<i>Problems</i>	124
6. THREE RESPONSE ANALYSIS OF CONTROL SYSTEMS	127–215
6.1 Transient and Steady State Response	128
6.2 Input Test Signals	128
6.3 Time Response of a First Order Control System	130
6.3.1 Time Response of a First Order Control System Subjected to Unit Step Input Function	130
6.3.2 Demarcation between the Transient Part and Steady State Part of the Time Response in terms of Time Constant	131
6.3.3 Time Response of a First Order Control System Subjected to Unit Ramp Input Function	132
6.3.4 Time Response of a First Order Control System Subjected to Unit Impulse Input Function	133
6.4 Time Response of a Second Order Control System	134
6.4.1 Time Response of a Second Order Control System Subjected to Unit Step Input Function	134
6.4.2 Critical Damping	139
6.4.3 Characteristic Equation	140
6.4.4 Transient Response Specifications of Second Order Control System	141
6.4.5 Time Response of a Second Order Control System Subjected to Unit Ramp Input Function	144
6.4.6 Time Response of a Second Order Control System Subjected to Impulse Input Function	146
6.5 Time Response of a Third Order Control System	147
6.5.1 Effect of First Order Term Time Constant on Time Response of Third Order Control System	151
6.6 Time Response of Higher Order Control System	152
6.6.1 An Example of Third Order Unstable Control System	152
6.7 Steady State Error	153
6.7.1 Static Error Coefficients	155
6.7.2 Type of Transfer Functions and Steady State Error	156
6.7.3 Generalized Error Coefficients	159
6.7.4 Performance Indices	162

<i>Chapter</i>		<i>Pages</i>	
6.8	Sensitivity	...	168
6.8.1	Effect of Transfer Function Parameter variations in an Open-loop Control System	...	168
6.8.2	Effect of Forward Path Transfer Function Parameter variations in a Closed-loop Control System	...	168
6.8.3	Sensitivity of Overall Transfer Function $M(s)$ with respect to Forward Path Transfer Function $G(s)$...	169
6.8.4	Sensitivity of Overall Transfer Function $M(s)$ with respect to Feedback Path Transfer Function	...	170
6.8.5	Effect of Feedback on Time Constant of a Control System	...	172
6.9	Control Actions	...	173
6.9.1	Proportional Control	...	173
6.9.2	Derivative Control	...	173
6.9.3	Integral Control	...	178
6.9.4	Proportional Plus Derivative Plus Integral Control (PID Control)	...	181
6.9.5	Derivative Feedback Control	...	181
6.10	Solved Examples	...	185
	<i>Problems</i>	...	212

7. STABILITY ANALYSIS OF CONTROL SYSTEMS		216–362	
7.1	Stability in Terms of Characteristic Equation of a Control System	...	218
7.1.1	Definition of Stability	...	218
7.1.2	Absolute and Relative Stability	...	218
7.1.3	Location of the Roots of Characteristic Equation in s -plane as related to Time Response and Prediction of Absolute Stability therefrom	...	218
7.2	To Determine the Number of Roots having Positive Real Parts for a Polynomial	...	219
7.3	Hurwitz Determinants of a Polynomial	...	220
7.4	Routh-Hurwitz Criterion	...	221
7.5	Solved Examples : Routh-Hurwitz Criterion	...	222
7.6	Nyquist Criterion	...	229
7.6.1	Procedure for Mapping from s -plane to $G(s) H(s)$ -plane	...	229
7.6.2	Determination from the Nyquist Plot the Number of Zeros of $G(s) H(s)$ which are located inside a Specified Region in s -plane	...	231
7.6.3	Application of Nyquist Criterion to Determine Stability of a Closed-loop Control System	...	231
7.7	Gain Margin and Phase Margin	...	242
7.8	Relative Stability from Nyquist Plot	...	244
7.9	Gain Phase Plot	...	245
7.10	Closed-loop Frequency Response of a Unity Feedback Control System from Nyquist Plot	...	246

Chapter	Pages
7.11 Constant Magnitude Loci : M -Circles	247
7.12 Constant Phase Angles Loci : N -Circles	249
7.13 Closed-Loop Frequency Response of Control System from M and N -Circles	250
7.14 Gain Adjustment using M -Circle	251
7.15 Nichols Chart	254
7.16 Cutoff Frequency and Bandwidth	257
7.17 Solved Examples : Nyquist Criterion	259
7.17.1 Inverse Nyquist Plot	284
7.17.2 $1/M$ -Circles (Inverse M -Circles) and Constant Phase Angle Loci	287
7.17.3 Gain Adjustment Using Inverse Polar Plot	288
7.18 Bode Plot	289
7.18.1 Bode Plot (Logarithmic Plot) for Transfer Functions	289
7.18.2 Graphs for the Gain Term K	290
7.18.3 Graphs for the Term $\frac{1}{(j\omega)^N}$	290
7.18.4 Graphs for the Term $(1 + j\omega T)$	291
7.18.5 Graphs for the Term $\frac{1}{(1 + j\omega T)}$	292
7.18.6 Initial Slope of Bode Plot	295
7.18.7 Determination of Static Error Coefficients from Initial Slope of Bode Plot	297
7.18.8 Graphs for Quadratic Term $\frac{\omega_n^2}{[(\omega_n^2 \omega^2) + j2\zeta \omega_n \omega]}$	298
7.18.9 Procedure for Drawing Bode Plot and Determination of Gain Margin, Phase Margin and Stability	301
7.18.10 Bode Plot for Time Delay Element : e^{-sT}	304
7.18.11 Minimum Phase, Non-minimum Phase and all Pass Transfer Function	306
7.19 Solved Examples : Bode Plot	308
7.20 Correlation between Transient Response and Frequency Response	315
7.21 Root Locus	319
7.22 Salient Features of Root Locus Plot	323
7.23 The Procedure for Plotting Root Locus	323
7.24 Solved Examples : Root Locus	324
7.25 Root Contours	351
<i>Problems</i>	357
8. COMPENSATION OF CONTROL SYSTEMS	363–381
8.1 Phase-Lead Compensation	364
8.2 Phase-Lag Compensation	366
8.3 Phase-Lag-lead Compensation	368

<i>Chapter</i>	<i>Pages</i>
8.4 Concluding Remarks	... 369
8.5 Feedback Compensation	... 369
8.6 Solved Examples	... 372
<i>Problems</i>	... 380
9. INTRODUCTION TO STATE SPACE ANALYSIS OF CONTROL SYSTEMS	382—463
9.1 State Space Representation	... 383
9.2 The Concept of State	... 384
9.3 State Space Representation of Systems	... 385
9.4 Block Diagram for State Equation	... 392
9.5 Transfer Function Decomposition	... 393
9.5.1 Direct Decomposition	... 394
9.5.2 Cascade Decomposition	... 396
9.5.3 Parallel Decomposition	... 397
9.5.4 Parallel Decomposition of a Transfer Function having Repeated Roots of its Characteristic Equation	... 398
9.5.5 Transfer Function Decomposition Using State Signal Flow Graph	... 399
9.6 Solution of State Equation	... 403
9.6.1 Determination of State Transition Matrix	... 405
9.6.2 Properties of State Transition Matrix	... 415
9.7 Transfer Matrix	... 415
9.8 Controllability	... 416
9.9 Observability	... 419
9.10 Solved Examples	... 421
9.11 State Variable Feedback	... 455
<i>Problems</i>	... 459
10. SAMPLED DATA CONTROL SYSTEMS	464—510
10.1 Sampler	... 466
10.2 Sampling Process	... 467
10.3 Laplace Transform of Sampled Function	... 468
10.4 z -Transform	... 469
10.5 z -transform of Some Useful Functions	... 470
10.6 Inverse z -transform	... 473
10.7 Hold Circuit	... 480
10.8 Reconstruction of Signal : Minimum Sampling Frequency	... 483
10.9 Pulse Transfer Function (z -transfer Function)	... 484
10.10 Stability Analysis of Sampled-data Control Systems	... 490
10.11 Solved Examples	... 497
<i>Problems</i>	... 509

<i>Chapter</i>	<i>Pages</i>
11. SOLUTION OF PROBLEMS USING COMPUTER	511–565
11.1. Introduction to MATLAB	... 511
11.2. Running MATLAB	... 514
Solved Examples	... 514
12. CLASSIFIED SOLVED EXAMPLES	566–654
Contains highly graded solved problems selected from question papers of various Technical Universities, Competitive Examinations and self developed variety of questions for clearly and deeper understanding of matter/topics covered in text.	
Appendix-I : Answers to Selected Problems of Text	655–659
Appendix-II : Objective Type of Questions with Answers	660–701
Appendix-III : Short Answer Questions	702–724
Appendix-IV : Key Formulae, Tables and Charts	725–734
References	735
Index	737–740



CHAPTER
1

Introduction

This chapter deals with basic ideas about the open-loop and closed-loop control systems. The differential equations describe the dynamic operation of control systems. The Laplace transform transforms the differential equation into an algebraic equation, the solution is obtained in the transform domain. The time domain solution is determined by taking the inverse Laplace transform.

CONTENTS

- *An Example of Control Action*
- *Open-Loop Control System*
- *Closed-Loop Control System*
- *Use of Laplace Transformation in Control Systems*
- *Laplace Transform*
- *Solved Examples*

CONTROL SYSTEM

A control system is a combination of elements arranged in a planned manner wherein each element causes an effect to produce a desired output. This cause and effect relationship is governed by a mathematical relation.

If the aforesaid mathematical relation is linear the control system is termed as linear control system. For a linear system the cause (independent variable or input) and the effect (dependent variable or output) are proportionally related and principle of superposition is applicable throughout the operating range of a system.

In a control system the cause acts through a control process which in turn results into an effect.

There may be variety of systems based on the principle mentioned above but all the systems have many features in common and as such common approach for the study and analysis of control systems is possible.

Control systems are used in many applications for example, systems for the control of position, velocity, acceleration, temperature, pressure, voltage and current etc.

1.1 AN EXAMPLE OF CONTROL ACTION

Control of a room temperature is achieved by switching ON and switching OFF of a power supply to a heating appliance. Thus power supply to an appliance is switched ON, when the room temperature is felt low and switched OFF, when the desired temperature is reached.

The above system can be modified, if the duration of application of power is predetermined to achieve the room temperature within desired limits.

However, a further refinement can be made by measuring the difference between the actual room temperature and the desired room temperature and this difference being the error is used to control the element which in turn controls the output, *i.e.* room temperature.

The above description indicates that in the former case the output (room temperature) has no control on the input and the control action is purely based on a sort of predetermined calibration only, where as in the latter case the control action is affected by a feedback received from the output to the input.

1.2 OPEN-LOOP CONTROL SYSTEM

Having explained the concept of control action, a control system can be described by a block diagram as shown in Fig. 1.2.1.



The input r controls the output c through a control action process. In the block diagram shown in Fig. 1.2.1, it is observed that the output has no effect on the control action. Such a system is termed as open-loop control system.

In an open-loop control system the output is neither measured nor feedback for comparison with the input. Faithfulness of an open-loop control system depends on the accuracy of input calibration.

1.3 CLOSED-LOOP CONTROL SYSTEM

In a closed-loop control system the output has an effect on control action through a feedback as shown in Fig. 1.3.1 and hence closed-loop control systems are also termed as feedback control systems. The control action is actuated by an error signal e which is the difference between the input signal r and the output signal c . This process of comparison between the output and input maintains the output at a desired level through control action process.

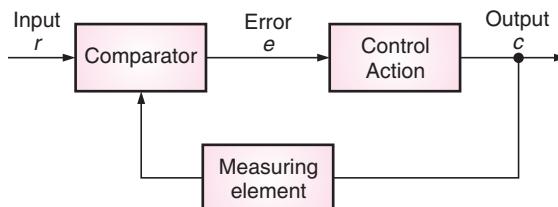


Fig. 1.3.1. Closed-loop control system.

The control systems without involving human intervention for normal operation are called automatic control systems.

A closed-loop (feedback) control system using a power amplifying device prior to controller and the output of such a system being mechanical *i.e.* position, velocity, acceleration is called servomechanism.

Comparison of Open-Loop and Closed-Loop Control System

Open-Loop	Closed-Loop
1. The accuracy of an open-loop system depends on the calibration of the input. Any departure from pre-determined calibration affects the output.	1. As the error between the reference input and the output is continuously measured through feedback, the closed-loop system works more accurately.
2. The open-loop system is simple to construct and cheap.	2. The closed-loop system is complicated to construct and costly.
3. The open-loop systems are generally stable.	3. The closed-loop systems can become unstable under certain conditions.
4. The operation of open-loop system is affected due to the presence of non-linearities in its elements.	4. In terms of the performance the closed-loop system adjusts to the effects of non-linearities present in its elements.

1.4 USE OF LAPLACE TRANSFORMATION IN CONTROL SYSTEMS

The control action for a dynamic control system whether electrical, mechanical, thermal, hydraulic etc. can be represented by a differential equation and the output response of such a dynamic system to a specified input can be obtained by solving the said differential equation. The system differential equation is derived according to physical laws governing a system in question.

In order to facilitate the solution of a differential equation describing a control system, the equation is transformed into an algebraic form. The differential equation wherein time being the independent variable is transformed into a corresponding algebraic equation by using Laplace transformation technique and the differential equation thus transformed is known as the equation in frequency domain. Hence, Laplace transform technique transforms a time domain differential equation into a frequency domain algebraic equation.

1.5 LAPLACE TRANSFORM

In order to transform a given function of time $f(t)$ into its corresponding Laplace transform first multiply $f(t)$ by e^{-st} , s being a complex number ($s = \sigma + j\omega$). Integrate this product w.r.t. time with limits as zero and infinity. This integration results in Laplace transform of $f(t)$, which is denoted by $F(s)$ or $\mathcal{L}f(t)$.

The mathematical expression for Laplace transform is,

$$\mathcal{L}f(t) = F(s) \quad t \geq 0$$

or

$$F(s) = \int_0^{\infty} f(t) \cdot e^{-st} dt \quad \dots(1.1)$$

The term “Laplace transform of $f(t)$ ” is used for the letter $\mathcal{L}f(t)$.

The time function $f(t)$ is obtained back from the Laplace transform by a process called inverse Laplace transformation and denoted as \mathcal{L}^{-1} thus

$$\mathcal{L}^{-1} [\mathcal{L}f(t)] = \mathcal{L}^{-1} [F(s)] = f(t)$$

The time function $f(t)$ and its Laplace transform $F(s)$ are a transform pair.

Table 1.5 gives transform pairs of some commonly used functions and Laplace transform pairs for some functions are derived here under.

1.5.1 Derivation of Laplace transform

1. Laplace transform of e^{at}

$$\begin{aligned}\mathcal{L} e^{at} &= \int_0^\infty e^{at} \cdot e^{-st} dt = \int_0^\infty e^{(a-s)t} dt = \frac{1}{(s-a)} \\ \therefore \quad \mathcal{L} e^{at} &= \frac{1}{(s-a)} \quad \dots(1.2)\end{aligned}$$

As the inverse Laplace transform is denoted by the letter \mathcal{L}^{-1} and, therefore, the inverse Laplace transform of $\frac{1}{(s-a)}$ is e^{at} and expressed as below,

$$\mathcal{L}^{-1} \left[\frac{1}{(s-a)} \right] = e^{at} \quad \dots(1.3)$$

2. In the function $f(t) = e^{at}$ put $a = 0$

$$\therefore \quad e^{at} = e^{0t} = 1. \quad \text{Hence, } f(t) = 1$$

$$\text{Therefore, using Eq. (1.2) } \mathcal{L}[1] = \frac{1}{(s-0)}$$

$$\text{or} \quad \mathcal{L}[1] = \frac{1}{s} \quad \dots(1.4)$$

$$\text{and} \quad \mathcal{L}^{-1} \left[\frac{1}{s} \right] = 1 \quad \dots(1.5)$$

3. In the function $f(t) = e^{at}$ put $a = j\omega$

$$\therefore \quad e^{at} = e^{j\omega t}. \quad \text{Hence, } f(t) = e^{j\omega t}$$

$$\text{Therefore, using Eq. (1.2) } \mathcal{L} e^{j\omega t} = \frac{1}{(s-j\omega)}$$

$$\therefore \quad e^{j\omega t} = (\cos \omega t + j \sin \omega t)$$

$$\therefore \quad \mathcal{L}(\cos \omega t + j \sin \omega t) = \frac{1}{(s-j\omega)} = \frac{s+j\omega}{(s^2+\omega^2)}$$

Separating into real and imaginary parts,

$$\mathcal{L} \cos \omega t = \frac{s}{(s^2+\omega^2)} \quad \dots(1.6)$$

$$\mathcal{L} \sin \omega t = \frac{\omega}{(s^2+\omega^2)} \quad \dots(1.7)$$

$$\text{and} \quad \mathcal{L}^{-1} \left[\frac{s}{(s^2+\omega^2)} \right] = \cos \omega t \quad \dots(1.8)$$

$$\mathcal{L}^{-1} \left[\frac{\omega}{(s^2+\omega^2)} \right] = \sin \omega t \quad \dots(1.9)$$

4. In the function $f(t) = e^{at}$ put $a = (-\alpha + j\omega)$

$$\therefore \quad e^{at} = e^{(-\alpha + j\omega)t}$$

$$\text{Hence,} \quad f(t) = e^{(-\alpha + j\omega)t}$$

Therefore, using Eq. (1.2)

$$\begin{aligned}\mathcal{L} e^{(-\alpha + j\omega)t} &= \frac{1}{s - (-\alpha + j\omega)} = \frac{1}{(s + \alpha) - j\omega} \\ \therefore e^{(-\alpha + j\omega)t} &= e^{-\alpha t} (\cos \omega t + j \sin \omega t) \\ \therefore \mathcal{L} e^{-\alpha t} (\cos \omega t + j \sin \omega t) &= \frac{1}{(s + \alpha) - j\omega} = \frac{(s + \alpha) + j\omega}{(s + \alpha)^2 + \omega^2}\end{aligned}$$

Separating into real and imaginary parts,

$$\mathcal{L} e^{-\alpha t} \cdot \cos \omega t = \frac{(s + \alpha)}{(s + \alpha)^2 + \omega^2} \quad \dots(1.10)$$

$$\mathcal{L} e^{-\alpha t} \cdot \sin \omega t = \frac{\omega}{(s + \alpha)^2 + \omega^2} \quad \dots(1.11)$$

and $\mathcal{L}^{-1} \left[\frac{(s + \alpha)}{(s + \alpha)^2 + \omega^2} \right] = e^{-\alpha t} \cdot \cos \omega t \quad \dots(1.12)$

$$\mathcal{L}^{-1} \left[\frac{\omega}{(s + \alpha)^2 + \omega^2} \right] = e^{-\alpha t} \cdot \sin \omega t \quad \dots(1.13)$$

5. In the function $f(t) = e^{at}$ put $a = 1$

$$\therefore e^{at} = e^{1 \cdot t} = e^t. \text{ Hence, } f(t) = e^t$$

Therefore, using Eq. (1.2) $\mathcal{L} e^t = \frac{1}{(s - 1)}$

$$\therefore e^t = 1 + t + \frac{t^2}{2} + \frac{t^3}{3} + \dots$$

and $\frac{1}{(s - 1)} = \frac{1}{s} + \frac{1}{s^2} + \frac{1}{s^3} + \frac{1}{s^4} + \dots$

Table 1.5. Table of Laplace transform pairs

S.No.	$f(t)$	$F(s) = \mathcal{L}[f(t)]$
1	$\delta(t)$ unit impulse at $t = 0$	1
2	$u(t)$ unit step at $t = 0$	$\frac{1}{s}$
3	$u(t - T)$ unit step at $t = T$	$\frac{1}{s} e^{-sT}$
4	t	$\frac{1}{s^2}$
5	$\frac{t^2}{2}$	$\frac{1}{s^3}$
6	t^n	$\frac{\angle n}{s^{n+1}}$
7	e^{-at}	$\frac{1}{s + a}$
8	e^{at}	$\frac{1}{s - a}$

9	te^{-at}	$\frac{1}{(s+a)^2}$
10	te^{at}	$\frac{1}{(s-a)^2}$
11	$t^n e^{-at}$	$\frac{\angle n}{(s+a)^{n+1}}$
12	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
13	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
14	$e^{-\alpha t} \sin \omega t$	$\frac{\omega}{(s+\alpha)^2 + \omega^2}$
15	$e^{-\alpha t} \cos \omega t$	$\frac{(s+\alpha)}{(s+\alpha)^2 + \omega^2}$
16	$\sinh \alpha t$	$\frac{\alpha}{s^2 - \alpha^2}$
17	$\cosh \alpha t$	$\frac{s}{s^2 - \alpha^2}$

∴ Comparing the terms

$$\begin{aligned} \mathcal{L}[1] &= \frac{1}{s}, \quad \mathcal{L}[t] = \frac{1}{s^2} \\ \mathcal{L}\left[\frac{t^2}{\angle 2}\right] &= \frac{1}{s^2} \\ \mathcal{L}\left[\frac{t^n}{\angle n}\right] &= \frac{1}{s^{n+1}} \quad \text{or} \quad \mathcal{L}[t^n] = \frac{\angle n}{s^{n+1}} \end{aligned} \quad \dots(1.14)$$

and $\mathcal{L}^{-1}\left[\frac{\angle n}{s^{n+1}}\right] = t^n \quad \dots(1.15)$

1.5.2 Basic Laplace Transform Theorems

Basic theorems of Laplace transform are given below :

1. Laplace transform of linear combination

$$\mathcal{L}[a f_1(t) + b f_2(t)] = a F_1(s) + b F_2(s) \quad \dots(1.16)$$

where $f_1(t), f_2(t)$ are functions of time and a, b are constants.

2. If the Laplace transform of $f(t)$ is $F(s)$, then

$$\begin{aligned} (i) \quad \mathcal{L}\left[\frac{df(t)}{dt}\right] &= [s F(s) - f(0+)] \\ (ii) \quad \mathcal{L}\left[\frac{d^2 f(t)}{dt^2}\right] &= [s^2 F(s) - s f(0+) - f'(0+)] \\ (iii) \quad \mathcal{L}\left[\frac{d^3 f(t)}{dt^3}\right] &= [s^3 F(s) - s^2 f(0+) - s f'(0+) - f''(0+)] \end{aligned} \quad \dots(1.17)$$

where $f(0+)$, $f'(0+)$, $f''(0+)$... are the values of $f(t)$, $\frac{df(t)}{dt}$, $\frac{d^2f(t)}{dt^2}$... at $t = (0+)$

3. If the Laplace transform of $f(t)$ is $F(s)$, then

$$\begin{aligned} (i) \mathcal{L} \int f(t) dt &= \left[\frac{F(s)}{s} + \frac{f^{-1}(0+)}{s} \right] \\ (ii) \mathcal{L} \int \int f(t) dt &= \left[\frac{F(s)}{s^2} + \frac{f^{-1}(0+)}{s^2} + \frac{f^{-2}(0+)}{s} \right] \\ (iii) \mathcal{L} \int \int \int f(t) dt &= \left[\frac{F(s)}{s^3} + \frac{f^{-1}(0+)}{s^3} + \frac{f^{-2}(0+)}{s^2} + \frac{f^{-3}(0+)}{s} \right] \end{aligned} \quad \dots(1.18)$$

where $f^{-1}(0+)$, $f^{-2}(0+)$, $f^{-3}(0+)$... are the values of $\int f(t) dt$, $\int \int f(t) dt$, $\int \int \int f(t) dt$... at $t = (0+)$.

4. If the Laplace transform of $f(t)$ is $F(s)$, then

$$\mathcal{L}e^{-at} f(t) = F(s+a)$$

5. If the Laplace transform of $f(t)$ is $F(s)$, then

$$\mathcal{L}t f(t) = -\frac{d}{ds} F(s)$$

6. Initial value theorem

$$\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} s \mathcal{L}f(t) \quad \dots(1.19 \ a)$$

or $\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} s F(s) \quad \dots(1.19 \ b)$

7. Final value theorem

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s \mathcal{L}f(t) \quad \dots(1.20 \ a)$$

or $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s F(s) \quad \dots(1.20 \ b)$

The final value theorem gives the final value ($t \rightarrow \infty$) of a time function using its Laplace transform and as such very useful in the analysis of control systems. However, if the denominator of $s F(s)$ has any root having real part as zero or positive, then the final value theorem is not valid.

1.6 SOLVED EXAMPLES

Example 1.6.1. Find the inverse Laplace transform of the following functions :

$$(i) F(s) = \frac{1}{s(s+1)}$$

$$(ii) F(s) = \frac{s+6}{s(s^2+4s+3)}$$

$$(iii) F(s) = \frac{1}{s^2+4s+8}$$

$$(iv) F(s) = \frac{s+2}{s^2+4s+6}$$

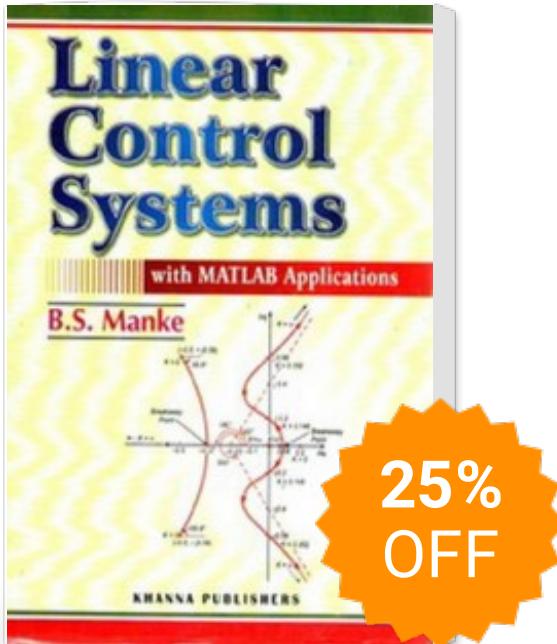
$$(v) F(s) = \frac{5}{s^2+4s+5}$$

$$(vi) F(s) = \frac{s^2+2s+3}{s^3+6s^2+12s+8}$$

Solution. (i)

$$F(s) = \frac{1}{s(s+1)}$$

Linear Control Systems with MATLAB Applications eBook By B.S. Manke



Publisher : KHANNA
PUBLISHERS

ISBN : 9788174093103

Author : B.S Manke

Type the URL : <http://www.kopykitab.com/product/190>



Get this eBook