Linear Control Systems

with MATLAB Applications

B.S. Manke

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with MATLAB Applications

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To my wife
Sulbha
and sons
Manish, Shailesh.

Preface to the First Edition

This book has been written to explain the basic principles of Linear Control Systems and an effort is made to present the subject in a simple and sequential manner to enable the students to acquire a good grasp of fundamentals of the subject.

The text presented covers the course content of the subject Linear Control Systems of Indian Universities and is meant for pre-final/final year students of electrical, electronics and mechanical engineering.

The material given in this book has been thoroughly class tested by the author while teaching the subject of control systems at undergraduate level for the past several years.

This book is divided in 9 chapters. The first four chapters give the basic concepts of the subject from the view point of control system representation. Chapter 5 presents the modelling of control systems and the respective mathematical models derived therein. The time response and steady state analysis is given in Chapter 6. Necessary derivations have been derived from the first principles. The stability analysis is described in Chapter 7. The methods of ascertaining stability: Routh-Hurwitz criterion, Nyquist criterion, Bode plot and root locus plot have been explained step by step in a simplified manner to make the explanation easily understandable. The compensation methods and introduction to state space analysis is described in chapters 8 and 9 respectively.

Suitable illustrative examples as well as solved examples have been incorporated in the text to make the subject clear and interesting. A list of references is given at the end.

Selective unsolved problems have been included at the end of each chapter to help the student to judge himself whether he has gained sufficient workable knowledge of basic principles involved. Answers to odd numbered problems being given in Appendix I.

The salient feature of this book is the inclusion of objective type multiple choice questions given in Appendix II covering the entire text which would be of great help for the students preparing for competitive examinations.

The author hopes that this book will serve the purpose of introducing basic principles of Linear Control Systems to undergraduate students for whom it is written.

The author would welcome any comments and suggestions to further improve the usefulness of this book.

The author acknowledges his indebtedness to Miss Saroj Rangnekar, Asst. Prof. in Elect. Engg., Maulana Azad College of Technology, Bhopal who thoroughly checked the manuscript and made useful suggestions.

Bhopal
October, 1987

B.S. Manke

Preface to the Tenth Edition

The text written in the book deals with the concepts of feed-back control theory. The first five chapters stress on the fundamental concepts regarding representation and modelling of a control system. The subsequent chapters deal with the time response analysis, stability analysis, compensation method, state variable approach, and sampled data/discrete data systems.

Each chapter contains solved examples to support the theory developed. Unsolved problems have been included as an exercise.

The answers to graphical solutions may slightly deviate due to graphical errors.

The chapter on computer solutions to control problems gives the use of MATLAB* solftware. The examples on various topics in the text have been solved using MATLAB software. This verifies the answers obtained using analytical solution.

Appendices given at the end of the book include :

Appendix I : Answers to Selected Problems

Appendix II : A Set of Objective Questions

Appendix III : Short Answer Type Questions

Appendix IV : List of Key Formulae, Charts and Calculation Tables

The author wishes to acknowledge the outcome of discussions with Dr. D.M. Deshpande, Prof. M.A.N.I.T., Bhopal towards the revision of this edition.

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*MATLAB is registered trade mark of Mathwork, Inc.

Contents

Contains highly graded solved problems selected from question papers of various Technical Universities, Competitive Examinations and self developed variety of questions for clearly and deeper understanding of matter/topics covered in text.

CHAPTER 1

Introduction

This chapter deals with basic ideas about the open-loop and closed-loop control systems. The differential equations describe the dynamic operation of control systems. The Laplace transform transforms the differential equation into an algebraic equation, the solution is obtained in the transform domain. The time domain solution is determined by taking the inverse Laplace transform.

CONTENTS

CONTROL SYSTEM

A control system is a combination of elements arranged in a planned manner wherein each element causes an effect to produce a desired output. This cause and effect relationship is governed by a mathematical relation.

If the aforesaid mathematical relation is linear the control system is termed as linear control system. For a linear system the cause (independent variable or input) and the effect (dependent variable or output) are proportionally related and principle of superposition is applicable throughout the operating range of a system.

In a control system the cause acts through a control process which in turn results into an effect.

There may be variety of systems based on the principle mentioned above but all the systems have many features in common and as such common approach for the study and analysis of control systems is possible.

Control systems are used in many applications for example, systems for the control of position, velocity, acceleration, temperature, pressure, voltage and current etc.

1.1 AN EXAMPLE OF CONTROL ACTION

Control of a room temperature is achieved by switching ON and switching OFF of a power supply to a heating appliance. Thus power supply to an appliance is switched ON, when the room temperature is felt low and switched OFF, when the desired temperature is reached.

The above system can be modified, if the duration of application of power is predetermined to achieve the room temperature within desired limits.

However, a further refinement can be made by measuring the difference between the actual room temperature and the desired room temperature and this difference being the error is used to control the element which in turn controls the output, *i.e.* room temperature.

The above description indicates that in the former case the output (room temperature) has no control on the input and the control action is purely based on a sort of predetermined calibration only, where as in the latter case the control action is affected by a feedback received from the output to the input.

1.2 OPEN-LOOP CONTROL SYSTEM

Having explained the concept of control action, a control system can be described by a block diagram as shown in Fig. 1.2.1.

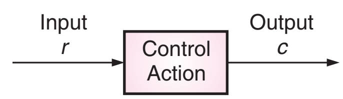

Fig. 1.2.1. Open-loop control system.

The input r controls the output c through a control action process. In the block diagram shown in Fig. 1.2.1, it is observed that the output has no effect on the control action. Such a system is termed as open-loop control system.

In an open-loop control system the output is neither measured nor fedback for comparison with the input. Faithfulness of an open-loop control system depends on the accuracy of input calibration.

1.3 CLOSED-LOOP CONTROL SYSTEM

In a closed-loop control system the output has an effect on control action through a feedback as shown in Fig. 1.3.1 and hence closed-loop control systems are also termed as feedback control systems. The control action is actuated by an error signal e which is the difference between the input signal r and the output signal c. This process of comparison between the output and input maintains the output at a desired level through control action process.

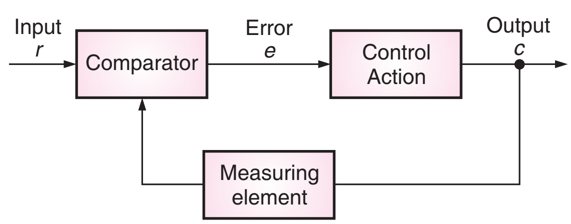

Fig. 1.3.1. Closed-loop control system.

The control systems without involving human intervention for normal operation are called automatic control systems.

A closed-loop (feedback) control system using a power amplifying device prior to controller and the output of such a system being mechanical *i.e.* position, velocity, acceleration is called servomechanism.

Comparison of Open-Loop and Closed-Loop Control System

Open-Loop	*Closed-Loop*
1. The accuracy of an open-loop system depends on the calibration of the input. Any departure from pre-determined calibration affects the output.	1. As the error between the reference input and the output is continuously measured through feedback, the closed-loop system works more accurately.
2. The open-loop system is simple to construct and cheap.	2. The closed-loop system is complicated to construct and costly.
3. The open-loop systems are generally stable.	3. The closed-loop systems can become unstable under certain conditions.
4. The operation of open-loop system is affected due to the presence of non-linearities in its elements.	4. In terms of the performance the closed-loop system adjusts to the effects of non-linearities present in its elements.

1.4 USE OF LAPLACE TRANSFORMATION IN CONTROL SYSTEMS

The control action for a dynamic control system whether electrical, mechanical, thermal, hydraulic etc. can be represented by a differential equation and the output response of such a dynamic system to a specified input can be obtained by solving the said differential equation. The system differential equation is derived according to physical laws governing a system in question.

In order to facilitate the solution of a differential equation describing a control system, the equation is transformed into an algebraic form. The differential equation wherein time being the independent variable is transformed into a corresponding algebraic equation by using Laplace transformation technique and the differential equation thus transformed is known as the equation in frequency domain. Hence, Laplace transform technique transforms a time domain differential equation into a frequency domain algebraic equation.

1.5 LAPLACE TRANSFORM

In order to transform a given function of time $f(t)$ into its corresponding Laplace transform first multiply $f(t)$ by e^{-st}, s being a complex number ($s = \sigma + j\omega$). Integrate this product w.r.t. time with limits as zero and infinity. This integration results in Laplace transform of $f(t)$, which is denoted by $F(s)$ or $\mathcal{L} f(t)$.

The mathematical expression for Laplace transform is,

$$\mathcal{L} f(t) = F(s) \qquad t \geq 0$$

or

$$F(s) = \int_0^\infty f(t) \cdot e^{-st}\, dt \qquad \text{...(1.1)}$$

The term "Laplace transform of $f(t)$" is used for the letter $\mathcal{L} f(t)$.

The time function $f(t)$ is obtained back from the Laplace transform by a process called inverse Laplace transformation and denoted as $\mathcal{L}^{-1}$ thus

$$\mathcal{L}^{-1} [\mathcal{L} f(t)] = \mathcal{L}^{-1} [F(s)] = f(t)$$

The time function $f(t)$ and its Laplace transform $F(s)$ are a transform pair.

Table 1.5 gives transform pairs of some commonly used functions and Laplace transform pairs for some functions are derived here under.

1.5.1 Derivation of Laplace transform

1. Laplace transform of e^{at}

$$\mathcal{L}\, e^{at} = \int_0^\infty e^{at} \cdot e^{-st}\, dt = \int_0^\infty e^{(a-s)t}\, dt = \frac{1}{(s-a)}$$

$$\therefore \qquad \mathcal{L}\, e^{at} = \frac{1}{(s-a)} \qquad \text{...(1.2)}$$

As the inverse Laplace transform is denoted by the letter $\mathcal{L}^{-1}$ and, therefore, the inverse Laplace transform of $\frac{1}{(s-a)}$ is e^{at} and expressed as below,

$$\mathcal{L}^{-1}\left[\frac{1}{(s-a)}\right] = e^{at} \qquad \text{...(1.3)}$$

2. In the function $f(t) = e^{at}$ put $a = 0$

$\therefore$ $e^{at} = e^{0t} = 1$. Hence, $f(t) = 1$

Therefore, using Eq. (1.2) $\mathcal{L}[1] = \frac{1}{(s-0)}$

or
$$\mathcal{L}[1] = \frac{1}{s} \qquad \text{...(1.4)}$$

and
$$\mathcal{L}^{-1}\left[\frac{1}{s}\right] = 1 \qquad \text{...(1.5)}$$

3. In the function $f(t) = e^{at}$ put $a = j\omega$

$\therefore$ $e^{at} = e^{j\omega t}$. Hence, $f(t) = e^{j\omega t}$

Therefore, using Eq. (1.2) $\mathcal{L}\, e^{j\omega t} = \frac{1}{(s-j\omega)}$

$\because$ $e^{j\omega t} = (\cos \omega t + j \sin \omega t)$

$$\therefore \quad \mathcal{L}(\cos \omega t + j \sin \omega t) = \frac{1}{(s-j\omega)} = \frac{s+j\omega}{(s^2+\omega^2)}$$

Separating into real and imaginary parts,

$$\mathcal{L} \cos \omega t = \frac{s}{(s^2+\omega^2)} \qquad \text{...(1.6)}$$

$$\mathcal{L} \sin \omega t = \frac{\omega}{(s^2+\omega^2)} \qquad \text{...(1.7)}$$

and
$$\mathcal{L}^{-1}\left[\frac{s}{(s^2+\omega^2)}\right] = \cos \omega t \qquad \text{...(1.8)}$$

$$\mathcal{L}^{-1}\left[\frac{\omega}{(s^2+\omega^2)}\right] = \sin \omega t \qquad \text{...(1.9)}$$

4. In the function $f(t) = e^{at}$ put $a = (-\alpha + j\omega)$

$\therefore$ $e^{at} = e^{(-\alpha + j\omega)\, t}$

Hence, $f(t) = e^{(-\alpha + j\omega)\, t}$

Therefore, using Eq. (1.2)

$$\mathcal{L}\, e^{(-\alpha + j\omega)t} = \frac{1}{s - (-\alpha + j\omega)} = \frac{1}{(s+\alpha) - j\omega}$$

$\because$ $$e^{(-\alpha + j\omega)t} = e^{-\alpha t}(\cos \omega t + j \sin \omega t)$$

$\therefore$ $$\mathcal{L}\, e^{-\alpha t}(\cos \omega t + j \sin \omega t) = \frac{1}{(s+\alpha) - j\omega} = \frac{(s+\alpha) + j\omega}{(s+\alpha)^2 + \omega^2}$$

Separating into real and imaginary parts,

$$\mathcal{L} e^{-\alpha t} \cdot \cos \omega t = \frac{(s+\alpha)}{(s+\alpha)^2 + \omega^2} \quad \text{...(1.10)}$$

$$\mathcal{L} e^{-\alpha t} \cdot \sin \omega t = \frac{\omega}{(s+\alpha)^2 + \omega^2} \quad \text{...(1.11)}$$

and $$\mathcal{L}^{-1}\left[\frac{(s+\alpha)}{(s+\alpha)^2 + \omega^2}\right] = e^{-\alpha t} \cdot \cos \omega t \quad \text{...(1.12)}$$

$$\mathcal{L}^{-1}\left[\frac{\omega}{(s+\alpha)^2 + \omega^2}\right] = e^{-\alpha t} \cdot \sin \omega t \quad \text{...(1.13)}$$

5. In the function $f(t) = e^{at}$ put $a = 1$

$\therefore$ $e^{at} = e^{1 \cdot t} = e^t$. Hence, $f(t) = e^t$

Therefore, using Eq. (1.2) $\mathcal{L} e^t = \dfrac{1}{(s-1)}$

$\because$ $$e^t = 1 + t + \frac{t^2}{\angle 2} + \frac{t^3}{\angle 3} + \ldots$$

and $$\frac{1}{(s-1)} = \frac{1}{s} + \frac{1}{s^2} + \frac{1}{s^3} + \frac{1}{s^4} + \ldots$$

Table 1.5. Table of Laplace transform pairs

S.No.	$f(t)$	$F(s) = \mathcal{L}[f(t)]$
1	$\delta(t)$ unit impulse at $t = 0$	1
2	$u(t)$ unit step at $t = 0$	$\frac{1}{s}$
3	$u(t - T)$ unit step at $t = T$	$\frac{1}{s} e^{-sT}$
4	t	$\frac{1}{s^2}$
5	$\frac{t^2}{2}$	$\frac{1}{s^3}$
6	t^n	$\frac{\angle n}{s^{n+1}}$
7	e^{-at}	$\frac{1}{s+a}$
8	e^{at}	$\frac{1}{s-a}$

9	te^{-at}	$\frac{1}{(s+a)^2}$
10	te^{at}	$\frac{1}{(s-a)^2}$
11	$t^n e^{-at}$	$\frac{\angle n}{(s+a)^{n+1}}$
12	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
13	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
14	$e^{-\alpha t} \sin \omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
15	$e^{-\alpha t} \cos \omega t$	$\frac{(s+\alpha)}{(s+\alpha)^2+\omega^2}$
16	$\sinh \alpha t$	$\frac{\alpha}{s^2-\alpha^2}$
17	$\cosh \alpha t$	$\frac{s}{s^2-\alpha^2}$

∴ Comparing the terms

$$\mathcal{L}[1] = \frac{1}{s}, \quad \mathcal{L}[t] = \frac{1}{s^2}$$

$$\mathcal{L}\left[\frac{t^2}{\angle 2}\right] = \frac{1}{s^2}$$

$$\mathcal{L}\left[\frac{t^n}{\angle n}\right] = \frac{1}{s^{n+1}} \quad \text{or} \quad \mathcal{L}[t^n] = \frac{\angle n}{s^{n+1}} \qquad ...(1.14)$$

and
$$\mathcal{L}^{-1}\left[\frac{\angle n}{s^{n+1}}\right] = t^n \qquad ...(1.15)$$

1.5.2 Basic Laplace Transform Theorems

Basic theorems of Laplace transform are given below :

1. Laplace transform of linear combination

$$\mathcal{L}[a\, f_1(t) + bf_2(t)] = aF_1(s) + bF_2(s) \qquad ...(1.16)$$

where $f_1(t)$, $f_2(t)$ are functions of time and a, b are constants.

2. If the Laplace transform of $f(t)$ is $F(s)$, then

(i) $\mathcal{L}\, \frac{df(t)}{dt} = [s\, F(s) - f(0+)]$

(ii) $\mathcal{L}\, \frac{d^2 f(t)}{dt^2} = [s^2 F(s) - s\, f(0+) - f'(0+)]$

(iii) $\mathcal{L}\, \frac{d^3 f(t)}{dt^3} = [s^3 F(s) - s^2 f(0+) - sf'(0+) - f''(0+)]$ ...(1.17) where $f(0+), f'(0+), f''(0+)$ ... are the values of $f(t), \dfrac{df(t)}{dt} \dfrac{d^2 f(t)}{dt^2}$ ... at $t = (0+)$

3. If the Laplace transform of $f(t)$ is $F(s)$, then

$$(i)\ \mathcal{L} \int f(t) = \left[\frac{F(s)}{s} + \frac{f^{-1}(0+)}{s}\right]$$

$$(ii)\ \mathcal{L} \iint f(t) = \left[\frac{F(s)}{s^2} + \frac{f^{-1}(0+)}{s^2} + \frac{f^{-2}(0+)}{s}\right]$$

$$(iii)\ \mathcal{L} \iiint f(t) = \left[\frac{F(s)}{s^3} + \frac{f^{-1}(0+)}{s^3} + \frac{f^{-2}(0+)}{s^2} + \frac{f^{-3}(0+)}{s}\right] \quad ..(1.18)$$

where $f^{-1}(0+), f^{-2}(0+), f^{-3}(0+)$... are the values of $\int f(t), \iint f(t), \iiint f(t)$ ... at $t = (0+)$.

4. If the Laplace transform of $f(t)$ is $F(s)$, then

$$\mathcal{L} e^{-at} f(t) = F(s+a)$$

5. If the Laplace transform of $f(t)$ is $F(s)$, then

$$\mathcal{L} t\, f(t) = -\frac{d}{ds} F(s)$$

6. Initial value theorem

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} s\, \mathcal{L} f(t) \quad ...(1.19\ a)$$

or

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} s\, F(s) \quad ...(1.19)$$

7. Final value theorem

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s\, \mathcal{L} f(t) \quad ...(1.20\ a)$$

or

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s\, F(s) \quad ...(1.20)$$

The final value theorem gives the final value ($t \to \infty$) of a time function using its Laplace transform and as such very useful in the analysis of control systems. However, if the denominator of $s\,F(s)$ has any root having real part as zero or positive, then the final value theorem is not valid.

1.6 SOLVED EXAMPLES

Example 1.6.1. *Find the inverse Laplace transform of the following functions :*

$(i)\ F(s) = \dfrac{1}{s(s+1)}$ $\qquad (ii)\ F(s) = \dfrac{s+6}{s(s^2+4s+3)}$

$(iii)\ F(s) = \dfrac{1}{s^2+4s+8}$ $\qquad (iv)\ F(s) = \dfrac{s+2}{s^2+4s+6}$

$(v)\ F(s) = \dfrac{5}{s(s^2+4s+5)}$ $\qquad (vi)\ F(s) = \dfrac{s^2+2s+3}{s^3+6s^2+12s+8}$.

Solution. (*i*)

$$F(s) = \frac{1}{s(s+1)}$$

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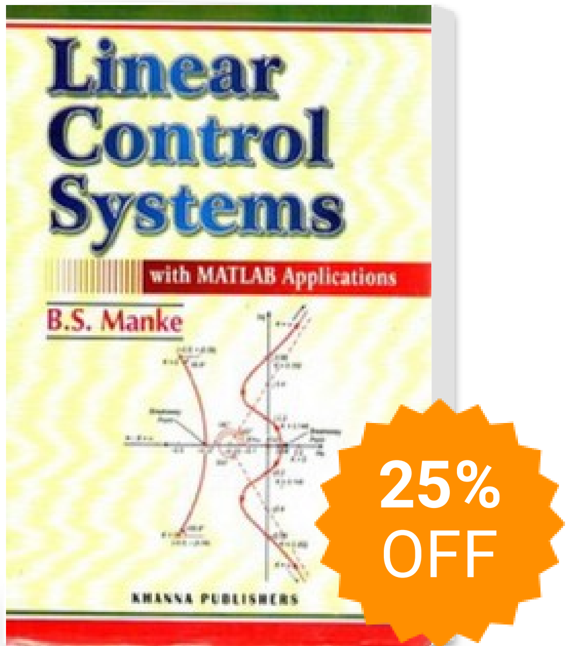

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